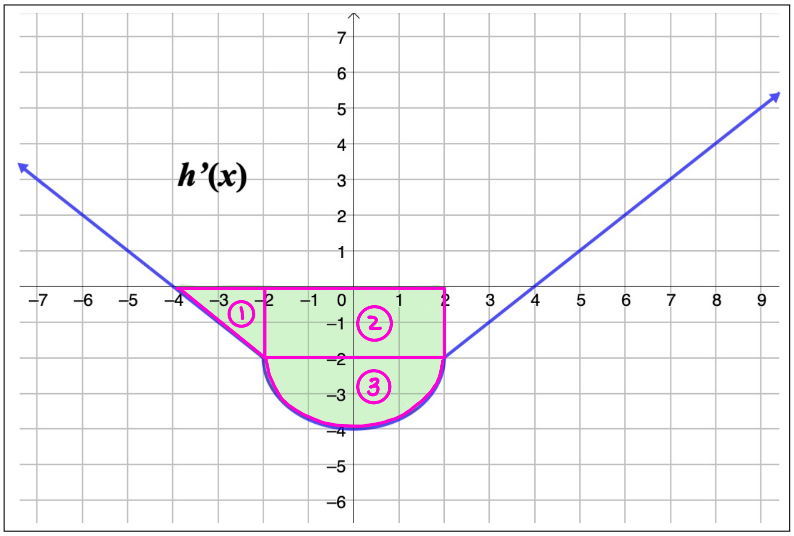
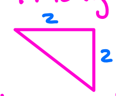
SESSION 10: REVIEW EXAM #3

1. Use the graph of $h'(x)$ below to find $\int_{-4}^{2} h'(x)\, dx$.



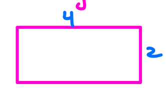
① Triangle



$A_T = \frac{1}{2} \cdot b \cdot h = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot 2$

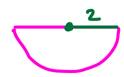
$= 2$

② Rectangle



$A_R = b \cdot h = 4 \cdot 2 = 8$

③ Semi circle



$A_c = \frac{\pi r^2}{2} = \frac{\pi (2)^2}{2} = \frac{4\pi}{2} = 2\pi$

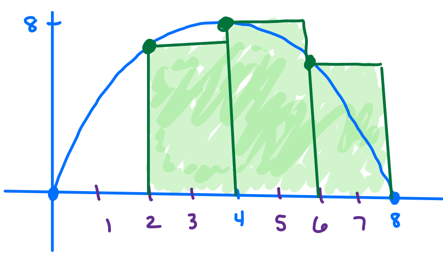
$\int_{-4}^{2} h'(x)\,dx = -2 + -8 - 2\pi$

$= \boxed{-10 - 2\pi}$

2. Given $f(x) = 4x - \frac{1}{2}x^2$. Use the Riemann Sums indicated to find the approximation of the area under the curve.

$= \frac{1}{2}x(8-x)$

(a) Left-hand Riemann sum on the interval $[2, 8]$, using 3 equal subintervals.



$\Delta x = \frac{8-2}{3} = \frac{6}{3} = 2$

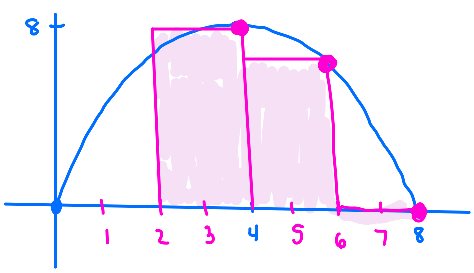
$S_{RL} = 2\left(f(2) + f(4) + f(6)\right)$

$= 2(6 + 8 + 6)$

$= 2(20)$

$= \boxed{40}$

(b) Right-hand Riemann sum on the interval $[2, 8]$, using 3 equal subintervals.



