Math 150 - Week-In-Review 9

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Problem Statements

1. Solve using elimination: $\begin{cases} 4x - 5y = 8 \\ -8x + 10y = -16 \end{cases} \xrightarrow{\times 2} \begin{cases} 8x - 10y = 16 \\ -8x + 10y = -16 \end{cases}$

add $\quad 0 = 0$ (fact)

$\Rightarrow \infty$-ly

∞-ly many answers choose a parameter "t"

if $x = t \Rightarrow y = ? \rightsquigarrow 4t - 5y = 8 \qquad y = \frac{4t-8}{5}$

$(x, y) = (t, \frac{4}{5}t - \frac{8}{5})$

2. Solve using whichever method you choose: $\begin{cases} (x+4)^2 + y^2 = 4 \\ y - \sqrt{x} = 0 \end{cases} \longrightarrow y = \sqrt{x}$

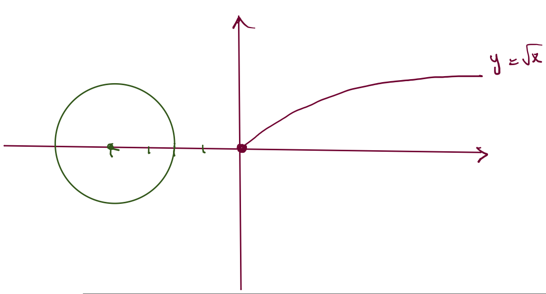
$(x+4)^2 + (\sqrt{x})^2 = 4 \longrightarrow x^2 + 8x + 16 + x = 4$

$x^2 + 9x + 12 = 0$

$x = \frac{-9 - \sqrt{33}}{2} < 0$

$x = \frac{-9 + \sqrt{33}}{2} < 0$

} extraneous since from $\sqrt{x}$ $x \geq 0$



$\Rightarrow$ No solutions!

3. Find all solutions to the system of equations

$$\begin{cases} x^2 + y^2 = 25 \\ xy = 12 \end{cases}$$

$xy = 12 \rightarrow y = \frac{12}{x}$

$x^2 + \frac{144}{x^2} = 25$

$x^4 + 144 = 25x^2$

$(x^2)^2 - 25x^2 + 144 = 0$

$u^2 - 25u + 144 = 0$

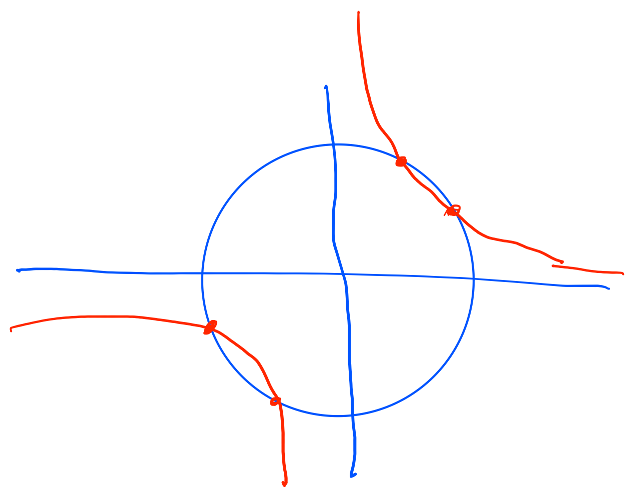
$(u-16)(u-9) = 0 \rightarrow u = 16 \rightarrow x^2 = 16 \rightarrow x = \pm 4$

$u = 9 \rightarrow x^2 = 9 \rightarrow x = \pm 3$

$x = 4 \rightarrow y = 3$
$x = -4 \rightarrow y = -3$
$x = 3 \rightarrow y = 4$
$x = -3 \rightarrow y = -4$

Solutions:

$(4,3)$, $(-4,-3)$, $(3,4)$, $(-3,-4)$



4. Determine all solutions to the system:

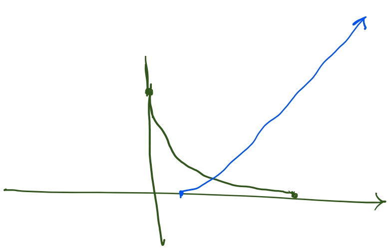
$$\begin{cases} \sqrt{x} + \sqrt{y} = 5 \\ \sqrt{x} - \sqrt{y} = 1 \end{cases}$$

