$\boxed{\cos(-u) = \cos u}$

1. Evaluate the integral

a) $\displaystyle\int_0^{\pi/12} \sin(\underbrace{3x-2}_{u})\, \underbrace{dx}_{\frac{du}{3}}$

$u = 3x-2$
$du = 3dx \Rightarrow dx = \frac{du}{3}$
$x=0 \to u = 3\cdot(0)-2 = -2$
$x = \frac{\pi}{12} \to u = \frac{\pi}{12}\cdot 3 - 2 = \frac{\pi}{4} - 2$

$$= \int_{-2}^{\pi/4-2} \sin u \, \frac{du}{3} = \frac{1}{3}\left(-\cos u\right)\Big|_{-2}^{\pi/4-2}$$

$$= -\frac{1}{3}\left(\cos\left(\frac{\pi}{4}-2\right) - \cos(-2)\right)$$

$$= \boxed{-\frac{1}{3}\left(\cos\left(\frac{\pi}{2}-2\right) - \cos 2\right)}$$

b) $\displaystyle\int x(4x^2+1)^6\, dx$

$u = 4x^2+1$
$du = 8x\,dx \to x\,dx = \frac{du}{8}$

$$= \int u^6 \frac{du}{8} = \frac{1}{8}\cdot\frac{u^7}{7} + C$$

$$= \boxed{\frac{(4x^2+1)^7}{56} + C}$$

c) $\displaystyle\int x^3(x^2+3)^3\, dx$

$$= \int x \underbrace{(x^2)}_{u-3}\underbrace{(x^2+3)}_{u}^3\, dx$$

$u = x^2+3 \to x^2 = u-3$
$du = 2x\,dx \to x\,dx = \frac{du}{2}$

$$= \int (u-3)\, u^3 \frac{du}{2} = \frac{1}{2}\int (u^4 - 3u^3)\, du$$

$$= \frac{1}{2}\left(\frac{u^5}{5} - \frac{3u^4}{4}\right) + C = \boxed{\frac{1}{2}\left(\frac{(x^2+3)^5}{5} - \frac{3(x^2+3)^4}{4}\right) + C}$$

d) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}\, dx \to \frac{dx}{\sqrt{x}} = 2du$

$$= \int \sin u\,(2du) = 2\int \sin u\, du$$

$$= -2\cos u + C = \boxed{C - 2\cos\sqrt{x}}$$

e) $\displaystyle\int_0^1 x^2 e^{x^3}\, dx$

$u = x^3$
$du = 3x^2 dx \to x^2 dx = \frac{du}{3}$
$x=0 \to u = 0^3 = 0$
$x=1 \to u = 1^3 = 1$

$$= \int_0^1 e^u \frac{du}{3} = \frac{1}{3} e^u\Big|_0^1 = \boxed{\frac{1}{3}(e-1)}$$

f) $\displaystyle\int \frac{x + \arcsin x}{\sqrt{1-x^2}}\, dx = \int \frac{x\,dx}{\sqrt{1-x^2}} + \int \frac{\arcsin x}{\sqrt{1-x^2}}\, dx$

$u = 1-x^2$
$du = -2x\,dx$
$x\,dx = -\frac{du}{2}$

$v = \arcsin x$
$dv = \frac{dx}{\sqrt{1-x^2}}$

$$= \int \frac{-\frac{du}{2}}{\sqrt{u}} + \int v\,dv = -\frac{1}{\cancel{2}}\frac{u^{-1/2+1}}{\cancel{-1/2+1}} + \frac{v^2}{2} + C$$

$$= -u^{1/2} + \frac{v^2}{2} + C = \boxed{-\sqrt{1-x^2} + \frac{\arcsin^2 x}{2} + C}$$

g) $\displaystyle\int \frac{2x^2+4x}{x^3+3x^2-4}\, dx = \int \frac{2(x^2+2x)}{x^3+3x^2-4}\, dx$

h) $\displaystyle\int \frac{e^x}{e^x+1}\, dx$

$u = e^x+1$

g) $\int \frac{2x^2+4x}{x^3+3x^2-4}dx = \int \frac{2(x^2+2x)\,dx}{x^3+3x^2-4}$

$u = x^3+3x^2-4$
$du = (3x^2+6x)dx$
$du = 3(x^2+2x)dx \longrightarrow (x^2+2x)dx = \frac{du}{3}$

$= 2\int \frac{\frac{du}{3}}{u} = \frac{2}{3}\ln|u|+C = \boxed{\frac{2}{3}\ln|x^3+3x^2-4|+C}$

h) $\int \frac{e^x}{e^x+1}dx$

$u = e^x+1$
$du = e^x dx$

$= \int \frac{du}{u} = \ln|u|+C$
$= \ln(e^x+1)+C$

i) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x$
$du = \frac{dx}{x}$
$e \to u = \ln e = 1$
$e^4 \to u = \ln e^4 = 4$

