2024 Fall Math 140 Week-In-Review
Week 12: Sections 6.1 and 6.2

Some Key Words and Terms: Interest, Simple Interest, Principal/Present Value, Accumulated Amount/Future Value, Interest Rate, Time, Compound Interest, Compounding Periods, TVM Solver, Savings/Investing, Total Interest Earned, Continuously Compounded, Effective Interest Rate/Effective Yield, Annuity, Loan, Down Payment, Total Paid on the Loan, Total Interest Paid, Total Amount Paid.

Interest: the money that an account earns, this can be good or bad depending on the type of account

- Simple interest (no compounding)
- Compound Interest (non-continuously)
- Compound Interest (continuously)

Simple Interest:

The interest is calculated once: Interest $(I) = Prt$

P = principal/present value/ initial amount

r = interest rate (as a decimal)

t = time (in years, sometimes we have to convert)

Principal/Present Value:

- initial deposit or initial loan amount
- price of an item we want (loan)

   at time zero, what is the money amount

Accumulated Amount/Future Value:

- the amount at a later time
- total in a savings account
- for a loan, this is generally zero unless we refinance

at some specific time, what is the money amount

For Simple Interest: Accumulated [1]Value = Present Value + Interest

$A = P + I$ again where $I = Prt$

<u>Interest Rate:</u> this is the % interest applied to the money already in the account

- when working with equation, we want the decimal form (simple interest, continuously compounded, ___)

- when working with TVM Solver, we want the percent

⭐ one of the Most impactful things for an account ⭐

<u>Time:</u> we want years; generally given in a problem

- Sometimes, we're given "monthly payments" & "for ___ months"

  this is the N in the TVM Solver

To solve for t with

① <u>equations:</u> simple interest $A = P + Prt$ & use algebra

continuously compound $A = Pe^{rt}$ & will use "ln( )"

② <u>TVM:</u> solve for "N" then solve $N = m \cdot t$

<u>Compound Interest:</u>

<u>Non-Continuous</u>: "daily", "weekly", "monthly" ----

these tell us the number of compounding period (m)

⭐<u>Continuous</u>: then there is no m b/c the account is being constantly updated (the "e" formulas)

↳ must know these formulas: $A = Pe^{rt}$ & $r_{eff} = e^r - 1$

<u>Compounding Periods:</u>

You just have to know the m-value associated w/ the words

monthly ⟶ m = 12

daily ⟶ m = 365

weekly ⟶ m = 52

quarterly ⟶ m = 4

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<u>TVM Solver:</u> where we spend most of our time in 6.1 & 6.2

N = total number of compounding periods = m·t

I% = interest rate as <u>a percent</u>

PV = P = present value/initial amount/initial deposit/loan amount

PMT = any <u>regular</u> payment (loan) or deposit (savings)

FV = A = accumulated amount/future value → (loan = 0)

C/Y = # of times compounded per year ⎫
                                       ⎬ = m
P/Y = # of payments per year   ⎭

(End (always))

depending on the situation, some #s are positive & some are negative

<u>Savings/Investing:</u> building up money over time
- often an initial deposit to open an account (FV but as a negative)
- often a regular payment into account (PMT is a negative)
- FV/accumulated amount is positive

<u>Total Interest Earned:</u>
Final Value/Accumulated Amount = (All payment) + (Initial Deposit) + (Interest Earned)
$$FV = (PMT)(N) + PV + Interest$$
when we plug in #s from TVM Solver, they are all positive

<u>Continuously Compounded:</u> Just have to know
$$A = Pe^{rt}$$

a common usage is "double" or "tripling" b/c we don't need to know A or P in that situation

"<u>double</u>": $A = 2P^3$ "<u>triple</u>": $A = 3P$

Effective Interest Rate/Effective Yield: How we compare accounts w/ different percentage rates & different compounding periods

Non-continuous: $r_{eff} = (1 + \frac{r}{m})^m - 1$ calculator will do this

→ Eff (% interest, m)

Continuous: $r_{eff} = e^r - 1$

Just have to know

APPS → Finance → ▶Eff(

Annuity: an account w/ a regular payment

this can come in many different forms as long as there is a regular deposit/payment

Loan: When we borrow money officially; usually from a bank or loan company

TVM: PV = loan amount
FV = 0 (unless we refinance)

• applies after any down payment

Down Payment: An initial amount of money paid up front on the price to reduce the amount the loan is taken for

☆ you never pay interest on a down payment ☆
☆ often a percent of price

Total Paid on the Loan:

