2024 Fall Math 140 Week-In-Review

Week 7: Sections 4.2 and 4.4

Some Key Words and Terms: Sample Space, Uniform, Event, Probability, Probability Distribution, Union Rule, Complement Rule, Venn Diagram, Random Variable, Histogram, Expected Value, Fair Game.

Sample Space: Is the set of all outcomes of an experiment.

Roll 6-sided: $S = \{1, 2, 3, 4, 5, 6\}$

Uniform: A sample space has outcomes that are all equally likely aka same probability for each outcome

is uniform if the die is "fair"

Event: Any subset of a sample space for an experiment

$S = \{1, 2, 3, 4, 5, 6\}$ Events: E = an even number rolled

Z = a number 1-3 is rolled

T = any number is rolled

⋮

total # of possible events is 2^n where n is the # of outcomes in the sample space

Probability:

1) $\text{Probability} = \dfrac{\text{\# of favorable results}}{\text{total \# of results}}$ $S = \{1, 2, 3, 4, 5, 6\}$ $P(\text{even number}) = \dfrac{3}{6} = \dfrac{1}{2}$

2) All probability values must be between 0 & 1 (inclusive)

★ $0 \leq \text{probability} \leq 1$ ★

Probability Distribution: Just a table of outcomes & their probabilities.

① Structure:

Outcomes	outcome 1	outcome 2	----
Probabilities	probability 1	probability 2	----

② All probabilities must add to 1!

1

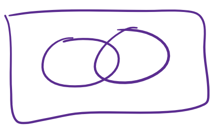
Union Rule: $P(A \cup B) = P(A) + P(B) \underbrace{- P(A \cap B)}$

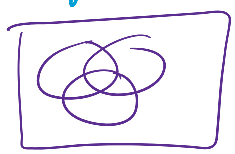
account for any double counting that occurs

★ Note: if A & B are "mutually exclusive", then
$P(A \cup B) = P(A) + P(B)$ → (b/c $A \cap B = \emptyset$ if mutually exclusive)

Complement Rule: Comes from the idea that all probabilities must add to 1 → $P(A) + P(A^c) = 1$ ① $P(A^c) = 1 - P(A)$ OR ② $P(A) = 1 - P(A^c)$

Venn Diagram: Convenient graphical representation of data or probabilities, usually involving two to three different groups.





Random Variable: A variable assigned to specific events in an experiment OR a value for a real-life application

↳ Ex: roll 2 6-sided die & count the number of 5s rolled

$X = 0, 1, 2$ ← the # of 5s I could roll

Ex: "net winnings" OR "insurance premium" etc....

Histogram: a bar graph showing results of an experiment on the horizontal & their probabilities on the vertical. Every histogram can be converted into a probability distribution.

Expected Value: A calculation from a probability distribution

X	x_1	x_2	x_3	x_4
$P(X)$	p_1	p_2	p_3	p_4

all these letters are capitalized

$E(X) = \underbrace{(x_1)(p_1) + (x_2)(p_2) + (x_3)(p_3) + (x_4)(p_4)}$

multiply each column's entries & add them all up

★ if on FRQ, be sure to show setup ★

Fair Game: ① We calculate $E(X)$ for a game using net winnings

② If $E(X) = 0 \rightarrow$ game is fair
If $E(X) \neq 0 \rightarrow$ game is not fair

Examples:

1. For the following experiments, write the sample space and determine if the sample space is uniform or not.

   (a) Flipping a fair coin recording a result of heads or tails.

   $S = \{\text{heads}, \text{tails}\}$, uniform

   (b) Rolling a fair 6-sided die and recording the number rolled.

   $S = \{1, 2, 3, 4, 5, 6\}$, uniform

   ☆ (c) Rolling a fair 10-sided die and recording a result of $\underbrace{1\text{-}6}_{6 \text{ things}}$ as "A" and a result of $\underbrace{7\text{-}10}_{4 \text{ things}}$ as "B".

   $S = \{A, B\}$, not uniform $P(A) = \frac{6}{10} \neq P(B) = \frac{4}{10}$

   (d) Drawing a card from a $\boxed{\text{well-shuffled standard 52}}$ card deck and recording the suit of the card.

