# STAT 201 - Week-In-Review 11
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# Problem Solutions

1. Frame the appropriate null hypothesis $H_0$ and alternative hypothesis $H_A$ for each of the following scenarios.

   (A) When a new medicine is manufactured, the pharmaceutical company must subject it to testing before receiving the necessary permission from FDA to market the medicine. Formulate the appropriate hypotheses.

   **Solution:** $H_0$ : "the medicine is unsafe."   vs.   $H_A$ : "the medicine is safe."

   (B) A trucking firm doubts a tire manufacturer's claim that certain tires last, on average, more than 28000 miles. Formulate the appropriate hypotheses.

   **Solution:** Let $\mu$ denote the average distance those particular tires can last.

   - $H_0$: $\mu \leq 28000$   vs.   $H_A$: $\mu > 28000$

   (C) It is widely known that alcohol related car accidents can have a big impact on people's lives. One widely reported statistic is that the mean number of "years of potential life lost" among men is 32, but it was computed several years ago. Researchers want to know if this statistic has changed. Formulate the appropriate hypotheses.

   **Solution:** Let the parameter $\mu$ denote the mean number of years of potential life lost among men. Then

   - $H_0$ : $\mu = 32$   vs.   $H_A$ : $\mu \neq 32$

   (D) Suppose the University president wants to know if more than half of the students support sport passes being included in tuition. Formulate the appropriate hypotheses.

   **Solution:** Let p be the population proportion of students who support sport passes being included in tuition.

   - $H_0$ : $p \leq 0.50$   vs.   $H_A$ : $p > 0.50$

   (E) A polling organization conducts an exit poll and somehow it appears that Candidate A is likely to receive less than one third of the votes. Formulate the appropriate hypotheses.

   **Solution:** Let p be the population proportion of voters who are in favor of voting Candidate A.

   - $H_0$ : $p \geq 1/3$   vs.   $H_A$ : $p < 1/3$.

2. A resistor supposedly is produced with an average resistance of 1.000m■. Production variability is known to exist with a standard deviation of $\sigma = 0.005$m■, and the resistance distribution is approximately normal. For quality control, 10 resistors are sampled at a time to test the target value of 1.000m■. A deviation in the average resistance in either direction is considered important to notice. Suppose the average resistor in the sample is 1.00239m■ with a sample standard deviation of 0.00468m■.

Test whether there is sufficient evidence in the data to conclude that there has been a change in the mean resistance at level of significance $\alpha = 0.02$.

**Solution:**

**Framing the appropriate null and alternative hypotheses:**

To test $\mathrm{H_0} : \mu = 1.000$ vs $\mathrm{H_A} : \mu \neq 1.000$.

**Checking the Assumptions:**

The population distribution is approximately normal with $\sigma = 0.005$.

**Choosing the appropriate test statistic:**

$$\mathsf{T} = \frac{\overline{\mathsf{X}} - \mu_0}{\sigma/\sqrt{\mathsf{n}}} \overset{\mu=\mu_0}{\sim} \mathsf{N}(0, 1) \text{ (approximately), with } \mu_0 = 1.000.$$

**Calculating the observed value of the test statistic:**

According to the problem, $\overline{\mathsf{X}} = 1.00239$, $\sigma = 0.005$, and n = 10.

$$\text{Observed test statistic: } \mathsf{t} = \frac{1.00239 - 1.000}{0.005/\sqrt{10}} = 1.5115687 \approx 1.51.$$

**Estimating the p–value:**

p–value $= 2\mathrm{P}\,(Z > |1.51|) = 2(1 - 0.93448) = 0.13104 > \alpha = 0.02.$

Thus, we fail to reject $\mathrm{H_0}$ at 2% level of significance, and conclude that there is not sufficient evidence in the data to believe that there has been a change in the mean resistance value.

3. It is suspected that a machine used for filling plastic bottles with a net volume of 16.0 oz is not performing to specifications. We don't want to overfill the bottles as this will waste product.

We collect 35 measurements and will reset the machine if there is evidence that the population mean volume is greater than 16 oz at level of significance $\alpha = 0.02$. Suppose the sample mean volume is 16.0367 oz. Assume that the population standard deviation is 0.0943 oz.

**Solution:**

**Framing the appropriate null and alternative hypotheses:**

We want to test $H_0 : \mu = 16$ vs $H_A : \mu > 16$.

**Checking the Assumptions:**

The underlying population distribution is not known to be normal.

But, the sample size n = 35 is large enough.

**Choosing the appropriate test statistic:**

$T = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \overset{\mu=\mu_0}{\sim} N(0, 1) (\text{approximately}), \text{ with } \mu_0 = 16.$

**Calculating the observed value of the test statistic:**

According to the problem, $\overline{X}$ = 16.0367, $\sigma$ = 0.0943, and n = 35.

$\text{Observed test statistic: } t = \dfrac{16.0367 - 16}{0.0943/\sqrt{35}} = 2.30244 \approx 2.30.$

**Estimating the p–value:**

$\text{p–value} = P(Z > 2.30) = 1 - 0.98928 = 0.01072 < \alpha = 0.02.$

Therefore, we reject $H_0$, and conclude that the machine isn't performing to specifications at 2% level of significance.

4. Suppose we are testing the hypotheses $H_0 : \mu = 100$ vs $H_A : \mu > 100$, where the population standard deviation $\sigma$ is given to be known. Which of the two sample means $\overline{X}$ = 110 and $\overline{X}$ = 115 based on two different samples of the same size (say, n) will have a smaller p-value?

   **Solution:** The one with the sample mean of $\overline{X}$ = 115 because it is further away from $\overline{X}$ = 110 towards the right tail. The former will provide more evidence against the null hypothesis $H_0$ (that is, it will provide more support to the alternative hypothesis $H_A$).

5. A topic of recent clinical interest is the possibility of using a new medicine to reduce infarct size of patients who have myocardial infarction within the past 24 hours. Suppose we know that in untreated patients the mean infarct size is 25 (ck - g-EQ/m). Furthermore, in a random sample of 8 patients treated with the new medicine, the mean infarct size is 18.5 (ck - g-EQ/m) with a standard deviation of 5.89 (ck - g-EQ/m).

   We would like to know whether there is statistical evidence that the new medicine is effective in reducing infarct size. What would be your conclusion at level of significance $\alpha = 0.01$?

Assume that infarct sizes of patients being treated with the new medicine is approximately normally distributed.

**Solution:**

**Framing the appropriate null and alternative hypotheses:**

We want to test $H_0 : \mu = 25$ vs $H_A : \mu < 25$.

**Checking the Assumptions:**

The underlying population distribution is approximately normal with an unknown $\sigma$.

**Choosing the appropriate test statistic:**

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} \overset{\mu=\mu_0}{\sim} t_{n-1} \equiv t_7 \text{ (approximately)}, \text{ with } \mu_0 = 25.$$

**Calculating the observed value of the test statistic:**

According to the problem, $\overline{X} = 18.5$, $S = 5.89$, and $n = 8$.

$$\text{Observed test statistic: } t = \frac{18.5 - 25}{5.89/\sqrt{8}} = -3.12135.$$

**Estimating the p–value:**

$$\text{p–value} = P\left(T_7 < -3.12135\right) < P\left(T_7 < -2.9980\right) = 0.01 = \alpha$$

$\Rightarrow$ Reject $H_0$ at $\alpha = 0.01$

Thus, we reject $H_0$ at 1% level of significance, and conclude that the new medicine is effective in reducing the infarct size of patients.

6. An engineer designs a new product, then tests it under stress. To be acceptable, the average lifetime, say, $\mu$, must be more than 50 hours. Towards that he tested 40 items, and the observed average lifetime is 53.3 hours, with a sample standard deviation of 8.71 hours.