Total Paid on Loan = Loan + Interest Charged for Loan

N · PMT = PV + Interest

no down payment b/c
just talking about loan

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<u>Total Interest Paid:</u>

$$\star \ \text{Interest} = \text{Total Payments} - \text{Loan Amount} \ \star$$

$$\text{Interest} = N \cdot PMT - PV$$

<u>Total Amount Paid:</u>

$$\text{Total Paid} = \text{All Payments} + \text{Down Payment}$$

$$= N \cdot PMT + \text{Down Payment}$$

<u>Examples:</u>

1. Your car breaks down and you have to take out a short term loan. The loan offers a simple interest rate of 23% for 8 months. If you need to borrow $1,800, how much will you owe at the end of the loan?

$$I = Prt \qquad A = P + I$$

$$A = P + Prt$$

$$A = 1800 + 1800(.23)\left(\tfrac{2}{3}\right)$$

$$\boxed{A = \$2076}$$

$P = 1800$

$r = 23\% \rightarrow .23$

$t = 8\text{months} \rightarrow \tfrac{8}{12}\text{ years} = \tfrac{2}{3}\text{ years}$

$A = ?$

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2. You borrow $20 from your friend while you're out and tell them you'll pay them back $30 in two weeks. If this is treated as a loan with simple interest, what is the simple interest rate you offered to pay?

$$A = P + I \qquad I = Prt \qquad P = \$20$$
$$A = \$30$$
$$t = 2 \text{ weeks} = \frac{2}{52} \text{ years} = \frac{1}{26} \text{ years}$$

$$30 = 20 + 20(r)\left(\frac{1}{26}\right)$$
$$\frac{10}{20} = \frac{20(r)\left(\frac{1}{26}\right)}{20}$$
$$\left(\frac{1}{2}\right) = \left(\frac{1}{26}r\right)26$$
$$13. = r \quad (\text{as a decimal})$$

so 1300% interest

3. You decide to deposit $2,500 in a savings account that earns 4.2% annual interest compounded weekly. If you make no other deposits, how much money will be in the account after 15 years?

N = $m \cdot t$ = (52)(15) (your TVM will multiply these)

I% = 4.2 — must use the negative button to the left of ENTER

PV = −2500 (b/c I have to give this up)

PMT = 0 (b/c no other deposits)

FV = ? (Alpha→Enter)

C/Y = 52

P/Y = 52

End (always)

there will be $4,692.83 after 15 years

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4. How long will it take a one-time investment of $10,000 to grow to $45,000 at an annual interest rate of 6% compounded continuously? (Round your final answer to 2 decimal places)

$$A = Pe^{rt}$$

$P = \$10,000$
$A = \$45,000$
$r = 6\% \rightarrow 0.06$
$t = ?$

$$\frac{45000}{10000} = \frac{10000 e^{0.06t}}{10000} \qquad \text{(isolate } e^{0.06t} \text{ first)}$$

$$4.5 = e^{0.06t}$$

$$\ln(4.5) = \ln(e^{0.06t})$$

$$\frac{\ln(4.5)}{0.06} = \frac{0.06t}{0.06}$$

$$t = \frac{\ln(4.5)}{0.06} \approx \boxed{25.07 \text{ years}}$$

exact value

5. What would be the minimum interest rate needed to triple any initial deposit in 10 years if the account has an annual interest rate that is compounded continuously? (Round your final answer to 3 decimal places before converting to a percent)

$$A = Pe^{rt}$$

$A = 3P$
$P = P$
$r = ?$
$t = 10 \text{ years}$

$$\frac{3P}{P} = \frac{Pe^{(r)(10)}}{P}$$

$$3 = e^{10r}$$

$$\ln(3) = \ln(e^{10r})$$

$$\frac{\ln(3)}{10} = \frac{10r}{10}$$

$$r = \frac{\ln(3)}{10} \approx 0.10986\cdots$$

$$\boxed{0.110 \quad \text{or} \quad 11\% \text{ interest}}$$

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6. From the following accounts, which would be the best for a savings account?

- Account A: 5.6% annual interest, compounded monthly
- Account B: 5.5% annual interest, compounded weekly
- Account A: 5.7% annual interest, compounded continuously

▸Eff(

largest r_{eff}

$e^r - 1$

A: ▸Eff(5.6, 12) → r_{eff} = 5.745992838......%

B: ▸Eff(5.5, 52) → r_{eff} = 5.65099......%

C: $e^{0.057} - 1$ ⇒ r_{eff} = 0.0586558...

or 5.865....%

we should choose account C

7. From the following accounts, which would be the best for a loan?

- Account A: 6.2% annual interest, compounded daily
- Account B: 6.23% annual interest, compounded weekly
- Account A: 6.18% annual interest, compounded quarterly

smallest r_{eff}

A: ▸Eff(6.2, 365) → r_{eff} = 6.39567...%

B: ▸Eff(6.23, 52) → r_{eff} = 6.424189...%

C: ▸Eff(6.18, 4) → r_{eff} = 6.3247...%

we should choose account C

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8. You decide to start putting money aside for emergencies. You open a savings account with an initial deposit of $1,000 and make monthly deposits of $125. The account earns an annual interest of 2.55% compounded monthly. How much money will you have saved after 10 years?