$= \int_1^4 \frac{du}{\sqrt{u}}$

$= \frac{u^{1/2}}{1/2}\Big|_1^4 = 2\sqrt{u}\Big|_1^4 = 2(\sqrt{4}-1)$
$= 2(2-1)$
$= \boxed{2}$

j) $\int \frac{x\,dx}{\sqrt{1+x^4}} = \int \frac{x\,dx}{\sqrt{1+(x^2)^2}}$

$u = x^2$
$du = 2x\,dx \longrightarrow x\,dx = \frac{du}{2}$

$= \int \frac{\frac{du}{2}}{\sqrt{1+u^2}} = \frac{1}{2}\int \frac{du}{\sqrt{1+u^2}} = \frac{1}{2}\ln|u+\sqrt{1+u^2}|+C$

$= \boxed{\frac{1}{2}\ln|x^2+\sqrt{1+x^2}|+C}$

k) $\int_1^4 \frac{1}{x^2}\sqrt{\frac{1}{x}+1}\,dx$

$u = \frac{1}{x}+1$
$du = -\frac{dx}{x^2}$
$x=1 \longrightarrow u = \frac{1}{1}+1 = 2$
$x=4 \longrightarrow u = \frac{1}{4}+1 = \frac{5}{4}$

$= -\int_2^{5/4} \sqrt{u}\,du = \int_{5/4}^2 \sqrt{u}\,du$

$= \left(\frac{u^{3/2}}{3/2}\right)\Big|_{5/4}^2 = \frac{2}{3}\left(2^{3/2} - \left(\frac{5}{4}\right)^{3/2}\right)$

$= \frac{2}{3}\left(2\sqrt{2} - \frac{5}{4}\sqrt{\frac{5}{4}}\right)$

$= \boxed{\frac{2}{3}\left(2\sqrt{2} - \frac{5}{8}\sqrt{5}\right)}$

$u = \tan(\cos x)$

l) $\int \tan x \ln(\cos x)\,dx = \int \frac{\sin x}{\cos x}\ln(\cos x)dx$

$u = \cos x$
$du = -\sin x\,dx$

$= -\int \frac{\ln u}{u}du$

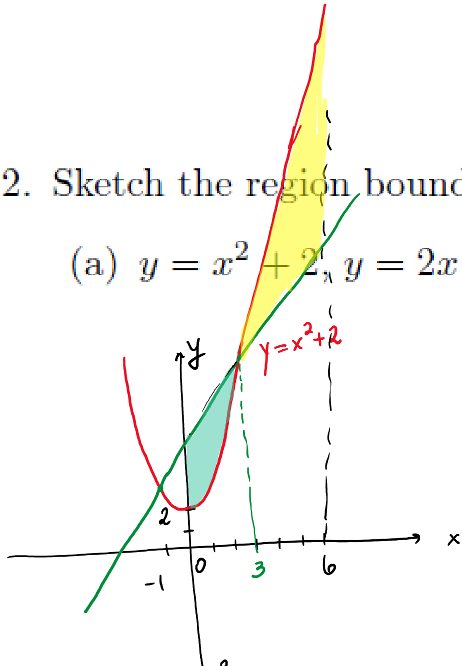
$\ln u = v$
$dv = \frac{du}{u}$

$= -\int v\,dv = -\frac{v^2}{2}+C = -\frac{\ln^2 u}{2}+C$

$= \boxed{-\frac{\ln^2(\cos x)}{2}+C}$

2. Sketch the region bounded by the given curves and find the area of the region.

(a) $y = x^2 + 2$, $y = 2x + 5$, $x = 0$, $x = 6$.



Points of intersection

$x^2+2 = 2x+5$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x_1 = 3, \quad x_2 = -1$

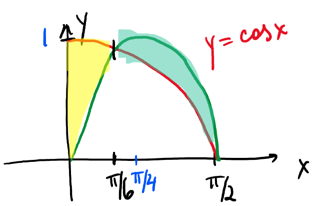
$$A = \int_0^3 \left[(2x+5) - (x^2+2)\right] dx + \int_3^6 \left[(x^2+2) - (2x+5)\right] dx$$

$$= \int_0^3 \left(2x - x^2 + 3\right) dx + \int_3^6 \left(x^2 - 2x - 3\right) dx$$

$$= \left(x^2 - \frac{x^3}{3} + 3x\right)\Big|_0^3 + \left(\frac{x^3}{3} - x^2 - 3x\right)\Big|_3^6$$

$$= \cancel{9} - \cancel{9} + 9 + \frac{216}{3} - 36 - 18 - \cancel{9} + \cancel{9} + 9 = \boxed{36}$$

(b) $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$.