$S_{2R} = 2\left(f(4) + f(6) + f(8)\right)$

$= 2(8 + 6 + 0)$

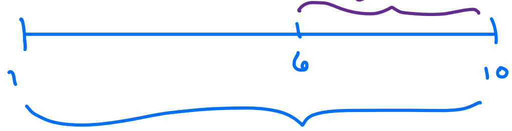
$= 2(14)$

$= \boxed{28}$

$$\int_6^{10} f(x)\,dx = 25$$

3. Given that $f(x)$ is a continuous function, $\int_1^{10} f(x)\,dx = -4$, and $\int_{10}^{6} f(x)\,dx = -25$, find $\int_1^{6} (7 + f(x))\,dx$.

$$\int_6^{10} f(x)\,dx = 25$$



$$\int_1^{10} f(x)\,dx = -4$$

$$\int_1^{10} f(x)\,dx = \int_1^{6} f(x)\,dx + \int_6^{10} f(x)\,dx$$

$$-4 = \int_1^{6} f(x)\,dx + 25$$

$$-29 = \int_1^{6} f(x)\,dx$$

$$\int_1^{6} (7 + f(x))\,dx$$

$$= \int_1^{6} 7\,dx + \int_1^{6} f(x)\,dx$$

$$= 7x \Big]_1^{6} - 29$$

$$= 7(6) - 7(1) - 29 = 42 - 7 - 29$$

$$= 42 - 36 = \boxed{6}$$

4. For a continuous and differentiable function $f(x)$, if $\int_a^b f'(x)\,dx = 17$ and $f(b) = 9$, find $f(a)$.

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$
$$17 = 9 - f(a)$$
$$8 = -f(a)$$
$$\boxed{-8 = f(a)}$$

5. Evaluate $\int_1^4 \left(3e^x - \frac{6}{\sqrt{x}}\right)\,dx$.

$$\int_1^4 \left(3e^x - 6x^{-1/2}\right)dx = 3e^x - 6(2)x^{1/2}\Big]_1^4$$
$$= 3e^x - 12\sqrt{x}\Big]_1^4$$
$$= 3e^4 - 12\sqrt{4} - (3e^1 - 12\sqrt{1}$$
$$= 3e^4 - 24 - 3e + 12$$
$$= \boxed{3e^4 - 3e - 12}$$

6. Find $\int_0^3 x\sqrt[3]{8+x^2}\,dx$.

$u = 8 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

- when $x = 0$
  $u = 8 + 0^2 = 8$
- when $x = 3$
  $u = 8 + 3^2 = 8 + 9$
  $= 17$

$$\int_8^{17} \frac{1}{2}u^{1/3}\,du$$
$$= \frac{1}{2}\cdot\frac{3}{4}u^{4/3}\Big]_8^{17}$$
$$= \frac{3}{8}(17)^{4/3} - \frac{3}{8}(8)^{4/3}$$
$$= \frac{3}{8}(17)^{4/3} - \frac{3}{\cancel{8}}(\cancel{16}\,2)$$
$$= \boxed{\frac{3}{8}(17^{4/3}) - 6 \approx 10.39}$$

OR

$$\int \frac{1}{2}u^{1/3}\,du$$
$$= \frac{1}{2}\cdot\frac{3}{4}u^{4/3}$$
$$= \frac{3}{8}(8+x^2)^{4/3}\Big]_0^3$$
$$= \frac{3}{8}(8+3^2)^{4/3} - \frac{3}{8}(8+0^2)^{4/3}$$
$$= \frac{3}{8}\cdot(17)^{4/3} - \frac{3}{8}(8)^{4/3}$$

same

7. Find $\int_0^2 x^2 e^{x^3+1} dx$

$u = x^3 + 1$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

when $x = 0$
$u = 0^3 + 1 = 1$

when $x = 2$
$u = 2^3 + 1 = 8 + 1$
$= 9$

$\int_1^9 \frac{1}{3} e^u du$

$= \frac{1}{3} e^u \Big]_1^9$

$= \boxed{\frac{1}{3} e^9 - \frac{1}{3} e}$

OR

$\int \frac{1}{3} e^u du$

$= \frac{1}{3} e^u$

$= \frac{1}{3} e^{x^3+1} \Big]_0^2$

$= \frac{1}{3} e^{2^3+1} - \frac{1}{3} e^{0^3+1}$

same $= \frac{1}{3} e^9 - \frac{1}{3} e^1$

8. The marginal cost function for a company is given by $C'(x) = 20x + \dfrac{26}{\sqrt{13x + 160}}$ dollars per item, where x is the number of items produced. Calculate the change in total cost when production increases from 36 to 50 items. Round your answer to the nearest cent.

$$C(50) - C(36) = \int_{36}^{50} C'(x)\, dx$$

$$= \int_{36}^{50} \left(20x + \frac{26}{\sqrt{13x + 160}} \right) dx$$

in calculator....