   Help him address whether the new design is acceptable at level of significance $\alpha = 0.05$ based on the above sample information.

   **Solution:**

   **Framing the appropriate null and alternative hypotheses:**

   We want to test $H_0 : \mu \leq 50$ vs $H_A : \mu > 50$.

   **Checking the Assumptions:**

   The underlying population distribution is not known to be normal.

The population standard deviation $\sigma$ is also not known.

But, the sample size n = 40 is large enough.

**Choosing the appropriate test statistic:**

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} \overset{\mu=\mu_0}{\sim} t_{n-1} \equiv t_{39} \text{ (approximately), with } \mu_0 = 50.$$

**Calculating the observed value of the test statistic:**

According to the problem, $\overline{X}$ = 53.3, S = 8.71, and n = 40.

Observed test statistic: $t = \dfrac{53.3 - 50}{8.71/\sqrt{40}} = 2.39621.$

**Estimating the p–value:**

p–value = $P(T_{39} > 2.39621) < P(T_{39} > 1.6849) < 0.05 = \alpha$

$\Rightarrow$ Reject $H_0$ at $\alpha = 0.01$

Thus, we reject $H_0$ at 5% level of significance, and conclude that the new tire design is acceptable.

7. A sanitation supervisor is interested in testing to see if the mean amount $\mu$ of garbage per bin is significantly different from 50 pounds. In a random sample of 36 bins, the sample mean amount was 49.12 pounds and the sample standard deviation was 3.9 pounds. Conduct the appropriate hypothesis test using a 0.05 level of significance.

**Solution:**

**Framing the appropriate null and alternative hypotheses:**

We want to test $H_0 : \mu = 50$ vs $H_A : \mu \neq 50$.

**Checking the Assumptions:**

The underlying population distribution is not known to be normal.

The population standard deviation $\sigma$ is also not known.

But the sample size n = 36 is large enough.

**Choosing the appropriate test statistic:**

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} \overset{\mu=\mu_0}{\sim} t_{n-1} \equiv t_{35} \text{ (approximately), with } \mu_0 = 50.$$

**Calculating the observed value of the test statistic:**

According to the problem, $\overline{X}$ = 49.12, S = 3.9 and n = 36.

Observed test statistic: $t = \dfrac{49.12 - 50}{3.9/\sqrt{36}} = -1.353846$.

**Estimating the p–value:**

p–value = $2P\left(T_{35} > |-1.353846|\right)$ = $2P\left(T_{35} > 1.353846\right)$ > $2P\left(T_{35} > 2.0301\right)$ = 0.05 = $\alpha$

$\Rightarrow$ Fail to reject $H_0$ at $\alpha$ = 0.05.

Thus, we fail to reject $H_0$ at 5% level of significance, and conclude that the mean amount $\mu$ of garbage per bin is not significantly different from 50 pounds.

8. Meteorologists in Texas want to increase the amount of rain delivered by thunderheads by seeding the clouds. Without a seeding, thunderheads produce, on average, 300 acre-feet. The meteorologists randomly selected 25 clouds which they seeded with silver iodide to test their theory that the average acre-feet is more than 300. The sample mean is 370.4 with a sample standard deviation of 300.1.
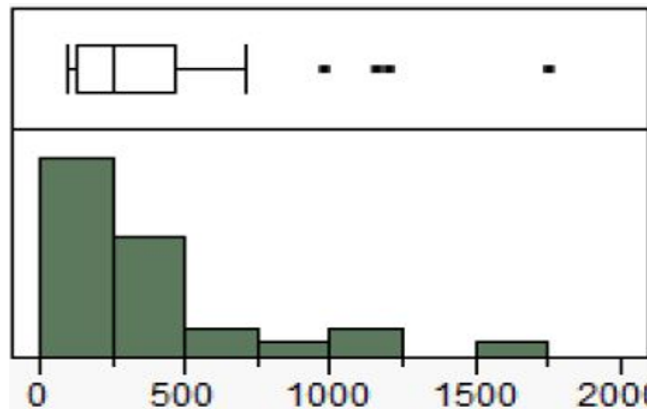


(A) State the appropriate null and alternative hypotheses in this context.

   **Solution:** Let $\mu$ be the average acre-feet caused by thunderheads after seeding the clouds.

   We want to test $H_0 : \mu$ = 300 vs $H_A : \mu > 300$.

(B) What conclusion can be drawn when at a significance level $\alpha$ = 0.05?

   (a) The data does not provide statistical evidence that the average acre-feet from seeded clouds is more than 300.

   (b) The data does provide statistical evidence that the average acre-feet from seeded clouds is more than 300.

   (c) The data does not provide statistical evidence that the sample average acre-feet from seeded clouds is more than 300.

(d) Two of the above are correct.

(e) *** We cannot draw conclusions based on the p-value because the conditions are not met due to the extreme outliers.

9. Suppose, you wish to test a claim that $\mu \neq 38$ at a level of significance of $\alpha = 0.05$, and you are given the following information: n = 35, $\overline{X}$ = 37.1, and S = 2.7. The observations are plotted using the following histogram and Normal QQ plot.
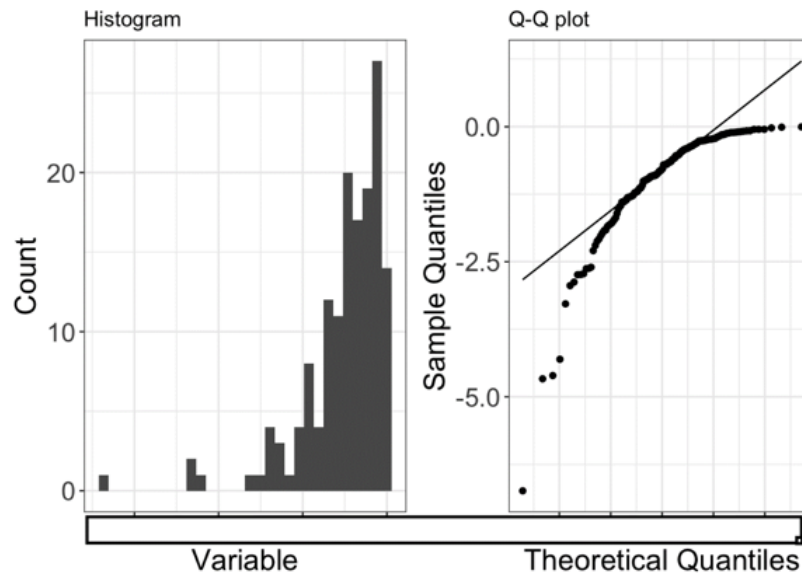


(A) State the appropriate null and alternative hypotheses in this context.

**Solution:** We want to test $H_0 : \mu = 38$ vs $H_A : \mu \neq 38$.

(B) Find the observed value of an appropriately chosen level $\alpha = 0.05$ test.

**Solution:** $T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} \overset{\mu=\mu_0}{\sim} t_{n-1} \equiv t_{34}$ (approximately), with $\mu_0 = 38$.

According to the problem, n = 35, $\overline{X}$ = 37.1, and S = 2.7.

Observed test statistic: $t = \dfrac{37.1 - 38}{2.7/\sqrt{35}} = -1.972$.

(C) Estimate the p-value. What should be your conclusion at $\alpha = 0.05$?

**Solution:** Since the alternative hypothesis is two-sided,

$$\text{p–value} = 2P\left(T_{34} > |-1.972|\right) = 2P\left(T_{34} > 1.972\right) > 2P\left(T_{34} > 2.0322\right) = 0.05 = \alpha$$

$\Rightarrow$ Fail to reject $H_0$ at $\alpha = 0.05$.

Thus, we fail to reject $H_0$ at 5% level of significance, and conclude that the population mean $\mu$ is not significantly different from 38.

(D) Is your conclusion a valid one?