Point of intersection

$\cos x = \sin 2x$

$\boxed{\sin 2x = 2 \sin x \cos x}$

$\cos x = 2 \sin x \cos x$

$2 \sin x \cos x - \cos x = 0$

$\cos x (2 \sin x - 1) = 0$

$\cos x = 0$ or $2 \sin x - 1 = 0$

$\boxed{x = \pi/2}$ $\quad \sin x = \frac{1}{2} \rightarrow \boxed{x = \frac{\pi}{6}}$

$$A = \int_0^{\pi/6} (\cos x - \sin 2x)\, dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\, dx$$

$$= \int^{\pi/6} \cos x\, dx - \int_0^{\pi/6} \sin 2x\, dx + \int_{\pi/6}^{\pi/2} \sin 2x\, dx \quad \int_{\pi/6}^{\pi/2} \cos x\, dx$$

$$= \int_0^{\pi/6} \cos x\, dx - \int_0^{\pi/6} \sin 2x\, dx + \int_{\pi/6}^{\pi/2} \sin 2x\, dx \quad \int_{\pi/6} \cos x\, dx$$

$u = 2x$

$du = 2dx \Rightarrow dx = \frac{du}{2}$

$x = 0 \to u = 2(0) = 0$

$x = \frac{\pi}{6} \to u = 2 \cdot \frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{\pi}{2} \to u = 2 \cdot \frac{\pi}{2} = \pi$

$$= \sin x \Big|_0^{\pi/6} - \frac{1}{2}\int_0^{\pi/3} \sin u\, du + \frac{1}{2}\int_{\pi/3}^{\pi} \sin u\, du - \sin x \Big|_{\pi/6}^{\pi/2}$$

$$= \overset{1/2}{\cancel{\sin\frac{\pi}{6}}} - \overset{0}{\cancel{\sin 0}} + \frac{1}{2}\cos u \Big|_0^{\pi/3} - \frac{1}{2}\cos u \Big|_{\pi/3}^{\pi} - \overset{1}{\cancel{\sin\frac{\pi}{2}}} + \overset{1/2}{\cancel{\sin\frac{\pi}{6}}}$$

$$= \cancel{\frac{1}{2}} + \frac{1}{2}\left(\overset{1/2}{\cancel{\cos\frac{\pi}{3}}} - \overset{1}{\cancel{\cos 0}}\right) - \frac{1}{2}\left(\overset{-1}{\cancel{\cos\pi}} - \overset{1/2}{\cancel{\cos\frac{\pi}{3}}}\right) - \cancel{\frac{1}{2}}$$

$$= \frac{1}{2}\left(-\frac{1}{2}\right) - \frac{1}{2}\left(-\frac{3}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

(c) $x + y^2 = 2$, $x + y = 0$.

$x = 2 - y^2$, $\quad x = -y$

integrate for y.

Points of intersection

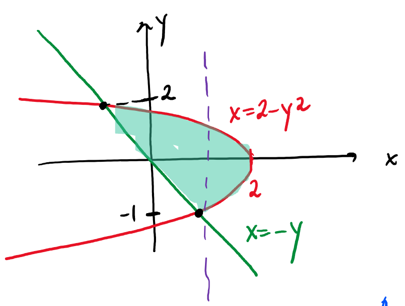
$2 - y^2 = -y$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$.

$y_1 = 2$, $\quad y_2 = -1$.

$-1 \le y \le 2$



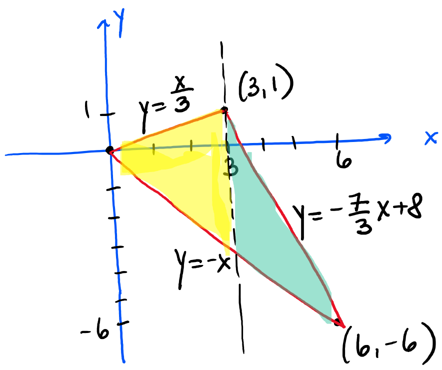
$$A = \int_{-1}^{2} \Big[\overset{[\text{right}]-[\text{left}]}{2 - y^2 - (-y)}\Big]\, dy = \int_{-1}^{2} \left(2 - y^2 + y\right) dy$$