$\boxed{\$12053.63}$

9. Find $\int_1^b \frac{2}{x^2}\,dx$, where b is a real number and $b > 1$?

$$\int_1^b 2x^{-2}\,dx = -2x^{-1}\Big]_1^b = \frac{-2}{x}\Big]_1^b = \frac{-2}{b} - \frac{-2}{1}$$

$$= \boxed{\frac{-2}{b} + 2}$$

10. Locate the values where $f(x) = x^3 - 3x^2 - 9x + 5$ attains the absolute maximum and absolute minimum on the interval

$$f'(x) = 3x^2 - 6x - 9 \qquad x = 3, -1$$
$$= 3(x^2 - 2x - 3)$$
$$= 3(x-3)(x+1)$$
$$0 = 3(x-3)(x+1)$$

(a) $[-2, 4]$

x	$f(x)$
-2	3
-1	10 ← abs max
3	-22 ← abs min
4	-15

- The absolute max is 10 at $x = -1$.
- The absolute min is -22 at $x = 3$.

(b) $(-5, 4]$

x	$f(x)$
-5	-150 * open circle No abs. min
-1	10
3	-22
4	-15

- The absolute max is 10 at $x = -1$.
- No absolute min because of open circle at $(-5, -150)$.

11. Determine two positive numbers whose sum is 12 and such that the product of one of them and the square of the other is a maximum.

① maximize product

② Let $x, y > 0$.

③ objective function: $xy^2 = M$

④ Constraint: $x + y = 12$

⑤ objective function in one variable

$$x = 12 - y \longrightarrow (12-y)y^2 = M$$

$$12y^2 - y^3 = M$$

⑥ Interval: $(0, 12)$

Because $x+y=12$ and $x, y > 0$

If $x=0$ then $y=12$.

⑦ Derivative + critical values

$$24y - 3y^2 = \frac{dM}{dy}$$

$$3y(8-y) = 0$$

$$y = 8, \cancel{0} \longleftarrow y > 0,\ y \neq 0$$

⑧ $$\frac{d^2M}{dy} = 24 - 6y$$

when $y = 8$, $24 - 6(8) < 0$

So $y = 8$ is absolute max on $(0, 12)$, per 2nd derivative test.

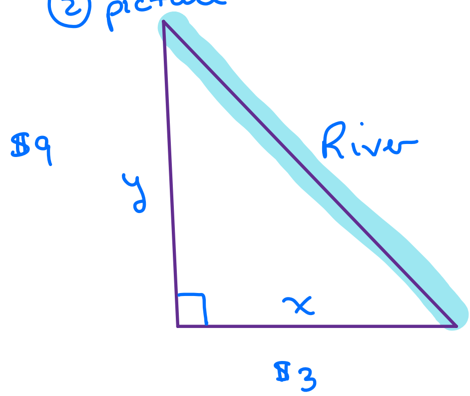
⑨ answer question

$$x = 12 - 8$$
$$= 4$$

$$\boxed{\begin{matrix} x = 4 \\ y = 8 \end{matrix}}$$

12. Bob needs to fence in a right-angled triangular region that will border a (mostly straight) river. The fencing for the left border costs \$9 per foot, and the lower border costs \$3 per foot. Bob doesn't need any fencing along the side of the river. He has \$630 to spend. Find the dimensions of the triangular region that allows Bob to fence as much area as possible, and then find the area of the region? (Hint: Design this region so the river is along the longest side of the triangular region.)

② picture



① maximize area

③ $A = \frac{1}{2}xy$

④ $3x + 9y = 630$ (constraint)

$x, y > 0$

⑤ $3x = 630 - 9y$

$x = \frac{630 - 9y}{3} = 210 - 3y$

⑥ $A = \frac{1}{2}(210 - 3y)y$

$= (105 - \frac{3y}{2})y$

$= 105y - \frac{3}{2}y^2$

⑦ $(0, 70) \longrightarrow$ A must be positive

$A = y\underbrace{(105 - \frac{3}{2}y)}$

$105 - \frac{3}{2}y = 0$

$y = -105(-\frac{2}{3})$

$= 70$

⑧ $\frac{dA}{dy} = 105 - 3y$

$0 = 105 - 3y$

$35 = \frac{-105}{-3} = y$

⑨ $\frac{d^2A}{dy} = -3$ (always c.c. down)

so $y = 35$ is absolute max

⑩ $x = 210 - 3y \longrightarrow x = 210 - 3(35)$

$= 105$

$$\boxed{\begin{array}{l}\text{dimensions} \\ \text{are } 105\text{ft} \times 35 \text{ feet.} \\ \text{area is} \\ \frac{1}{2}(105)(35) = 1837.5 \text{ ft}^2\end{array}}$$