> **Solution:** Yes, because the sample size is more than 30, and hence, the level $\alpha = 0.05$ t-test can be used here.

10. A research report claims that 20% of all individuals use Firefox to browse the web. A software company is trying to determine if the proportion of their users who use Firefox is significantly different from 0.2. In a sample of 200 of their users, 32 users stated that they used Firefox. Using this data, conduct the appropriate hypothesis test using a 0.05 level of significance.

    **Solution:**

    **Framing the appropriate null and alternative hypotheses:**

    We want to test $H_0 : p = 0.2$ vs $H_A : p \neq 0.2$.

    **Checking the Assumptions:**

    Here, n = 200, and $p_0 = 0.2$, whence $np_0 = 40$, & $n(1 - p_0) = 160$

    $\Rightarrow np_0 > 10, \text{ and } n(1 - p_0) > 10$.

    **Choosing the appropriate test statistic:**

    $$T = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \overset{p=p_0}{\sim} N(0, 1) \text{ (approximately)}.$$

    **Calculating the observed value of the test statistic:**

    According to the problem, $\hat{p} = \dfrac{32}{200} = 0.16$, $p_0 = 0.20$ and n = 200.

    Observed test statistic: $t = \dfrac{0.16 - 0.20}{\sqrt{0.20(1 - 0.20)/200}} = -1.414214 \approx -1.41$.

    **Estimating the p–value:**

    p–value = $2P (Z > |-1.41|) = 2(1 - 0.92073) = 0.15854 > \alpha = 0.05 \Rightarrow$ Fail to reject $H_0$ at $\alpha = 0.05$

    Thus, we fail to reject $H_0$ at 5% level of significance, and conclude that the true population proportion of Firefox web browser users is not significantly different from 0.2.

11. A polling organization conducts an exit poll and somehow it appears that Candidate A is likely to receive less than one third of the votes. The polling organization interviews 116 people to check their voting preferences, and 40 of them speak in favor of Candidate A. Formulate the appropriate hypotheses to test, and carry out the test at level of significance $\alpha = 0.05$.

**Solution:**

**Framing the appropriate null and alternative hypotheses:**

We want to test $H_0 : p \geq 1/3$ vs $H_A : p < 1/3$.

**Checking the Assumptions:**

Here, n = 116, and $p_0$ = 1/3, whence $np_0 \approx 38.67$, and $n(1 - p_0) \approx 77.33$

$\Rightarrow np_0 > 10,$ and $n(1 - p_0) > 10$.

**Choosing the appropriate test statistic:**

$$T = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \overset{p=p_0}{\sim} N(0, 1) \text{ (approximately)}.$$

**Calculating the observed value of the test statistic:**

According to the problem, $\hat{p} = \frac{40}{116}$, $p_0$ = 1/3 and n = 116.

Observed test statistic: $t = \dfrac{\frac{40}{116} - \frac{1}{3}}{\sqrt{\left(\frac{1}{3}\right)\left(1 - \frac{1}{3}\right)/116}} = 0.2626129 \approx 0.26.$

**Estimating the p–value:**

p–value = $P(Z < 0.26) = 0.60257 > \alpha = 0.05 \Rightarrow$ Fail to reject $H_0$ at $\alpha = 0.05$

Thus, we fail to reject $H_0$ at 5% level of significance, and conclude that Candidate A is likely to receive more than one third of the votes in the election.

12. Suppose an ornithologist is comparing the average wingspan $\mu$ of a new subspecies of butterfly to a known value of the species, $\mu_0$ = 2.980 cm. He specifically wants to know if $\mu$ significantly differs from $\mu_0$. Based a random sample of n = 25 new subspecies, the ornithologist calculated an average wingspan in the sample as 2.925 cm, with a standard deviation of 0.093 cm.

Assume that the distribution of wingspan of the new subspecies is approximately normal.

What should be his decision at 5% level of significance?

**Solution:**

**Framing the appropriate null and alternative hypotheses:**

$$H_0 : \mu = \mu_0 \text{ against } H_A : \mu \neq \mu_0, \text{ with } \mu_0 = 2.980.$$

The underlying population distribution is approximately normal having an unknown mean $\mu$ and an unknown standard deviation $\sigma$. Also, $\overline{X}$ = 2.925, S = 0.093, and n = 25.

Hence, an estimated 95% confidence interval for $\mu$ is:

$$\left[\overline{X} - t_{0.025;24}\frac{S}{\sqrt{n}}, \overline{X} + t_{0.025;24}\frac{S}{\sqrt{n}}\right]$$
$$= \left[2.925 - 2.0639 \times \frac{0.093}{\sqrt{25}}, 2.925 + 2.0639 \times \frac{0.093}{\sqrt{25}}\right]$$
$$\approx [2.8866, 2.9634],$$

which fails to contain the null hypothesized value $\mu$ = 2.980. Hence, we reject $H_0$ at 5% level of significance.

13. In testing $H_0 : \mu$ = 5 vs. $H_A : \mu > 5$, a sample of size n = 50 yielded a p-value of 0.014. If $\alpha$ = 0.01, and the true value of the mean was actually $\mu$ = 7, determine whether the experimenter has made a correct decision. If not, determine the type of error.

    **Solution:** Since p-value = 0.014 $> \alpha$ = 0.01, we fail to reject $H_0$ at level $\alpha$ = 0.01.

    Since the true value of $\mu$ = 7 > 5, $H_0$ is false.

    Thus, we fail to reject a false null hypothesis $\Rightarrow$ Type II error

14. In testing $H_0 : p \leq 0.5$ vs. $H_A : p > 0.5$, a sample of size n = 64 yielded a p-value of 0.017. If $\alpha$ = 0.05, and the true value of the population proportion p was actually p = 0.38, determine whether the experimenter has made a correct decision. If not, determine the type of error.

    **Solution:** Since p-value = 0.027 $< \alpha$ = 0.05, we reject $H_0$ at level $\alpha$ = 0.05.

    Since the true value of p = 0.38 < 0.5, $H_0$ is true.

    Thus, we reject a true null hypothesis $\Rightarrow$ Type I error

15. Dave is feeling sick, so he goes to the doctor and gets a chest X-ray to determine whether or not he has pneumonia. Let the null hypothesis be that Dave doesn't have pneumonia and the alternative hypothesis be that Dave has pneumonia. Assume

    $$H_0 : \text{"Dave doesn't have pneumonia" vs. } H_A : \text{"Dave has pneumonia"}$$

    What would be a type I error and a Type II error in this case?
    **Solution:**

    Type I error: Dave doesn't have pneumonia, but the doctors concluded that he has pneumonia

Type II error: Dave has pneumonia, but the doctors concluded that he doesn't have pneumonia

16. A group of doctors is deciding whether or not to perform an operation to remove a cancerous tumor. Suppose the null hypothesis is:

$$H_0 : \text{the surgical procedure successfully removes the tumor.}$$

State the Type I and Type II errors in this context.

**Solution:**

Type I error: The surgical procedure successfully removed the tumor, but the doctors felt that it wasn't successful, and hence they decided to operate the patient once again.

Type II error: The surgical procedure actually failed to remove the tumor, but the doctors felt that it was successful, and hence they decided not to operate the patient once again.

17. A credit union has received several customers' complaints about the long waiting time before their calls were answered. The average time a customer will wait before the call is answered is 15 minutes. The credit union increased the number of customer representatives and conducts a hypothesis test to see if the customer waiting time has significantly decreased.

$H_0$: The average waiting time has not decreased.

$H_A$: The average waiting time has decreased.

Describe the type I error, and type II error in this context.

**Solution:**

- Type I error: The credit union concludes that the average waiting time has decreased significantly when actually it has not.
- Type II error: The credit union concludes that the average waiting time has not decreased significantly when actually the average waiting time has decreased.