$$= \left(2y - \frac{y^3}{3} + \frac{y^2}{2}\right)\Big|_{-1}^{2}$$

$$= 4 - \frac{8}{3} + 2 + 2 - \frac{1}{3} - \frac{1}{2}$$

$$= 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \boxed{\frac{9}{2}}$$

3. Find the area of the triangle with vertices $(0,0)$, $(3,1)$, $(6,-6)$.



line from (3,1) to (6,-6)

slope is $\frac{-6-1}{6-3} = -\frac{7}{3}$

$$y-1 = -\frac{7}{3}(x-3)$$

$$y-1 = -\frac{7}{3}x + 7$$

$$y = -\frac{7}{3}x + 8$$

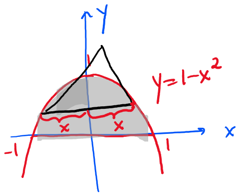
$$A = \int_0^3 \left[\frac{x}{3} - (-x)\right] dx + \int_3^6 \left[-\frac{7}{3}x + 8 - (-x)\right] dx$$

$$= \int_0^3 \frac{4x}{3}\, dx + \int_3^6 \left(8 - \frac{4}{3}x\right) dx$$

$$= \frac{\cancel{4}^{\,2}}{3}\frac{x^2}{\cancel{2}}\Big|_0^3 + \left(8x - \frac{\cancel{4}^{\,2}}{3}\frac{x^2}{\cancel{2}}\right)\Big|_3^6$$

$$= \frac{2}{3}(9-0) + 8(6-3) - \frac{2}{3}(36-9) = \dots$$

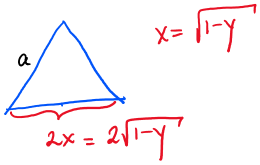
4. Find the volume of the solid S whose base is a region bounded by the parabola $y = 1 - x^2$ and the x-axis, and cross sections perpendicular to the y-axis are equilateral triangles.



$V = \int_a^b A(y)\,dy$

$0 \le y \le 1$

$A(y) = \frac{\sqrt{3}}{4}\left[2\sqrt{1-y}\right]^2 = \sqrt{3}(1-y)$

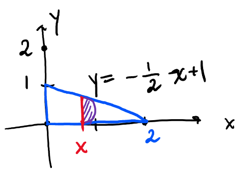


$\boxed{A = \frac{\sqrt{3}}{4}a^2}$

$V = \int_0^1 \sqrt{3}(1-y)\,dy = \sqrt{3}\left(y - \frac{y^2}{2}\right)\Big|_0^1$

$= \sqrt{3}\left(1 - \frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$

5. Find the volume of the solid S whose base is the triangular region with vertices (0,0), (2,0), (0,1), and cross sections perpendicular to the x-axis are semicircles.



$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{D}{2}\right)^2 = \frac{\pi}{8}D^2$

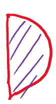
$0 \le x \le 2.$

$V = \int_0^2 A(x)\,dx = \frac{\pi}{8}\int_0^2 \left(1 - \frac{1}{2}x\right)^2 dx = \frac{\pi}{8}\int_0^2 \left(1 - x + \frac{1}{4}x^2\right) dx$

$A(x) = \frac{\pi}{8}\left(1 - \frac{1}{2}x\right)^2$

$= \frac{\pi}{8}\left(x - \frac{x^2}{2} + \frac{x^3}{12}\right)\Big|_0^2$

$= \frac{\pi}{8}\left(2 - 2 + \frac{8}{12}\right) = \boxed{\frac{\pi}{12}}$

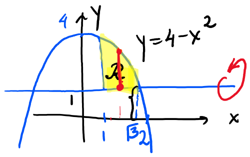


$D = \left(1 - \frac{1}{2}x\right)$

$$V_{ox} = \pi \int_a^b \left[r_{out}^2 - r_{in}^2 \right] dx \quad , \quad V_{oy} = \pi \int_c^d \left[r_{out}^2 - r_{in}^2 \right] dy.$$

6. Set up the integrals to find the volume of the solid obtained by rotating the region bounded by the curves $y = 4 - x^2$, $y = 1$, $x = 1$, $x = \sqrt{3}$ about the indicated lines using the method of disks/washers.