$2\sqrt{x} = 6 \rightarrow \sqrt{x} = 3 \rightarrow x = 9$

$\sqrt{9} + \sqrt{y} = 5$

$3 + \sqrt{y} = 5 \rightarrow \sqrt{y} = 2$

$y = 4$



$(9, 4)$

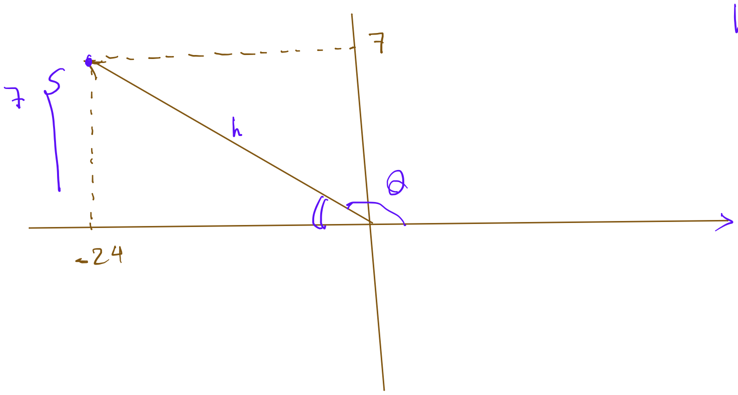
5. Convert 75° to radians.

$$75^\circ = 75 \cdot (1^\circ) = 75\left(\frac{\pi}{180}\text{ rad}\right) = \frac{5\pi}{12} \text{ Radians}$$

6. Convert $\dfrac{19\pi}{12}$ to degrees.

$$\frac{19\pi}{12}\text{ rad} = \frac{19\pi}{12}\cdot(1 \text{ rad}) = \frac{19\cancel{\pi}}{12}\left(\frac{180}{\cancel{\pi}}^\circ\right) = 19 \times 15^\circ = 285^\circ$$

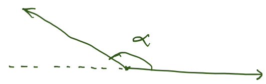
7. Let $(-24, 7)$ be a point on the terminal side of θ. Find the sine, cosine of θ.

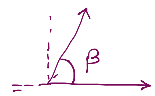


$$h^2 = (-24)^2 + (7)^2$$
$$h^2 = 576 + 49$$
$$h^2 = 625$$
$$h = 25$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp.}} = \frac{-27}{25} \qquad \sin\theta = \frac{\text{opp.}}{\text{hyp}} = \frac{7}{25}$$

8. Let $\alpha = 135°$ and $\beta = 55°$. Sketch α and β. Compute a supplementary angle for α. Compute a complementary angle for β.

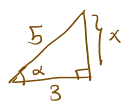




Supplementary angle for α : (means an angle that when added to α, results in $180°$) $\Rightarrow$ $180 - 135 = \boxed{45°}$

Complementary angle for β: (an angle that added to β, results in $90°$) $\Rightarrow$ $90 - 55 = 35°$

9. Suppose α is an acute angle with $\cos(\alpha) = \frac{3}{5}$. Determine $\sin(\alpha)$ and use this to plot α in standard position. State the sine and cosine of the following angles:

using 

$\rightarrow$ $9 + x^2 = 25 \Rightarrow x = 4$ $\Rightarrow$ $\boxed{\sin\alpha = \frac{4}{5}}$

(a) $\theta = \pi + \alpha$

$\sin(\pi + \alpha) = -\sin\alpha = -4/5$

$\cos(\pi + \alpha) = -\cos\alpha = -3/5$

(b) $\theta = 2\pi - \alpha$

$\sin(2\pi - \alpha) = -\sin\alpha = -4/5$

$\cos(2\pi - \alpha) = \cos\alpha = 3/5$

(c) $\theta = 3\pi - \alpha$

$\sin(3\pi - \alpha) = \sin\alpha = 4/5$

$\cos(3\pi - \alpha) = -\cos\alpha = -3/5$

(d) $\theta = 2\pi + \alpha$

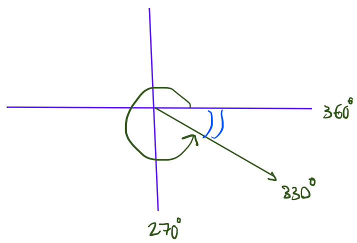
$\sin(2\pi + \alpha) = \sin\alpha = 4/5$