   $S = \{\underbrace{\text{clubs, diamonds, hearts, spades}}_{13 \text{ of each in deck}}\}$, uniform

   (e) Spinning a spinner with three equal regions that are blue, green, and orange, and recording the color the spinner lands on.

   $S = \{\text{blue}, \text{green}, \text{orange}\}$, uniform

   (f) Drawing a card from a well-shuffled standard 52 card deck and recording an Ace as a "1" and any other card as a "2".   4 aces , 48 other cards

   $S = \{1, 2\}$, not-uniform

   (g) Rolling a fair 9-sided die and recording an even or odd result.

   ☆ even: 4 results , odd: 5 results ☆

   $S = \{\text{even}, \text{odd}\}$, not-uniform

2. An experiment involves rolling a fair 6-sided die and a fair 4-sided and recording the result of each die.

   (a) Construct a dice chart representing this experiment.

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6

don't double count for part (b)

   (b) Determine the probability of rolling 4 on at least on die.

P(rolling at least one 4) = $\frac{9}{24}$ ← (6)(4) = 24 results

P(rolling at least one 4) = P(4 green) + P(4 purple) − P(rolling 4 on both)

$= \frac{6}{24} + \frac{4}{24} - \frac{1}{24} = \frac{9}{24}$

   (c) Determine the probability of rolling a double.

P(double) = $\frac{4}{24}$ (nothing crazy here)

   (d) Determine the probability of rolling an odd number on the 4-sided die or a 6 on the 6-sided die.

union

"and" & "but"

intersection

P(odd on 4-sided $\cup$ 6 on 6-sided)

= P(odd on 4-sided) + P(6 on 6-sided) − P(both at same time)

$= \frac{12}{24} + \frac{4}{24} - \frac{2}{24} = \frac{14}{24}$

3. A survey is conducted asking respondents which social media platform they use the most between Facebook, Snapchat, and Instagram. They table below shows the results.

	Facebook	Snapchat	Instagram	Total
Under 30	21	46	43	110
31 to 50	46	41	58	145
Over 50	27	8	13	48
Total	94	95	114	303

$\uparrow$ the total # of results

(a) Determine the probability that a respondent is not over 50.

$$P(\underline{\text{not}} \text{ over } 50) = 1 - P(\text{over } 50) = 1 - \frac{48}{303}$$

complement

$$= \boxed{85/101}$$

(b) Determine the probability that a respondent is a person under 30 and uses Facebook.

intersection

$$P(\text{under } 30 \cap \text{Facebook}) = \boxed{\frac{21}{303}}$$

(c) Determine the probability that a respondent uses Instagram or is 31 to 50 years of age.

$\leftarrow$ union

$$P(\text{Instagram} \cup 31 \text{ to } 50) = P(\text{Insta}) + P(31 \text{ to } 50) - P(\text{Insta} \cap 31 \text{ to } 50)$$

$$= \frac{114}{303} + \frac{145}{303} - \frac{58}{303} = \boxed{\frac{201}{303}}$$

(d) Determine the probability that a respondent is under 30 and uses Snapchat, or is over 50 and uses Facebook.

$\uparrow$ indicate a grouping w/ ( )

$$P\left(\underline{(\text{under } 30 \cap \text{Snapchat})} \cup \underline{(\text{over } 50 \cap \text{Facebook})}\right)$$

$P(\text{under } 30 \cap \text{Snapchat}) = \frac{46}{303}$

$P(\text{over } 50 \cap \text{Facebook}) = \frac{27}{303}$

★ is there an intersection between (under 30 ∩ Snapchat) & (over 50 ∩ Facebook)? ★

no b/c these are mutually exclusive

$$P((\text{under } 30 \cap \text{Snapchat}) \cup (\text{over } 50 \cap \text{Facebook})) = \frac{46}{303} + \frac{27}{303} = \boxed{\frac{73}{303}}$$

4. Given the following probability distribution, answer the following.

Outcome	A	B	C	D	E	F
Probability	0.17	0.22	0.08	0.11	0.31	0.11

★ All probabilities add to 1!

Let X be the event A, C, or E occurs; let Y be the event that A, B, or C occurs; and let Z be the event that B or F occurs.