**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| -3.9 | .00005 | .00005 | .00004 | .00004 | .00004 | .00004 | .00004 | .00004 | .00003 | .00003 |
| -3.8 | .00007 | .00007 | .00007 | .00006 | .00006 | .00006 | .00006 | .00005 | .00005 | .00005 |
| -3.7 | .00011 | .00010 | .00010 | .00010 | .00009 | .00009 | .00008 | .00008 | .00008 | .00008 |
| -3.6 | .00016 | .00015 | .00015 | .00014 | .00014 | .00013 | .00013 | .00012 | .00012 | .00011 |
| -3.5 | .00023 | .00022 | .00022 | .00021 | .00020 | .00019 | .00019 | .00018 | .00017 | .00017 |
| -3.4 | .00034 | .00032 | .00031 | .00030 | .00029 | .00028 | .00027 | .00026 | .00025 | .00024 |
| -3.3 | .00048 | .00047 | .00045 | .00043 | .00042 | .00040 | .00039 | .00038 | .00036 | .00035 |
| -3.2 | .00069 | .00066 | .00064 | .00062 | .00060 | .00058 | .00056 | .00054 | .00052 | .00050 |
| -3.1 | .00097 | .00094 | .00090 | .00087 | .00084 | .00082 | .00079 | .00076 | .00074 | .00071 |
| -3.0 | .00135 | .00131 | .00126 | .00122 | .00118 | .00114 | .00111 | .00107 | .00104 | .00100 |
| -2.9 | .00187 | .00181 | .00175 | .00169 | .00164 | .00159 | .00154 | .00149 | .00144 | .00139 |
| -2.8 | .00256 | .00248 | .00240 | .00233 | .00226 | .00219 | .00212 | .00205 | .00199 | .00193 |
| -2.7 | .00347 | .00336 | .00326 | .00317 | .00307 | .00298 | .00289 | .00280 | .00272 | .00264 |
| -2.6 | .00466 | .00453 | .00440 | .00427 | .00415 | .00402 | .00391 | .00379 | .00368 | .00357 |
| -2.5 | .00621 | .00604 | .00587 | .00570 | .00554 | .00539 | .00523 | .00508 | .00494 | .00480 |
| -2.4 | .00820 | .00798 | .00776 | .00755 | .00734 | .00714 | .00695 | .00676 | .00657 | .00639 |
| -2.3 | .01072 | .01044 | .01017 | .00990 | .00964 | .00939 | .00914 | .00889 | .00866 | .00842 |
| -2.2 | .01390 | .01355 | .01321 | .01287 | .01255 | .01222 | .01191 | .01160 | .01130 | .01101 |
| -2.1 | .01786 | .01743 | .01700 | .01659 | .01618 | .01578 | .01539 | .01500 | .01463 | .01426 |
| -2.0 | .02275 | .02222 | .02169 | .02118 | .02068 | .02018 | .01970 | .01923 | .01876 | .01831 |
| -1.9 | .02872 | .02807 | .02743 | .02680 | .02619 | .02559 | .02500 | .02442 | .02385 | .02330 |
| -1.8 | .03593 | .03515 | .03438 | .03362 | .03288 | .03216 | .03144 | .03074 | .03005 | .02938 |
| -1.7 | .04457 | .04363 | .04272 | .04182 | .04093 | .04006 | .03920 | .03836 | .03754 | .03673 |
| -1.6 | .05480 | .05370 | .05262 | .05155 | .05050 | .04947 | .04846 | .04746 | .04648 | .04551 |
| -1.5 | .06681 | .06552 | .06426 | .06301 | .06178 | .06057 | .05938 | .05821 | .05705 | .05592 |
| -1.4 | .08076 | .07927 | .07780 | .07636 | .07493 | .07353 | .07215 | .07078 | .06944 | .06811 |
| -1.3 | .09680 | .09510 | .09342 | .09176 | .09012 | .08851 | .08691 | .08534 | .08379 | .08226 |
| -1.2 | .11507 | .11314 | .11123 | .10935 | .10749 | .10565 | .10383 | .10204 | .10027 | .09853 |
| -1.1 | .13567 | .13350 | .13136 | .12924 | .12714 | .12507 | .12302 | .12100 | .11900 | .11702 |
| -1.0 | .15866 | .15625 | .15386 | .15151 | .14917 | .14686 | .14457 | .14231 | .14007 | .13786 |
| -0.9 | .18406 | .18141 | .17879 | .17619 | .17361 | .17106 | .16853 | .16602 | .16354 | .16109 |
| -0.8 | .21186 | .20897 | .20611 | .20327 | .20045 | .19766 | .19489 | .19215 | .18943 | .18673 |
| -0.7 | .24196 | .23885 | .23576 | .23270 | .22965 | .22663 | .22363 | .22065 | .21770 | .21476 |
| -0.6 | .27425 | .27093 | .26763 | .26435 | .26109 | .25785 | .25463 | .25143 | .24825 | .24510 |
| -0.5 | .30854 | .30503 | .30153 | .29806 | .29460 | .29116 | .28774 | .28434 | .28096 | .27760 |
| -0.4 | .34458 | .34090 | .33724 | .33360 | .32997 | .32636 | .32276 | .31918 | .31561 | .31207 |
| -0.3 | .38209 | .37828 | .37448 | .37070 | .36693 | .36317 | .35942 | .35569 | .35197 | .34827 |
| -0.2 | .42074 | .41683 | .41294 | .40905 | .40517 | .40129 | .39743 | .39358 | .38974 | .38591 |
| -0.1 | .46017 | .45620 | .45224 | .44828 | .44433 | .44038 | .43644 | .43251 | .42858 | .42465 |
| -0.0 | .50000 | .49601 | .49202 | .48803 | .48405 | .48006 | .47608 | .47210 | .46812 | .46414 |