$\cos(2\pi + \alpha) = \cos\alpha = 3/5$

10. Find the reference angle for:

(closest degree to x-axis)

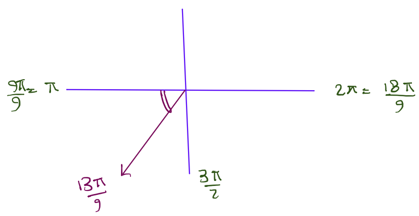
a) $\theta = 330°$



$360° - 330° = 30°$

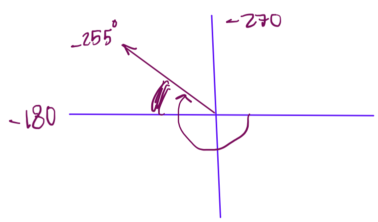
b) $\theta = \dfrac{13\pi}{9}$

$\left(\dfrac{13\pi}{9} = \dfrac{26\pi}{18} < \dfrac{3\pi}{2} = \dfrac{27\pi}{18} \right.$



$\dfrac{13\pi}{9} - \pi = \boxed{\dfrac{4\pi}{9}}$

c) $\theta = -255°$.

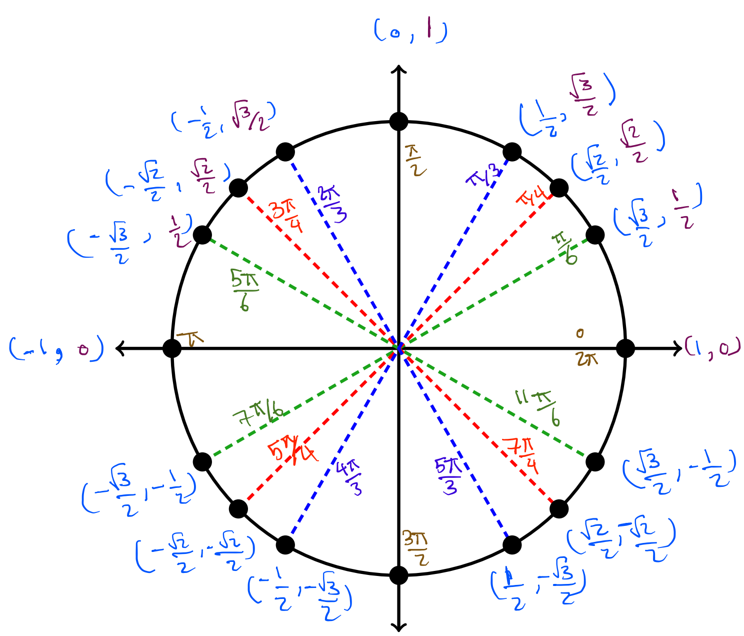


$-255° - (-180°) = -75°$

or

$-255° + 360° = 105°$

$180° - 105° = 75°$



$(\cos x, \sin x)$

11. Evaluate the following:

a) $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

$\sin\left(\pi + \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$

b) $\cos\frac{4\pi}{3} = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$

a) $\sin 315^\circ = \sin(360^\circ - 45^\circ) = \sin(-45^\circ)$

$= -\frac{\sqrt{2}}{2}$

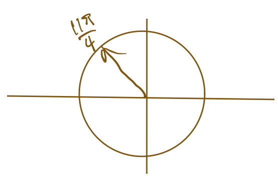
b) $\cos 315^\circ = \cos(-45^\circ) = \cos(45^\circ)$

$= \frac{\sqrt{2}}{2}$

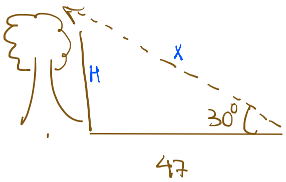
12. Use the reference angle to find the indicated trigonometric value for the specified angles.