13. Bike Tykes is a company that makes bikes for children. The company's weekly marginal profit function is given by $P'(x) = 30x - 0.3x^2 - 250$ dollars per bike when x bikes are sold. If the profit from selling 9 bikes is \$0 (i.e., the break-even quantity is 9 bikes), find the company's profit when 80 bikes are sold in a week.

$$P(x) = \int P'(x)\,dx = \int (30x - .3x^2 - 250)\,dx$$

$$P(x) = \frac{30}{2}x^2 - \frac{.3}{3}x^3 - 250x + C$$

$$= 15x^2 - .1x^3 - 250x + C$$

$P(9) = 0$, therefore

$$0 = 15(9)^2 - .1(9)^3 - 250(9) + C$$

$$0 = -1107.9 + C$$

$$1107.9 = C$$

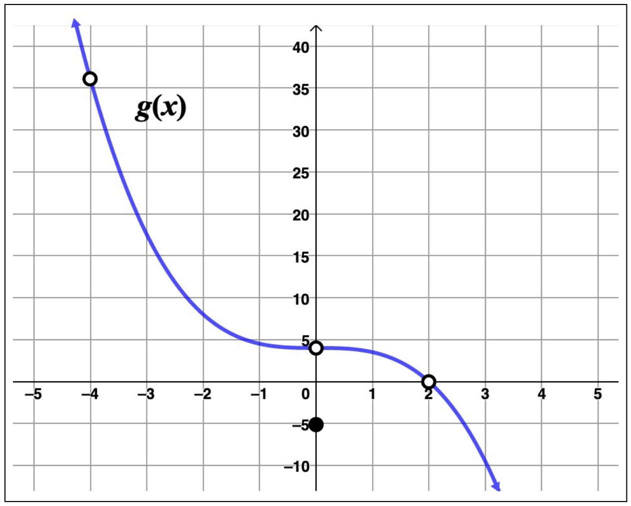
$$P(x) = 15x^2 - .1x^3 - 250x + 1107.9$$

$$P(80) = 15(80)^2 - .1(80)^3 - 250(80) + 1107.9$$

$$= 25907.9$$

$$\boxed{\$25{,}907.90}$$

14. Use the graph of $g(x)$ below to find the absolute extrema of $g(x)$ on $(-4, 2)$.



No absolute max

Absolute min. of -5 at $x=0$.

15. Evaluate $\int (x^2+1)(3x+4)\,dx$

$$\int (3x^3+4x^2+3x+4)\,dx = \frac{3}{4}x^4 + \frac{4}{3}x^3 + \frac{3}{2}x^2 + 4x + C$$

16. To evaluate the indefinite integral $\int \frac{e^{2x}}{(e^{2x}+1)^3}\,dx$ we can use the u-substitution method for integrals. When using this method, which equation is the best choice for u?

(a) $u = e^{2x}$
(b) $u = 2x$
(c) $u = e^{2x} + 1$
(d) $u = (e^{2x} + 1)^3$
(e) $u = x$

17. The indefinite integral $\int \frac{5}{x \ln x}\,dx$ is in terms of x and represents the most general antiderivative with respect to x. To evaluate the integral, we can use the u-substituion method for integrals. After making the substitution for u, which of the following integrals correctly represents the modified integral all in terms of u?

(a) $\int \frac{5}{u \ln u}\,du$
(b) $\int \frac{5}{u}\,du$
(c) $\int \frac{5}{x^2 u}\,du$
(d) $\int \frac{5}{xu}\,du$
(e) $\int \frac{5}{\ln u}\,du$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$5du = \frac{5}{x}\,dx$

$\int \frac{5}{x} \cdot (\ln x)^{-1}\,dx = \int 5u^{-1}\,du$