**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.0 | .50000 | .50399 | .50798 | .51197 | .51595 | .51994 | .52392 | .52790 | .53188 | .53586 |
| 0.1 | .53983 | .54380 | .54776 | .55172 | .55567 | .55962 | .56356 | .56749 | .57142 | .57535 |
| 0.2 | .57926 | .58317 | .58706 | .59095 | .59483 | .59871 | .60257 | .60642 | .61026 | .61409 |
| 0.3 | .61791 | .62172 | .62552 | .62930 | .63307 | .63683 | .64058 | .64431 | .64803 | .65173 |
| 0.4 | .65542 | .65910 | .66276 | .66640 | .67003 | .67364 | .67724 | .68082 | .68439 | .68793 |
| 0.5 | .69146 | .69497 | .69847 | .70194 | .70540 | .70884 | .71226 | .71566 | .71904 | .72240 |
| 0.6 | .72575 | .72907 | .73237 | .73565 | .73891 | .74215 | .74537 | .74857 | .75175 | .75490 |
| 0.7 | .75804 | .76115 | .76424 | .76730 | .77035 | .77337 | .77637 | .77935 | .78230 | .78524 |
| 0.8 | .78814 | .79103 | .79389 | .79673 | .79955 | .80234 | .80511 | .80785 | .81057 | .81327 |
| 0.9 | .81594 | .81859 | .82121 | .82381 | .82639 | .82894 | .83147 | .83398 | .83646 | .83891 |
| 1.0 | .84134 | .84375 | .84614 | .84849 | .85083 | .85314 | .85543 | .85769 | .85993 | .86214 |
| 1.1 | .86433 | .86650 | .86864 | .87076 | .87286 | .87493 | .87698 | .87900 | .88100 | .88298 |
| 1.2 | .88493 | .88686 | .88877 | .89065 | .89251 | .89435 | .89617 | .89796 | .89973 | .90147 |
| 1.3 | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| 1.4 | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| 1.5 | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |
| 1.6 | .94520 | .94630 | .94738 | .94845 | .94950 | .95053 | .95154 | .95254 | .95352 | .95449 |
| 1.7 | .95543 | .95637 | .95728 | .95818 | .95907 | .95994 | .96080 | .96164 | .96246 | .96327 |
| 1.8 | .96407 | .96485 | .96562 | .96638 | .96712 | .96784 | .96856 | .96926 | .96995 | .97062 |
| 1.9 | .97128 | .97193 | .97257 | .97320 | .97381 | .97441 | .97500 | .97558 | .97615 | .97670 |
| 2.0 | .97725 | .97778 | .97831 | .97882 | .97932 | .97982 | .98030 | .98077 | .98124 | .98169 |
| 2.1 | .98214 | .98257 | .98300 | .98341 | .98382 | .98422 | .98461 | .98500 | .98537 | .98574 |
| 2.2 | .98610 | .98645 | .98679 | .98713 | .98745 | .98778 | .98809 | .98840 | .98870 | .98899 |
| 2.3 | .98928 | .98956 | .98983 | .99010 | .99036 | .99061 | .99086 | .99111 | .99134 | .99158 |
| 2.4 | .99180 | .99202 | .99224 | .99245 | .99266 | .99286 | .99305 | .99324 | .99343 | .99361 |
| 2.5 | .99379 | .99396 | .99413 | .99430 | .99446 | .99461 | .99477 | .99492 | .99506 | .99520 |
| 2.6 | .99534 | .99547 | .99560 | .99573 | .99585 | .99598 | .99609 | .99621 | .99632 | .99643 |
| 2.7 | .99653 | .99664 | .99674 | .99683 | .99693 | .99702 | .99711 | .99720 | .99728 | .99736 |
| 2.8 | .99744 | .99752 | .99760 | .99767 | .99774 | .99781 | .99788 | .99795 | .99801 | .99807 |
| 2.9 | .99813 | .99819 | .99825 | .99831 | .99836 | .99841 | .99846 | .99851 | .99856 | .99861 |
| 3.0 | .99865 | .99869 | .99874 | .99878 | .99882 | .99886 | .99889 | .99893 | .99896 | .99900 |
| 3.1 | .99903 | .99906 | .99910 | .99913 | .99916 | .99918 | .99921 | .99924 | .99926 | .99929 |
| 3.2 | .99931 | .99934 | .99936 | .99938 | .99940 | .99942 | .99944 | .99946 | .99948 | .99950 |
| 3.3 | .99952 | .99953 | .99955 | .99957 | .99958 | .99960 | .99961 | .99962 | .99964 | .99965 |
| 3.4 | .99966 | .99968 | .99969 | .99970 | .99971 | .99972 | .99973 | .99974 | .99975 | .99976 |
| 3.5 | .99977 | .99978 | .99978 | .99979 | .99980 | .99981 | .99981 | .99982 | .99983 | .99983 |
| 3.6 | .99984 | .99985 | .99985 | .99986 | .99986 | .99987 | .99987 | .99988 | .99988 | .99989 |
| 3.7 | .99989 | .99990 | .99990 | .99990 | .99991 | .99991 | .99992 | .99992 | .99992 | .99992 |
| 3.8 | .99993 | .99993 | .99993 | .99994 | .99994 | .99994 | .99994 | .99995 | .99995 | .99995 |
| 3.9 | .99995 | .99995 | .99996 | .99996 | .99996 | .99996 | .99996 | .99996 | .99997 | .99997 |

# Statistics - T-Distribution Table

The critical values of t distribution are calculated according to the probabilities of two alpha values and the degrees of freedom. The Alpha (a) values 0.05 one tailed and 0.1 two tailed are the two columns to be compared with the degrees of freedom in the row of the table.