$P(D) = 1 - (\text{all other probabilities})$

$P(D) = 0.11$

5. Determine $P(Z^C)$

$P(Z^c) = P(A) + P(C) + P(D) + P(E)$

$P(Z^c) = \boxed{0.67}$

$X = \{A, C, E\} \rightarrow X^c = \{B, D, F\}$

$Y = \{A, B, C\} \rightarrow Y^c = \{D, E, F\}$

$Z = \{B, F\} \rightarrow Z^c = \{A, C, D, E\}$

6. Determine $P(X^C \cup Z)$

$X^c \cup Z = \{B, D, F\}$

$P(X^c \cup Z) = P(B) + P(D) + P(F)$

$P(X^c \cup Z) = \boxed{0.44}$

7. Determine $P(\boxed{(X \cup Y)^C} \cap Z)$

$(X \cup Y) = \{A, B, C, E\}$

$(X \cup Y)^c = \{D, F\}$

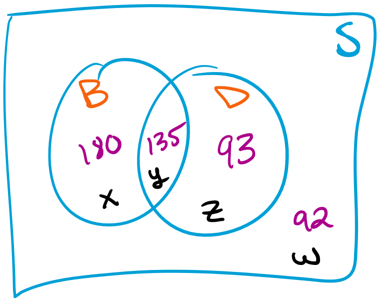
$(X \cup Y)^c \cap Z = \{D, F\} \cap \{B, F\}$

$= \{F\}$

$P((X \cup Y)^c \cap Z) = \boxed{0.11}$

8. A survey is conducted where students that live of-campus were asked if they drive to class or take the bus to class. Out of a survey of 500 students, 228 indicated that they drive to class and 315 indicated that the take the bus to class. Out of the 500 respondents, 92 indicated that they neither drive nor take the bus to class.

(a) Construct a Venn Diagram representing the results of the survey.



B = the student takes the Bus

D = the student Drives

☆ now label each unique region ☆

$D = 228 = y + z$ $\quad B = 315 = x + y$

$D^c \cap B^c = 92 = w$ $\quad$ ☆ $x + y + z + w = 500$ ☆

if working w/ probability, then we have $x + y + z + w = 1$

$x + y + z + w = 500$ & substitute $w = 92$ and either $y + z = 228$ oR $x + y = 315$

$315 + z + 92 = 500$

$z = 93$

$y + 93 = 228$

$y = 135$

$x + 135 = 315$

$x = 180$

(b) Determine the probability that a randomly selected student rides the bus to class and drives to class.

$$P(B \cap D) = P(y) = \boxed{\frac{135}{500}}$$

$B \cup D$

$B = \{x, y\} \cup D = \{y, z\}$

$= \{x, y, z\}$

(c) Determine the probability that a randomly selected student rides the bus to class but does not drive to class.

"and"

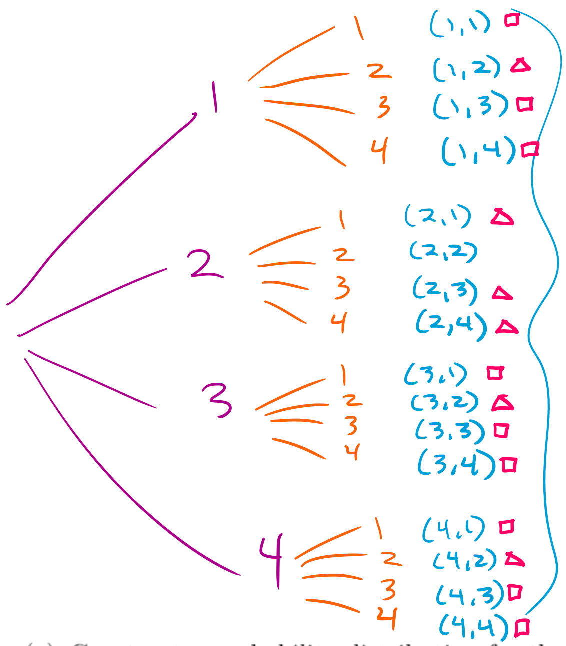
$$P(B \cap D^c) = P(x) = \boxed{\frac{180}{500}}$$

$B = \{x, y\}$

$D^c = \{x, w\}$

what is in common? x

9. An experiment consists of rolling a fair 4-sided die two times in a row and recording the result of each roll. ☆ Since we already constructed a dice chart, we'll use a tree diagram here ☆



0 2s → □

1 2 → △

sample space → 16 total outcomes

(a) Construct a probability distribution for the random variable X representing the number of 2's rolled.

