2024 Fall Math 140 Week-In-Review

Week 3: Sections 2.3-2.4

Section 2.3 and 2.4: Systems of Two Equations in Two Unknowns and Setting Up and Solving Systems of Linear Equations

Some Key Words and Terms: Number and Types of Solutions to Systems, Independent System, Inconsistent System, Dependent System, Solving with Substitution, Solving with the Addition/Elimination Method, Solving with Matrices, Augmented Matrix, Reduced Row Echelon Form Matrix, Parametric Solution, Break-Even Point, Equilibrium Point

Solutions to Systems:

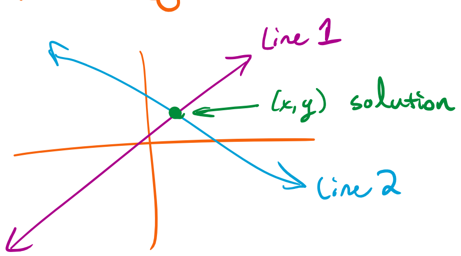
Independent System $\longleftrightarrow$ Exactly 1 solution
(x,y) (x,y,z) etc...

Inconsistent System $\longleftrightarrow$ No (zero) solutions

Dependent System $\longleftrightarrow$ Infinite Solutions
"parameterize"

Independent System:

Graphically:

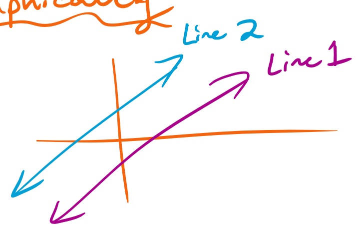


☆ the lines have different slopes ☆

Given a system of 2 equations w/ 2 variables, if the slopes of the lines are different there is exactly one solution (independent system)

Inconsistent System:

Graphically:

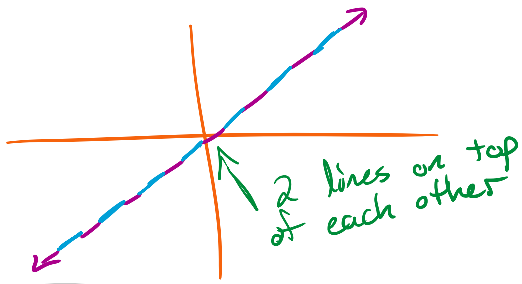


☆ the lines have same slope & different y-intercepts ☆

Given a system of 2 equs. w/ 2 variables, if slopes equal but y-intercepts don't then no solutions (inconsistent system)

Dependent System:

Graphically:



☆ the lines have same slope & same y-intercept ☆

Given a system of 2 equs. w/ 2 variables if the lines have the same slope & y-intercept then infinite solutions (dependent system)

Parametric Solution:

For dependent systems (∞ solutions) we take all the variables & put them it terms of a common variable "the parameter", usually t.

$(x, y) \rightarrow (\underset{w/t}{\_\_\_}, \underset{w/t}{\_\_\_})$ $\underbrace{\text{"where } t \text{ is any real number"}}_{\text{need this for the } \infty \text{ solution}}$

Solving Systems: For this course, we implement 1 of 3 strategies:

1) Substitution Method (solve one equation for one variable then substitute that into the other equation)
2) Addition/Elimination Method (multiply one or both equations by a constant so that one of the variables eliminate when we add the equations)
3) Matrices (setup an augmented matrix & RREF it)

Solving with Substitution:

- Pick one equation & one variable in that equation
- Solve for that variable & substitute for that variable in the other equation
- Simplify & interpret:
  1) we get a number $\rightarrow$ exactly one solution
  2) we get a false statement "$0 = \frac{1}{2}$" $\rightarrow$ no solutions

  3) we get a true statement "$-4 = -4$" $\rightarrow$ ∞ solutions

Solving with Addition/Elimination:

- Pick an equation & one variable from that equation
- Multiply that equation by a # so that the variable will have the opposite coefficient (negative) of the coefficient in the other equation
- Add the two equations & either solve or interpret:
  1) simplifies to something we can solve → exactly one solution
  2) give a false statement → no solutions
  3) give a true statement → ∞ solutions