| One Tail | 0.05 | 0.025 | 0.01 | 0.005 | 0.0025 | 0.001 | 0.0005 |
|---|---|---|---|---|---|---|---|
| Two Tails | 0.1 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
| df | | | | | | | |
| 1 | 6.3138 | 12.7065 | 31.8193 | 63.6551 | 127.3447 | 318.4930 | 636.0450 |
| 2 | 2.9200 | 4.3026 | 6.9646 | 9.9247 | 14.0887 | 22.3276 | 31.5989 |
| 3 | 2.3534 | 3.1824 | 4.5407 | 5.8408 | 7.4534 | 10.2145 | 12.9242 |
| 4 | 2.1319 | 2.7764 | 3.7470 | 4.6041 | 5.5976 | 7.1732 | 8.6103 |
| 5 | 2.0150 | 2.5706 | 3.3650 | 4.0322 | 4.7734 | 5.8934 | 6.8688 |
| 6 | 1.9432 | 2.4469 | 3.1426 | 3.7074 | 4.3168 | 5.2076 | 5.9589 |
| 7 | 1.8946 | 2.3646 | 2.9980 | 3.4995 | 4.0294 | 4.7852 | 5.4079 |
| 8 | 1.8595 | 2.3060 | 2.8965 | 3.3554 | 3.8325 | 4.5008 | 5.0414 |
| 9 | 1.8331 | 2.2621 | 2.8214 | 3.2498 | 3.6896 | 4.2969 | 4.7809 |
| 10 | 1.8124 | 2.2282 | 2.7638 | 3.1693 | 3.5814 | 4.1437 | 4.5869 |
| 11 | 1.7959 | 2.2010 | 2.7181 | 3.1058 | 3.4966 | 4.0247 | 4.4369 |
| 12 | 1.7823 | 2.1788 | 2.6810 | 3.0545 | 3.4284 | 3.9296 | 4.3178 |
| 13 | 1.7709 | 2.1604 | 2.6503 | 3.0123 | 3.3725 | 3.8520 | 4.2208 |
| 14 | 1.7613 | 2.1448 | 2.6245 | 2.9768 | 3.3257 | 3.7874 | 4.1404 |
| 15 | 1.7530 | 2.1314 | 2.6025 | 2.9467 | 3.2860 | 3.7328 | 4.0728 |
| 16 | 1.7459 | 2.1199 | 2.5835 | 2.9208 | 3.2520 | 3.6861 | 4.0150 |
| 17 | 1.7396 | 2.1098 | 2.5669 | 2.8983 | 3.2224 | 3.6458 | 3.9651 |
| 18 | 1.7341 | 2.1009 | 2.5524 | 2.8784 | 3.1966 | 3.6105 | 3.9216 |
| 19 | 1.7291 | 2.0930 | 2.5395 | 2.8609 | 3.1737 | 3.5794 | 3.8834 |
| 20 | 1.7247 | 2.0860 | 2.5280 | 2.8454 | 3.1534 | 3.5518 | 3.8495 |
| 21 | 1.7207 | 2.0796 | 2.5176 | 2.8314 | 3.1352 | 3.5272 | 3.8193 |
| 22 | 1.7172 | 2.0739 | 2.5083 | 2.8188 | 3.1188 | 3.5050 | 3.7921 |
| 23 | 1.7139 | 2.0686 | 2.4998 | 2.8073 | 3.1040 | 3.4850 | 3.7676 |
| 24 | 1.7109 | 2.0639 | 2.4922 | 2.7970 | 3.0905 | 3.4668 | 3.7454 |
| 25 | 1.7081 | 2.0596 | 2.4851 | 2.7874 | 3.0782 | 3.4502 | 3.7251 |
| 26 | 1.7056 | 2.0555 | 2.4786 | 2.7787 | 3.0669 | 3.4350 | 3.7067 |
| 27 | 1.7033 | 2.0518 | 2.4727 | 2.7707 | 3.0565 | 3.4211 | 3.68 |
| 28 | 1.7011 | 2.0484 | 2.4671 | 2.7633 | 3.0469 | 3.4082 | 3.6739 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 29 | 1.6991 | 2.0452 | 2.4620 | 2.7564 | 3.0380 | 3.3962 | 3.6594 |
| 30 | 1.6973 | 2.0423 | 2.4572 | 2.7500 | 3.0298 | 3.3852 | 3.6459 |
| 31 | 1.6955 | 2.0395 | 2.4528 | 2.7440 | 3.0221 | 3.3749 | 3.6334 |
| 32 | 1.6939 | 2.0369 | 2.4487 | 2.7385 | 3.0150 | 3.3653 | 3.6218 |
| 33 | 1.6924 | 2.0345 | 2.4448 | 2.7333 | 3.0082 | 3.3563 | 3.6109 |
| 34 | 1.6909 | 2.0322 | 2.4411 | 2.7284 | 3.0019 | 3.3479 | 3.6008 |
| 35 | 1.6896 | 2.0301 | 2.4377 | 2.7238 | 2.9961 | 3.3400 | 3.5912 |
| 36 | 1.6883 | 2.0281 | 2.4345 | 2.7195 | 2.9905 | 3.3326 | 3.5822 |
| 37 | 1.6871 | 2.0262 | 2.4315 | 2.7154 | 2.9853 | 3.3256 | 3.5737 |
| 38 | 1.6859 | 2.0244 | 2.4286 | 2.7115 | 2.9803 | 3.3190 | 3.5657 |
| 39 | 1.6849 | 2.0227 | 2.4258 | 2.7079 | 2.9756 | 3.3128 | 3.5581 |
| 40 | 1.6839 | 2.0211 | 2.4233 | 2.7045 | 2.9712 | 3.3069 | 3.5510 |
| 41 | 1.6829 | 2.0196 | 2.4208 | 2.7012 | 2.9670 | 3.3013 | 3.5442 |
| 42 | 1.6820 | 2.0181 | 2.4185 | 2.6981 | 2.9630 | 3.2959 | 3.5378 |
| 43 | 1.6811 | 2.0167 | 2.4162 | 2.6951 | 2.9591 | 3.2909 | 3.5316 |
| 44 | 1.6802 | 2.0154 | 2.4142 | 2.6923 | 2.9555 | 3.2861 | 3.5258 |
| 45 | 1.6794 | 2.0141 | 2.4121 | 2.6896 | 2.9521 | 3.2815 | 3.5202 |
| 46 | 1.6787 | 2.0129 | 2.4102 | 2.6870 | 2.9488 | 3.2771 | 3.5149 |
| 47 | 1.6779 | 2.0117 | 2.4083 | 2.6846 | 2.9456 | 3.2729 | 3.5099 |
| 48 | 1.6772 | 2.0106 | 2.4066 | 2.6822 | 2.9426 | 3.2689 | 3.5051 |
| 49 | 1.6766 | 2.0096 | 2.4049 | 2.6800 | 2.9397 | 3.2651 | 3.5004 |
| 50 | 1.6759 | 2.0086 | 2.4033 | 2.6778 | 2.9370 | 3.2614 | 3.4960 |
| 51 | 1.6753 | 2.0076 | 2.4017 | 2.6757 | 2.9343 | 3.2579 | 3.4917 |
| 52 | 1.6747 | 2.0066 | 2.4002 | 2.6737 | 2.9318 | 3.2545 | 3.4877 |
| 53 | 1.6741 | 2.0057 | 2.3988 | 2.6718 | 2.9293 | 3.2513 | 3.4838 |
| 54 | 1.6736 | 2.0049 | 2.3974 | 2.6700 | 2.9270 | 3.2482 | 3.4800 |
| 55 | 1.6730 | 2.0041 | 2.3961 | 2.6682 | 2.9247 | 3.2451 | 3.4764 |
| 56 | 1.6725 | 2.0032 | 2.3948 | 2.6665 | 2.9225 | 3.2423 | 3.4730 |
| 57 | 1.6720 | 2.0025 | 2.3936 | 2.6649 | 2.9204 | 3.2394 | 3.4696 |
| 58 | 1.6715 | 2.0017 | 2.3924 | 2.6633 | 2.9184 | 3.2368 | 3.4663 |
| 59 | 1.6711 | 2.0010 | 2.3912 | 2.6618 | 2.9164 | 3.2342 | 3.4632 |
| 60 | 1.6706 | 2.0003 | 2.3901 | 2.6603 | 2.9146 | 3.2317 | 3.4602 |
| 61 | 1.6702 | 1.9996 | 2.3890 | 2.6589 | 2.9127 | 3.2293 | 3.4573 |
| 62 | 1.6698 | 1.9990 | 2.3880 | 2.6575 | 2.9110 | 3.2269 | 3.4545 |
| 63 | 1.6694 | 1.9983 | 2.3870 | 2.6561 | 2.9092 | 3.2247 | 3.4518 |
| 64 | 1.6690 | 1.9977 | 2.3860 | 2.6549 | 2.9076 | 3.2225 | 3.4491 |
| 65 | 1.6686 | 1.9971 | 2.3851 | 2.6536 | 2.9060 | 3.2204 | 3.4466 |
| 66 | 1.6683 | 1.9966 | 2.3842 | 2.6524 | 2.9045 | 3.2184 | 3.4441 |
| 67 | 1.6679 | 1.9960 | 2.3833 | 2.6512 | 2.9030 | 3.2164 | 3.4417 |
| 68 | 1.6676 | 1.9955 | 2.3824 | 2.6501 | 2.9015 | 3.2144 | 3.4395 |
| 69 | 1.6673 | 1.9950 | 2.3816 | 2.6490 | 2.9001 | 3.2126 | 3.4372 |
| 70 | 1.6669 | 1.9944 | 2.3808 | 2.6479 | 2.8987 | 3.2108 | 3.4350 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 71 | 1.6666 | 1.9939 | 2.3800 | 2.6468 | 2.8974 | 3.2090 | 3.4329 |
| 72 | 1.6663 | 1.9935 | 2.3793 | 2.6459 | 2.8961 | 3.2073 | 3.4308 |
| 73 | 1.6660 | 1.9930 | 2.3785 | 2.6449 | 2.8948 | 3.2056 | 3.4288 |
| 74 | 1.6657 | 1.9925 | 2.3778 | 2.6439 | 2.8936 | 3.2040 | 3.4269 |
| 75 | 1.6654 | 1.9921 | 2.3771 | 2.6430 | 2.8925 | 3.2025 | 3.4250 |
| 76 | 1.6652 | 1.9917 | 2.3764 | 2.6421 | 2.8913 | 3.2010 | 3.4232 |