(a) about the line $y = 1$



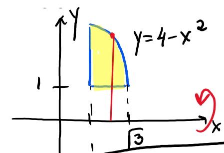
$r_{in} = 0$

$r_{out} = 4 - x^2 - 1 = 3 - x^2$

$1 \le x \le \sqrt{3}$

$$\boxed{V = \pi \int_1^{\sqrt{3}} (3 - x^2)^2 \, dx}$$

(b) about the x- axis



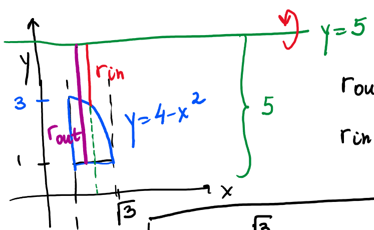
dx, $1 \le x \le \sqrt{3}$

$r_{in} = 1$

$r_{out} = 4 - x^2$

$$\boxed{V = \pi \int_1^{\sqrt{3}} \left[(4 - x^2)^2 - 1 \right] dx}$$

(c) about the line $y = 5$



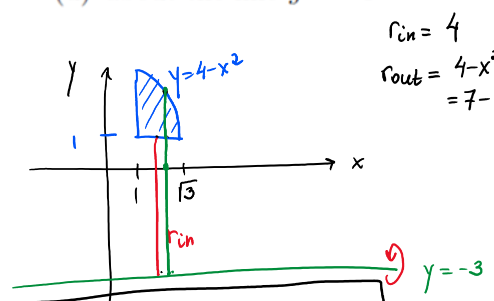
$r_{out} = 4$

$r_{in} = 5 - (4 - x^2)$

$= 1 + x^2$

$$\boxed{V = \pi \int_1^{\sqrt{3}} \left(16 - (1 + x^2)^2 \right) dx}$$

(d) about the line $y = -3$



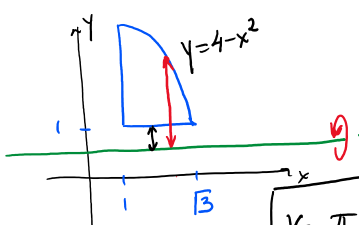
$r_{in} = 4$

$r_{out} = 4 - x^2 + 3$

$= 7 - x^2$

$$\boxed{V = \pi \int_1^{\sqrt{3}} \left[(7 - x^2)^2 - 16 \right] dx}$$

(e) about the line $y = 0.5$



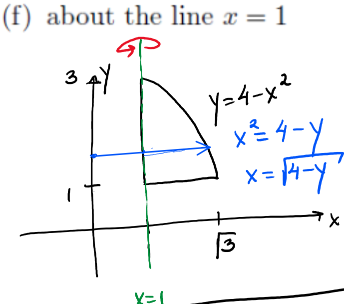
$r_{in} = 0.5$

$r_{out} = 4 - x^2 - 0.5$

$= 3.5 - x^2$

$$\boxed{V = \pi \int_1^{\sqrt{3}} \left[(3.5 - x^2)^2 - 0.25 \right] dx}$$

(f) about the line $x = 1$, dy, $1 \le y \le 3$

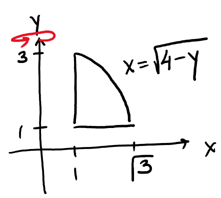


$r_{in} = 0$

$r_{out} = \sqrt{4-y} - 1$

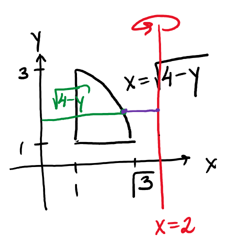
$$\boxed{V = \pi \int_1^3 \left(\sqrt{4-y} - 1 \right)^2 dy}$$

(g) about the y- axis



$r_{in} = 1$, $r_{out} = \sqrt{4-y}$

(h) about the line $x = 2$



$V = \pi \int_1^3 \left[(\sqrt{4-y})^2 - 1 \right] dy$

$\boxed{= \pi \int_1^3 (3 - y)\, dy}$

$r_{out} = 1$

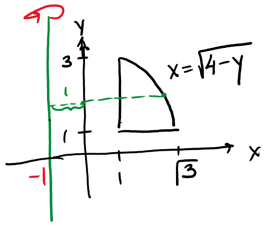
$r_{in} = 2 - \sqrt{4-y}$

$\boxed{V = \pi \int_1^3 \left[1 - \left(2 - \sqrt{4-y}\right)^2 \right] dy}$

(i) about the line $x = -1$.

$r_{in} = 2$

$r_{out} = 1 + \sqrt{4-y}$

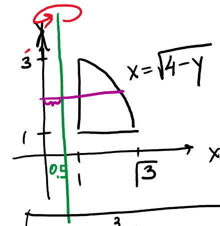


$\boxed{V = \pi \int_1^3 \left[\left(1 + \sqrt{4-y}\right)^2 - 4 \right] dy}$

(j) $x = 0.5$

$r_{in} = 0.5$

$r_{out} = \sqrt{4-y} - 0.5$



$\boxed{V = \pi \int_1^3 \left[\left(\sqrt{4-y} - 0.5\right)^2 - 0.25 \right] dy}$