$X = 0, 1, 2$

X	0	1	2
P(X)	9/16	6/16	1/16

(b) Determine $P(X = 0)$

$= \boxed{\frac{9}{16}}$

(c) Determine $P(X \geq 1)$ $= P(X = 1 \text{ OR } X = 2) = P(X=1 \cup X=2)$

$= \frac{6}{16} + \frac{1}{16} = \boxed{\frac{7}{16}}$

10. The table below shows the probability distribution for a random variable X. Determine the expected value of X. → $E(X) = ?$

X	1	2	3	4	5	6
P(X)	0.17	0.22	0.08	0.11	0.31	0.11

$$E(X) = (1)(0.17) + (2)(0.22) + (3)(0.08) + 4(0.11) + 5(0.31) + 6(0.11)$$

$$\boxed{E(X) = 3.5}$$

the number does not have to be on the table above

11. A company sells an advanced nano-machine for \$50,000 and offers a protection plan for the machine that costs an additional \$2,500. If the machine explodes, the company will pay back the full cost of the machine plus another \$10,000 in damages. If the machine short-circuits, the company will pay back 75% of the cost of the machine. If the machine "get cranky", the company will pay back 25% of the cost of the machine. There is a 0.5% chance the machine will explode, a 2% chance the machine will short-circuit, and an 5% chance the machine will "get cranky". How much profit can the company expect to make off the protection plans?

$E(X) = ?$

★ we will setup a probability distribution for X = profit for the company = (money in) − (money out)

Outcomes:

1. Explode: $(2500) - (50000 + 10000)$ = $-57{,}500$
2. Short-Circuits: $(2500) - (.75)(50000)$ = $-35{,}000$
3. Gets Cranky: $(2500) - (.25)(50000)$ = $-10{,}000$
4. Nothing Bad: $(2500) - (0)$ = 2500

X	-57,500	-35,000	-10000	2500
P(X)	0.005	0.02	0.05	.925

must add to 1

$$E(X) = (-57500)(.005) + (-35000)(0.02) + (-10000)(.05) + (2500)(.925)$$

$E(X) = 825$

the company can expect a profit of \$825 on each protection plan

12. The net winnings for the player in a new card game are given by the probability distribution given below. $\boxed{\text{Is the game fair?}}$ Explain how you know.

↳ $E(X) = 0$?

X	-\$6	-\$2	\$0	\$2	\$3	\$4
P(X)	0.1	0.25	0.25	0.15	0.15	0.1

$$E(X) = (-6)(.1) + (-2)(.25) + (0)(.25) + (2)(.15) + (3)(.15) + (4)(.1)$$

$E(X) = 0.05$ expected net winnings

$\boxed{E(X) \neq 0 \text{ so the game is not fair}}$

positive: unfair in player's 9 benefit

negative: unfair in casino's benefit

Don't forget: Chapter 3 is on Exam 2!

The Method of Corners (3.1-3.3)

- Be able to graph & shade for linear inequalities
- Determine a "feasible region" or "solution set" for a system of linear inequalities
- Define variables & setup an objective function & constraints based on those variables (don't forget units)
- Determine corner points of a feasible region by finding the intersection of lines (be sure to show work)
- Know the idea of bounded vs. unbound & sometimes unbounded regions don't have a solution
- Determine a solution for an objective by plugging in the corner points (if more than 1 pt gives the solution, there is more work to do b/c we need the line segment connecting them)

The Simplex Method (3.4)

- only applies to a "standard maximization" linear programming problem (determine if it is "standard max")
- we rewrite the constraints as equations by introducing slack variables that represent possible leftovers
- we rewrite the objective equation so that there is a zero on the right
- use them to construct an initial simplex tableau (don't forget to label the columns!)
- Know if/where to pivot for a simplex tableau
- Be able to classify all variables as basic or non-basic
- Be able to read a solution/corner point from any tableau including leftovers

slack variables