| 77 | 1.6649 | 1.9913 | 2.3758 | 2.6412 | 2.8902 | 3.1995 | 3.4214 |
| 78 | 1.6646 | 1.9909 | 2.3751 | 2.6404 | 2.8891 | 3.1980 | 3.4197 |
| 79 | 1.6644 | 1.9904 | 2.3745 | 2.6395 | 2.8880 | 3.1966 | 3.4180 |
| 80 | 1.6641 | 1.9901 | 2.3739 | 2.6387 | 2.8870 | 3.1953 | 3.4164 |
| 81 | 1.6639 | 1.9897 | 2.3733 | 2.6379 | 2.8859 | 3.1939 | 3.4147 |
| 82 | 1.6636 | 1.9893 | 2.3727 | 2.6371 | 2.8850 | 3.1926 | 3.4132 |
| 83 | 1.6634 | 1.9889 | 2.3721 | 2.6364 | 2.8840 | 3.1913 | 3.4117 |
| 84 | 1.6632 | 1.9886 | 2.3716 | 2.6356 | 2.8831 | 3.1901 | 3.4101 |
| 85 | 1.6630 | 1.9883 | 2.3710 | 2.6349 | 2.8821 | 3.1889 | 3.4087 |
| 86 | 1.6628 | 1.9879 | 2.3705 | 2.6342 | 2.8813 | 3.1877 | 3.4073 |
| 87 | 1.6626 | 1.9876 | 2.3700 | 2.6335 | 2.8804 | 3.1866 | 3.4059 |
| 88 | 1.6623 | 1.9873 | 2.3695 | 2.6328 | 2.8795 | 3.1854 | 3.4046 |
| 89 | 1.6622 | 1.9870 | 2.3690 | 2.6322 | 2.8787 | 3.1844 | 3.4032 |
| 90 | 1.6620 | 1.9867 | 2.3685 | 2.6316 | 2.8779 | 3.1833 | 3.4020 |
| 91 | 1.6618 | 1.9864 | 2.3680 | 2.6309 | 2.8771 | 3.1822 | 3.4006 |
| 92 | 1.6616 | 1.9861 | 2.3676 | 2.6303 | 2.8763 | 3.1812 | 3.3995 |
| 93 | 1.6614 | 1.9858 | 2.3671 | 2.6297 | 2.8755 | 3.1802 | 3.3982 |
| 94 | 1.6612 | 1.9855 | 2.3667 | 2.6292 | 2.8748 | 3.1792 | 3.3970 |
| 95 | 1.6610 | 1.9852 | 2.3662 | 2.6286 | 2.8741 | 3.1782 | 3.3959 |
| 96 | 1.6609 | 1.9850 | 2.3658 | 2.6280 | 2.8734 | 3.1773 | 3.3947 |
| 97 | 1.6607 | 1.9847 | 2.3654 | 2.6275 | 2.8727 | 3.1764 | 3.3936 |
| 98 | 1.6606 | 1.9845 | 2.3650 | 2.6269 | 2.8720 | 3.1755 | 3.3926 |
| 99 | 1.6604 | 1.9842 | 2.3646 | 2.6264 | 2.8713 | 3.1746 | 3.3915 |
| 100 | 1.6602 | 1.9840 | 2.3642 | 2.6259 | 2.8706 | 3.1738 | 3.3905 |
| 101 | 1.6601 | 1.9837 | 2.3638 | 2.6254 | 2.8700 | 3.1729 | 3.3894 |
| 102 | 1.6599 | 1.9835 | 2.3635 | 2.6249 | 2.8694 | 3.1720 | 3.3885 |
| 103 | 1.6598 | 1.9833 | 2.3631 | 2.6244 | 2.8687 | 3.1712 | 3.3875 |
| 104 | 1.6596 | 1.9830 | 2.3627 | 2.6240 | 2.8682 | 3.1704 | 3.3866 |
| 105 | 1.6595 | 1.9828 | 2.3624 | 2.6235 | 2.8675 | 3.1697 | 3.3856 |
| 106 | 1.6593 | 1.9826 | 2.3620 | 2.6230 | 2.8670 | 3.1689 | 3.3847 |
| 107 | 1.6592 | 1.9824 | 2.3617 | 2.6225 | 2.8664 | 3.1681 | 3.3838 |
| 108 | 1.6591 | 1.9822 | 2.3614 | 2.6221 | 2.8658 | 3.1674 | 3.3829 |
| 109 | 1.6589 | 1.9820 | 2.3611 | 2.6217 | 2.8653 | 3.1667 | 3.3820 |
| 110 | 1.6588 | 1.9818 | 2.3607 | 2.6212 | 2.8647 | 3.1660 | 3.3812 |
| 111 | 1.6587 | 1.9816 | 2.3604 | 2.6208 | 2.8642 | 3.1653 | 3.3803 |
| 112 | 1.6586 | 1.9814 | 2.3601 | 2.6204 | 2.8637 | 3.1646 | 3.3795 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 113 | 1.6585 | 1.9812 | 2.3598 | 2.6200 | 2.8632 | 3.1640 | 3.3787 |
| 114 | 1.6583 | 1.9810 | 2.3595 | 2.6196 | 2.8627 | 3.1633 | 3.3779 |
| 115 | 1.6582 | 1.9808 | 2.3592 | 2.6192 | 2.8622 | 3.1626 | 3.3771 |
| 116 | 1.6581 | 1.9806 | 2.3589 | 2.6189 | 2.8617 | 3.1620 | 3.3764 |
| 117 | 1.6580 | 1.9805 | 2.3586 | 2.6185 | 2.8612 | 3.1614 | 3.3756 |
| 118 | 1.6579 | 1.9803 | 2.3583 | 2.6181 | 2.8608 | 3.1607 | 3.3749 |
| 119 | 1.6578 | 1.9801 | 2.3581 | 2.6178 | 2.8603 | 3.1601 | 3.3741 |
| 120 | 1.6577 | 1.9799 | 2.3578 | 2.6174 | 2.8599 | 3.1595 | 3.3735 |
| 121 | 1.6575 | 1.9798 | 2.3576 | 2.6171 | 2.8594 | 3.1589 | 3.3727 |
| 122 | 1.6574 | 1.9796 | 2.3573 | 2.6168 | 2.8590 | 3.1584 | 3.3721 |
| 123 | 1.6573 | 1.9794 | 2.3571 | 2.6164 | 2.8585 | 3.1578 | 3.3714 |
| 124 | 1.6572 | 1.9793 | 2.3568 | 2.6161 | 2.8582 | 3.1573 | 3.3707 |
| 125 | 1.6571 | 1.9791 | 2.3565 | 2.6158 | 2.8577 | 3.1567 | 3.3700 |
| 126 | 1.6570 | 1.9790 | 2.3563 | 2.6154 | 2.8573 | 3.1562 | 3.3694 |
| 127 | 1.6570 | 1.9788 | 2.3561 | 2.6151 | 2.8569 | 3.1556 | 3.3688 |
| 128 | 1.6568 | 1.9787 | 2.3559 | 2.6148 | 2.8565 | 3.1551 | 3.3682 |
| 129 | 1.6568 | 1.9785 | 2.3556 | 2.6145 | 2.8561 | 3.1546 | 3.3676 |
| 130 | 1.6567 | 1.9784 | 2.3554 | 2.6142 | 2.8557 | 3.1541 | 3.3669 |
| 131 | 1.6566 | 1.9782 | 2.3552 | 2.6139 | 2.8554 | 3.1536 | 3.3663 |
| 132 | 1.6565 | 1.9781 | 2.3549 | 2.6136 | 2.8550 | 3.1531 | 3.3658 |
| 133 | 1.6564 | 1.9779 | 2.3547 | 2.6133 | 2.8546 | 3.1526 | 3.3652 |
| 134 | 1.6563 | 1.9778 | 2.3545 | 2.6130 | 2.8542 | 3.1522 | 3.3646 |
| 135 | 1.6562 | 1.9777 | 2.3543 | 2.6127 | 2.8539 | 3.1517 | 3.3641 |
| 136 | 1.6561 | 1.9776 | 2.3541 | 2.6125 | 2.8536 | 3.1512 | 3.3635 |
| 137 | 1.6561 | 1.9774 | 2.3539 | 2.6122 | 2.8532 | 3.1508 | 3.3630 |
| 138 | 1.6560 | 1.9773 | 2.3537 | 2.6119 | 2.8529 | 3.1503 | 3.3624 |
| 139 | 1.6559 | 1.9772 | 2.3535 | 2.6117 | 2.8525 | 3.1499 | 3.3619 |
| 140 | 1.6558 | 1.9771 | 2.3533 | 2.6114 | 2.8522 | 3.1495 | 3.3614 |
| 141 | 1.6557 | 1.9769 | 2.3531 | 2.6112 | 2.8519 | 3.1491 | 3.3609 |
| 142 | 1.6557 | 1.9768 | 2.3529 | 2.6109 | 2.8516 | 3.1486 | 3.3604 |
| 143 | 1.6556 | 1.9767 | 2.3527 | 2.6106 | 2.8512 | 3.1482 | 3.3599 |
| 144 | 1.6555 | 1.9766 | 2.3525 | 2.6104 | 2.8510 | 3.1478 | 3.3594 |
| 145 | 1.6554 | 1.9765 | 2.3523 | 2.6102 | 2.8506 | 3.1474 | 3.3589 |
| 146 | 1.6554 | 1.9764 | 2.3522 | 2.6099 | 2.8503 | 3.1470 | 3.3584 |
| 147 | 1.6553 | 1.9762 | 2.3520 | 2.6097 | 2.8500 | 3.1466 | 3.3579 |
| 148 | 1.6552 | 1.9761 | 2.3518 | 2.6094 | 2.8497 | 3.1462 | 3.3575 |
| 149 | 1.6551 | 1.9760 | 2.3516 | 2.6092 | 2.8494 | 3.1458 | 3.3570 |
| 150 | 1.6551 | 1.9759 | 2.3515 | 2.6090 | 2.8491 | 3.1455 | 3.3565 |
| 151 | 1.6550 | 1.9758 | 2.3513 | 2.6088 | 2.8489 | 3.1451 | 3.3561 |
| 152 | 1.6549 | 1.9757 | 2.3511 | 2.6085 | 2.8486 | 3.1447 | 3.3557 |
| 153 | 1.6549 | 1.9756 | 2.3510 | 2.6083 | 2.8483 | 3.1443 | 3.3552 |
| 154 | 1.6548 | 1.9755 | 2.3508 | 2.6081 | 2.8481 | 3.1440 | 3.3548 |

