Example 1 (15.6)**.** *Compute* $\iiint_E x^2(y+z)\,dV$*, where* $E = [0,2] \times [0,3] \times [-1,1]$.

Example 2 (15.6)**.** *Compute* $\int_1^2 \int_0^{3z} \int_0^{\ln x} xe^{-y}\,dy\,dx\,dz$.

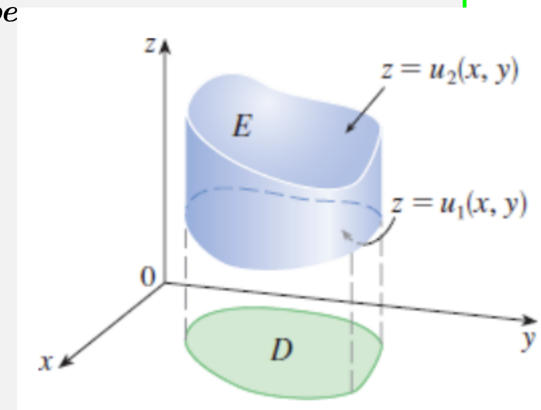
Definition (Type 1 Solid) *A solid region E is of **type 1** if it is of the form*

$$E = \{(x, y, z) \mid (x, y) \in D, u_1(x, y) \le z \le u_2(x, y)\}$$

where D is the projection of E onto the xy-plane.

In this case,

$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\, dz \right] dA$$



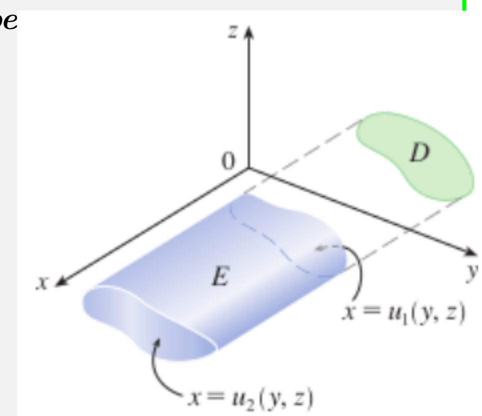
Definition (Type 2 Solid) *A solid region E is of **type 2** if it is of the form*

$$E = \{(x, y, z) \mid (y, z) \in D, u_1(y, z) \le x \le u_2(y, z)\}$$

where D is the projection of E onto the yz-plane.

In this case,

$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x, y, z)\, dx \right] dA$$



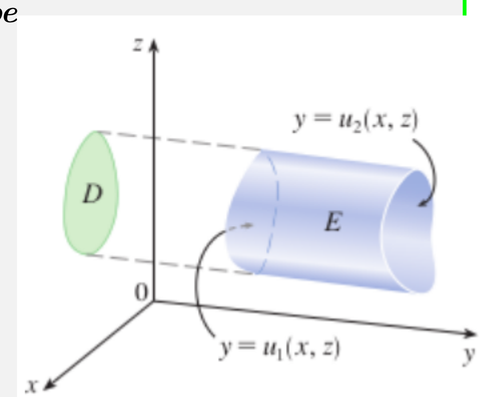
Definition (Type 3 Solid) *A solid region E is of **type 3** if it is of the form*

$$E = \{(x, y, z) \mid (x, z) \in D, u_1(x, z) \le y \le u_2(x, z)\}$$

where D is the projection of E onto the xz-plane.

In this case,

$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x, y, z)\, dy \right] dA$$



Example 3 (15.6). *Consider the triple integral* $\iiint_T 6y\,dV$, *where* T *is the tetrahedron with vertices at* $(0,0,0), (1,0,0), (0,1,0)$ *and* $(0,0,2)$.

(a) Write the iterated integral with all six possible orders.

(b) Evaluate the integral using one of the six orders.

Example 4 (15.6). *Evaluate* $\iiint\limits_E x^2\,dV$, *where* E *is the solid bounded by* $y = x^2$, $z = 0$, $y + z = 6$ *and* $y = 1$.

Example 5 (15.6). *Find the volume of the solid bounded by the parabolic cylinder* $y = x^2$ *and the planes* $z = 0$ *and* $y + z = 4$.

Example 6 (15.6)**.** *Find the volume of the solid bounded by the paraboloids* $z = x^2 + y^2$ *and* $z = 8 - x^2 - y^2$

(a) using a double integral.

(b) using a triple integral.

Example 7 (15.6). *Evaluate* $\iiint\limits_E 2y\,dV$, *where* E *is the solid bounded by the paraboloid* $y = 2x^2 + 2z^2$ *and the plane* $y = 2$.

Example 8 (15.7). *Evaluate* $\iiint\limits_E 2xz\,dV$, *where* E *is the solid in the first octant within the cylinder* $x^2 + y^2 = 1$, *below the plane* $z = 6$ *and above the paraboloid* $z = 1 - x^2 - y^2$.

Example 9 (15.7). *Evaluate the integral by changing into cylindrical coordinates.*

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{0}^{4-x^2-y^2} \sqrt{x^2+y^2}\, dz\, dx\, dy$$

Example 10 (15.7). *(Example 9 from WIR-6) Use cylindrical coordinates to find the volume of the solid between the cone* $z = \sqrt{x^2 + y^2}$ *and the ellipsoid* $2x^2 + 2y^2 + z^2 = 12$.