(a) $\sin\left(\frac{7\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

(b) $\cos\left(\frac{11\pi}{4}\right) = \cos\left(3\pi - \frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$



13. From a point on the ground 47 feet from the foot of a tree, the angle of elevation of the top of the tree is 30°. Find the height of the tree.



$\cos\theta = \frac{adj}{hyp}$

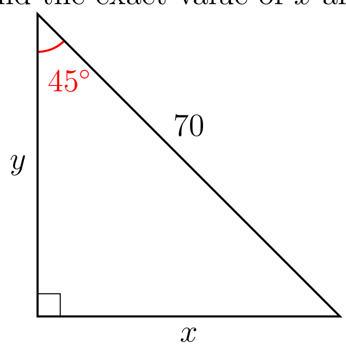
$\cos(30°) = \frac{47}{x}$

$\frac{\sqrt{3}}{2} = \frac{47}{x} \longrightarrow x = \frac{94}{\sqrt{3}}$

$\sin(30°) = \frac{opp.}{hyp.}$

$\frac{1}{2} = \frac{H}{\frac{94}{\sqrt{3}}} \longrightarrow H = \frac{94}{2\sqrt{3}} = \boxed{\frac{47}{\sqrt{3}}}$

14. Find the exact value of x and y.



$\sin(\theta) = \frac{opp.}{hyp.}$

$\sin(45°) = \frac{\sqrt{2}}{2}$

$\Downarrow$ ∥

$\frac{x}{70} = \frac{\sqrt{2}}{2} \longrightarrow x = \frac{70\sqrt{2}}{2} = 35\sqrt{2}$

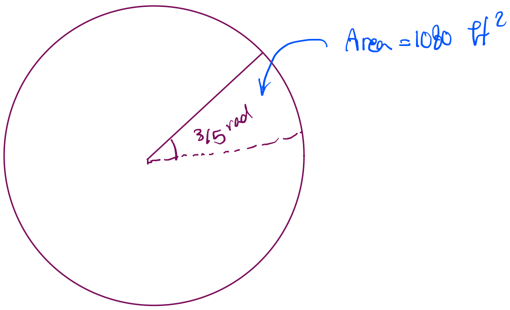
$\cos(45°) = \frac{\sqrt{2}}{2}$

∥

$\frac{y}{70}$

$\longrightarrow y = 35\sqrt{2}$

15. A circular sector created by a central angle of $\frac{3}{5}$ radians has an area of 1080 ft^2, determine the radius of the circle.



Area of a Circle $A = \pi r^2$

Area of a sector with angle θ: $A_{sec} = \dfrac{r^2 \cdot \theta}{2}$

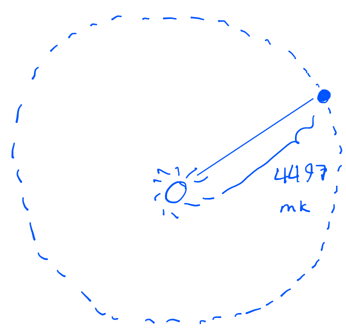
$$1080 = \frac{r^2 \cdot \left(\frac{3}{5}\right)}{2}$$

$$\frac{5}{3} \times 2160 = r^2$$

$$5 \times 720 = r^2$$

$$r = +\sqrt{3600} = +60$$

16. The planet Neptune has an orbit that is nearly circular. It orbits the Sun at a distance of 4497 million kilometers and completes one revolution every 165 years. How long is a full path of Nepture around the Sun? Then find the linear velocity of Neptune as it orbits the Sun.



full path $\Rightarrow$ one revolution

Circumference of circle $2\pi r$

$$C = 2\pi r$$
$$= 2\pi(4497) = 8894\pi \text{ million km}$$

$$\text{linear Velocity} = \frac{\text{distance traveled}}{\text{time passed}} = \frac{8894\pi}{165} = \frac{8894}{165}\pi \text{ million km/year}$$