N = (12)(10)

I% = 2.55

PV = -1000

PMT = -125

FV = ?

C/Y = 12

P/Y = 12

End (always)

I will have saved
$18,355.56

9. After completing your freshman year, you decide to start saving money for a summer trip at the end of you senior year, 3 years away, that will cost $12,000. How much money would you need to deposit in the account each week if you don't have any money to make an initial deposit and the account earns 3.2% annual interest compounded weekly?

N = (3)(52)

I% = 3.2

PV = 0

PMT = ? = -73.313485-----

FV = 12,000

C/Y = 52

P/Y = 52

End (always)

$73.31
per week
WA

☆ in the real world, we would deposit at least $73.32 each week to make ☆ sure

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10. Your car breaks down and you have to put the repairs on a credit card. The repairs cost $2,390 and the credit card charges an annual interest rate of 28% compounded monthly. Your credit card company requires you to make a minimum monthly payment of $61. How many years will it take to pay off the repairs if you only make the minimum payment every month? (Round your answer to 2 decimal places)

☆ credit cards are just loans ☆

$N = \cancel{(12)(t)} = ?$

$I\% = 28$

$PV = 2390$

$PMT = -61$

$FV = 0$

$C/Y = 12$

$P/Y = 12$

End (always)

$N = m \cdot t$

$106.4728541 = 12t$

$\boxed{t \approx 8.87 \text{ years}}$

11. You want to take out a personal loan to make some home repairs. You can handle a monthly payment of up to $230 per month and plan to take out a 4-year loan at an annual interest rate of 6.2% compounded monthly. What is the maximum loan amount you can apply for?

$PV = ?$

$N = (12)(4)$

$I\% = 6.2$

$PV = ?$

$PMT = -230$

$FV = 0$

$C/Y = 12$

$P/Y = 12$

End (always)

$\boxed{\text{the maximum loan is } \$9,755.34}$

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12. You decide to buy a house you plan to rent to generate passive income. The house you plan to buy has a listing price of $315,000. You find a bank that will finance a 20-year loan at 3.4% annual interest compounded monthly. You plan to make a down payment of 15% of the listing price.

(a) How much will the down payment be? How much will you have to take out as a loan?

$$\text{Down Payment} = (0.15)(315000) = \$47,250$$

$$\text{Loan} = (0.85)(315000) = \$267,750$$

(b) What will be the outstanding balance on the loan after 10-years? How much equity do you have at that point?

FV after 10 years?

we need the PMT on the loan before we can find equity

N = (12)(20)

I% = 3.4

PV = 267,750

PMT = ? = −1539.12

FV = 0

C/Y = 12

P/Y = 12

End (always)

N = (12)(10)

I% = 3.4

PV = 267750

PMT = −1539.12

FV = ? = −156385.58

P/Y = 12

C/Y = 12

END

outstanding balance is $156,385.58

☆ Equity = Value − Outstanding Balance

☆ Equity = $315,000 − $156,385.58

Equity = $158,614.42

(c) How much will you pay in interest over the life of the loan?

☆ Total Payments = Loan + Interest ☆

$$(\text{PMT})(N) = PV + \text{Interest}$$

$$(1539.12)(240) = 267750 + \text{Interest}$$

$$\text{Interest} = (1539.12)(240) − 267750$$

Interest = $101,638.80

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13. You take out a loan 3-year for $23,500 at an annual interest rate of 7.6% compounded monthly. How much of the 4th payment is applied to the principal and how much is applied to interest?

$N = (3)(12)$

$I\% = 7.6$

$PV = 23,500$

$PMT = ? = -732.08$

$FV = 0$

$C/Y = 12$

$P/Y = 12$

End (always)

※ Outstanding balance after 3rd Payment
※ Outstanding balance after 4th Payment

difference between this is how much of the 4th payment is applied to the principal

✓ ① Calculate PMT on whole loan
② Calculate FV after 3 Payments
③ Calculate FV after 4 payments

after 3 PMTs

$N = 3$
$I = 7.6$
$PV = 23500$
$PMT = -732.08$
$FV = ? = -21,739.15$
$C/Y = 12$
$P/Y = 12$

↓

$21,739.15$

After 4 PMTs

$N = 4$
$I = 7.6$
$PV = 23500$
$PMT = -732.08$
$FV = ? = -21,144.76$
$C/Y = 12$
$P/Y = 12$

↓

$21,144.76$

Amount 4th PMT to Principal $= 21739.15 - 21144.76$
$= \$594.39$ to principal

Payment $= \$732.08 = $ (Amount to Principal) + (Amount to Interest)

$732.08 = 594.39 + $ Amount Interest

Amount of 4th PMT to Interest $= \$137.69$

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