Solving with Matrices:

- Organize all equations in the same order: variables on left in same order & constant on right
- Setup augmented matrix
- RREF
- Interpret the result

Augmented Matrix:

- the coefficients of the variables & constant
- variables <u>must</u> be in the same order
- constant must be on the right of the equal sign

$$\begin{array}{l} 2x+3y=4 \\ 5x+6y=7 \end{array} \longrightarrow \begin{array}{ccc} x & y & \text{constant} \\ \end{array}$$
$$\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 5 & 6 & 7 \end{array}\right]$$

Reduced Row Echelon Form:

- The first non-zero in a row must be a 1 "leading 1"
- For any "leading 1", the rest of that column must be zeros (above & below the leading 1)

Break-Even Point:

↳ ① Revenue = Cost } solve one of these for x = break-even quantity
↳ ② Profit = 0 }

★ depending on context, this quantity might have to be a whole number ★

Solution: (x, y) → (break-even quantity, cost/revenue @ that quantity)

"cost/revenue/profit" "produced & sold"

Equilibrium Point:

↳ ① Supply = Demand } solve for x = equilibrium quantity

★ also might have to be a whole number ★

Solution: (x, y) → (equilibrium quantity, equilibrium price)

"supply/demand" "producers & consumers"

Examples:

1. For the system of linear equations shown below, determine the type of system and the number of solutions. Do not solve the system.

① Compare slopes

② if slopes equal, compare b-values

$$5y + 20 = 3x$$
$$21x - 147 = 35y$$

$5y + 20 = 3x$

$5y = 3x - 20$

$y = \frac{3}{5}x - 4$

$m_1 = 3/5 \quad b_1 = -4$

$21x - 147 = 35y$

$\frac{3}{5}x - \frac{21}{5} = y$

$m_2 = 3/5 \quad b_2 = -\frac{21}{5}$

since $m_1 = m_2$ & $b_1 \neq b_2$, then it is an incosistent system and has no solution

2. Solve the given system of equations using the Substituion Method.

$$2x + 5y = 8$$
$$-3x + 6 = 3y$$

pick one variable from one equation

★ smallest coefficient ★

$2x + 5y = 8$

$2x = 8 - 5y$

$x = 4 - \frac{5}{2}y$

$-3(4 - \frac{5}{2}y) + 6 = 3y$

$3 \to \frac{6}{2}$

$-12 + \frac{15}{2}y + 6 = 3y$

$-6 = 3y - \frac{15}{2}y$

$-\frac{2}{\cancel{9}_3} \cdot (-\cancel{6}^{-2}) = (-\frac{9}{2}y) \cdot -\frac{2}{9}$

$\frac{4}{3} = y$

$x = 4 - \frac{5}{\cancel{2}_1}(\frac{\cancel{4}^2}{3})$

$x = 4 - \frac{10}{3}$

$x = 2/3$

$\boxed{\text{Solution } (x,y) = (2/3, 4/3)}$

"Independent System"

3. Solve the given system of equations using the Addition/Elimination Method.

$$8y + 12 = 4x$$
$$\frac{1}{2}x = y + \frac{3}{2}$$

Multiply one equation by a # so that the variables cancel from the two equations

★ Reorder

$-4x + 8y = -12$

$(\frac{1}{2}x - y = \frac{3}{2}) \cdot 8$

$-4x + 8y = -12$

$+ \; 4x - 8y = 12$

$0x + 0y = 0$

$0 = 0$ True Statement $\to \infty$ solutions $\to$ parameterize

the two equations are the same, just different versions

★ w/ 2 variables, set $\underline{y = t}$ & substitute, then solve for x ★

$8t + 12 = 4x$

so $2t + 3 = x$

$\boxed{(x,y) = (2t+3, t) \text{ where } t \text{ is any real number}}$

4. Solve the given system of equations using Matrices. → all equations in the same order

$$1.25y + 3 = 0.5x$$
$$4x = 7 + 10y$$

$-0.5x + 1.25y = -3$

$4x - 10y = 7$

$$\rightarrow \begin{array}{ccc} x & y & \text{constant} \\ \end{array} \underbrace{\left[\begin{array}{cc|c} -0.5 & 1.25 & -3 \\ 4 & -10 & 7 \end{array}\right]}_{2\times 3}$$