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| 155 | 1.6547 | 1.9754 | 2.3507 | 2.6079 | 2.8478 | 3.1436 | 3.3544 |
| 156 | 1.6547 | 1.9753 | 2.3505 | 2.6077 | 2.8475 | 3.1433 | 3.3540 |
| 157 | 1.6546 | 1.9752 | 2.3503 | 2.6075 | 2.8472 | 3.1430 | 3.3536 |
| 158 | 1.6546 | 1.9751 | 2.3502 | 2.6073 | 2.8470 | 3.1426 | 3.3531 |
| 159 | 1.6545 | 1.9750 | 2.3500 | 2.6071 | 2.8467 | 3.1423 | 3.3528 |
| 160 | 1.6544 | 1.9749 | 2.3499 | 2.6069 | 2.8465 | 3.1419 | 3.3523 |
| 161 | 1.6544 | 1.9748 | 2.3497 | 2.6067 | 2.8463 | 3.1417 | 3.3520 |
| 162 | 1.6543 | 1.9747 | 2.3496 | 2.6065 | 2.8460 | 3.1413 | 3.3516 |
| 163 | 1.6543 | 1.9746 | 2.3495 | 2.6063 | 2.8458 | 3.1410 | 3.3512 |
| 164 | 1.6542 | 1.9745 | 2.3493 | 2.6062 | 2.8455 | 3.1407 | 3.3508 |
| 165 | 1.6542 | 1.9744 | 2.3492 | 2.6060 | 2.8452 | 3.1403 | 3.3505 |
| 166 | 1.6541 | 1.9744 | 2.3490 | 2.6058 | 2.8450 | 3.1400 | 3.3501 |
| 167 | 1.6540 | 1.9743 | 2.3489 | 2.6056 | 2.8448 | 3.1398 | 3.3497 |
| 168 | 1.6540 | 1.9742 | 2.3487 | 2.6054 | 2.8446 | 3.1394 | 3.3494 |
| 169 | 1.6539 | 1.9741 | 2.3486 | 2.6052 | 2.8443 | 3.1392 | 3.3490 |
| 170 | 1.6539 | 1.9740 | 2.3485 | 2.6051 | 2.8441 | 3.1388 | 3.3487 |
| 171 | 1.6538 | 1.9739 | 2.3484 | 2.6049 | 2.8439 | 3.1386 | 3.3483 |
| 172 | 1.6537 | 1.9739 | 2.3482 | 2.6047 | 2.8437 | 3.1383 | 3.3480 |
| 173 | 1.6537 | 1.9738 | 2.3481 | 2.6046 | 2.8435 | 3.1380 | 3.3477 |
| 174 | 1.6537 | 1.9737 | 2.3480 | 2.6044 | 2.8433 | 3.1377 | 3.3473 |
| 175 | 1.6536 | 1.9736 | 2.3478 | 2.6042 | 2.8430 | 3.1375 | 3.3470 |
| 176 | 1.6536 | 1.9735 | 2.3477 | 2.6041 | 2.8429 | 3.1372 | 3.3466 |
| 177 | 1.6535 | 1.9735 | 2.3476 | 2.6039 | 2.8427 | 3.1369 | 3.3464 |
| 178 | 1.6535 | 1.9734 | 2.3475 | 2.6037 | 2.8424 | 3.1366 | 3.3460 |
| 179 | 1.6534 | 1.9733 | 2.3474 | 2.6036 | 2.8423 | 3.1364 | 3.3457 |
| 180 | 1.6534 | 1.9732 | 2.3472 | 2.6034 | 2.8420 | 3.1361 | 3.3454 |
| 181 | 1.6533 | 1.9731 | 2.3471 | 2.6033 | 2.8419 | 3.1358 | 3.3451 |
| 182 | 1.6533 | 1.9731 | 2.3470 | 2.6031 | 2.8416 | 3.1356 | 3.3448 |
| 183 | 1.6532 | 1.9730 | 2.3469 | 2.6030 | 2.8415 | 3.1354 | 3.3445 |
| 184 | 1.6532 | 1.9729 | 2.3468 | 2.6028 | 2.8413 | 3.1351 | 3.3442 |
| 185 | 1.6531 | 1.9729 | 2.3467 | 2.6027 | 2.8411 | 3.1349 | 3.3439 |
| 186 | 1.6531 | 1.9728 | 2.3466 | 2.6025 | 2.8409 | 3.1346 | 3.3436 |
| 187 | 1.6531 | 1.9727 | 2.3465 | 2.6024 | 2.8407 | 3.1344 | 3.3433 |
| 188 | 1.6530 | 1.9727 | 2.3463 | 2.6022 | 2.8406 | 3.1341 | 3.3430 |
| 189 | 1.6529 | 1.9726 | 2.3463 | 2.6021 | 2.8403 | 3.1339 | 3.3428 |
| 190 | 1.6529 | 1.9725 | 2.3461 | 2.6019 | 2.8402 | 3.1337 | 3.3425 |
| 191 | 1.6529 | 1.9725 | 2.3460 | 2.6018 | 2.8400 | 3.1334 | 3.3422 |
| 192 | 1.6528 | 1.9724 | 2.3459 | 2.6017 | 2.8398 | 3.1332 | 3.3419 |
| 193 | 1.6528 | 1.9723 | 2.3458 | 2.6015 | 2.8397 | 3.1330 | 3.3417 |
| 194 | 1.6528 | 1.9723 | 2.3457 | 2.6014 | 2.8395 | 3.1328 | 3.3414 |
| 195 | 1.6527 | 1.9722 | 2.3456 | 2.6013 | 2.8393 | 3.1326 | 3.3411 |
| 196 | 1.6527 | 1.9721 | 2.3455 | 2.6012 | 2.8392 | 3.1323 | 3.3409 |

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|---|---|---|---|---|---|---|---|
| 197 | 1.6526 | 1.9721 | 2.3454 | 2.6010 | 2.8390 | 3.1321 | 3.3406 |
| 198 | 1.6526 | 1.9720 | 2.3453 | 2.6009 | 2.8388 | 3.1319 | 3.3403 |
| 199 | 1.6525 | 1.9720 | 2.3452 | 2.6008 | 2.8387 | 3.1317 | 3.3401 |
| 200 | 1.6525 | 1.9719 | 2.3451 | 2.6007 | 2.8385 | 3.1315 | 3.3398 |