Now put in calculator ★ double check your entries ★

★ $\boxed{2nd} \rightarrow \boxed{x^{-1}}$

RREF: $\boxed{2nd} \rightarrow \boxed{x^{-1}} \rightarrow$ "Math"

RREF

$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2.5 & 0 \\ 0 & 0 & 1 \end{array}\right] \rightarrow \begin{array}{l} 1x - 2.5y = 0 \\ 0x + 0y = 1 \end{array}$$

$0 = 1$ False

No Solution

5. Solve the given system of equations using Matrices.

$$20x + 8z - 22 = -6y$$
$$-1.5y - 2z = 5x - 5.5$$
$$9z = 11x - 4$$

$20x + 6y + 8z = 22$

$-5x - 1.5y - 2z = -5.5$

$-11x + 0y + 9z = -4$

$$\rightarrow \begin{array}{ccc} x & y & z \end{array} \left[\begin{array}{ccc|c} 20 & 6 & 8 & 22 \\ -5 & -1.5 & -2 & -5.5 \\ -11 & 0 & 9 & -4 \end{array}\right]$$

RREF (Math → Frac → Enter)

$$\left[\begin{array}{ccc|c} 1 & 0 & -9/11 & 4/11 \\ 0 & 1 & 134/33 & 27/11 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{array}{l} x - \frac{9}{11}z = \frac{4}{11} \\ y + \frac{134}{33}z = \frac{27}{11} \\ 0 = 0 \end{array}$$

parameterize w/ 3 variables we choose "t" to be the variable that is present in all equations

★ choose $z = t$

$x - \frac{9}{11}t = \frac{4}{11}$ & $y + \frac{134}{33}t = \frac{27}{11}$

$x = \frac{9}{11}t + \frac{4}{11}$ $y = \frac{-134}{33}t + \frac{27}{11}$

$$\boxed{(x, y, z) = \left(\frac{9}{11}t + \frac{4}{11}, \frac{-134}{33}t + \frac{27}{11}, t\right) \text{ where } t \text{ is any real number}}$$

6. A new company produces a rare product: "Ultima Vitamins". The function for the total production cost per day is $C(x) = 110x + 55,860$ where x represents the quantity of bottles of Ultima Vitamins and C is measured in dollars. The company sells each bottle of vitamins for $\$355$ each. Determine the daily break-even point for [redacted] Ultima Vitamins for the company and interpret it in the context of the scenario.

$R = C$ or $P = 0$

$C(x) = 110x + 55860$

Revenue = (selling price) $\cdot$ x

$R(x) = 355x$

$110x + 55860 = 355x$

$55860 = 245x$

$x = \dfrac{55860}{245} = 228$

break even quantity & we need the cost/revenue that goes w/ this

$R(228) = 355(228)$
$= \$80,940$

$\boxed{(228, 80940)}$ so when 228 bottles of vitamins are made & sold the company will have a cost & revenue of $80,940 & no profit

7. The linear demand equation for a particular vehicle, the City Cruiser, in a large city, Suburbia, is given by $D(x) = p(x) = -250x + 70000$. The linear supply equation for the same vehicle in the same city is given by $S(x) = p(x) = 165x + 2355$. Determine the equilibrium point for City Cruisers in Suburbia and interpret the equilibrium point in the context of the scenario.

Supply = Demand

$-250x + 70000 = 165x + 2355$
$+250x \quad -2355 \quad +250x \quad -2355$

$67645 = 415x$

$x = \dfrac{67645}{415} = 163$

equilibrium quantity & we need a price to go with this

price = $D(163) = S(163)$

price = $-250(163) + 70000$

price = $29,250$

$\boxed{(163, 29250)}$ so when 163 vehicles are sold at $29,250 each, then consumers and producers are both satisfied and the market is in equilibrium

8. For the following scenario, setup and do not solve a system of linear equations, including defining variable.

Last year, you decided to invest a total of \$65,000 in two different products: Epsilon and Gamma. The percent return on Epsilon was determined to be 8% and the percent return on Gamma was 13%. You made a total return on your investment of \$7,100. How much did the you invest in each product last year?

↳ variables are amount of each investment

x = the amount of money, in dollars, invested in Epsilon

y = the amount of money, in dollars, invested in Gamma

★ return = (% return as decimal) · \$ invested

(investment) $x + y = 65000$

(return) $0.08x + 0.13y = 7100$

9. For the following scenario, setup and do not solve a system of linear equations, including defining variable.

A company makes and sells 3 types of fans: Light-Breeze, Brisk-Draft, and Roaring-Gale. To make each Light-Breeze, it takes 2 units of electronics, 2 units of gears, and 3 units of plastic. To make each Brisk-Draft, it takes 3 units of electronics, 4 units of gears, and 3 units of plastic. To make each Roaring-Gale it takes 5 units of electronics, 5 units of gears, and 4 units of plastic. Each week, the company uses 935 units of electronics, 1055 units of gears, and 960 units of plastic. How many of each fan does the company make each week?

x = number of Light-Breeze fans made & sold

y = number of Brisk-Draft fans made & sold

z = number of Roaring-Gale fans made & sold

(electronics) $2x + 3y + 5z = 935$

(gears) $2x + 4y + 5z = 1055$

(plastic) $3x + 3y + 4z = 960$

10. The following augmented matrix shows the solution to a system of linear equations for a company that makes 3 types of plant fertilizer and is determining how much Nitrogen, Phosphorus, and Potassium to order. For this system, x represents the amount of Nitrogen, y represents the amount of Phosphorus, and z represents the amount of Potassium, in pounds. State the solution in the context of the scenario.

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & 108 \\ 0 & 1 & \frac{2}{5} & 80 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left.\begin{array}{l} x + \frac{3}{2}z = 108 \\ y + \frac{2}{5}z = 80 \\ 0 = 0 \end{array}\right\} \infty \text{ solutions}$$

so $z = t$ then $x + \frac{3}{2}t = 108$ $y + \frac{2}{5}t = 80$

$x = -\frac{3}{2}t + 108$ $y = -\frac{2}{5}t + 80$

$(x, y, z) = \left(-\frac{3}{2}t + 108, -\frac{2}{5}t + 80, t\right)$ ★"infinite" solutions w/ real-world context★

Answer: Based the number of pounds of Potassium $(z = t)$ we can have a variable number of pounds of Nitrogen and Phosphorus

★Follow-Up★

11. For the situation above, are there any values of the parameter that should be excluded?

★ x, y & z cannot be negative b/c of context ★

$x \geq 0$ $y \geq 0$ $z \geq 0$

$-\frac{3}{2}t + 108 \geq 0$ $-\frac{2}{5}t + 80 \geq 0$ $t \geq 0$ ★

$+\frac{3}{2}t$ $+\frac{3}{2}t$ $+\frac{2}{5}t$ $+\frac{2}{5}t$

$\frac{2}{3} \cdot (108) \geq \left(\frac{3}{2}t\right) \cdot \frac{2}{3}$ $\frac{5}{2}(80) \geq \left(\frac{2}{5}t\right)\frac{5}{2}$

$72 \geq t$ ★ $200 \geq t$ ★

all 3 must be true simultaneously

so $\boxed{0 \leq t \leq 72}$

Exam 1: Covers sections 1.1, 1.2, 2.1, 2.2, 2.3, and 2.4

Minimum Skills:

- Can you make a line w/ either 2 points or 1 point & slope?
  ★ we might have to "pull out" the two points from a word problem ★
- Can you add/subtract/multiply/transpose matrices?
- Can you solve systems of equations?
  ★ substitution/addition & elimination/matrices
- Translating Word Problems into workable math.

Recommendations: Practice, practice, practice....

- Free textbook (Canvas) w/ questions @ the end of every section
- Another WIR on Tuesday Night
- MLC tutoring session (mlc.tamu.edu)
- Office Hours
- Reworking Quizzes/Group Work that your Prof. wrote