Final Exam Review

Tuesday, April 29, 2025 7:24 PM

FINAL EXAM REVIEW

Pr 1. You wish to buy a car for $25,000. The dealership offers you three different loans. Loan A has a monthly APR of 5%, Loan B has an annual interest rate of 7%, compounded quarterly, and Loan C has an annual interest rate of 6%, compounded continuously. Which loan has the smallest effective interest rate?

effective interest rate of A:
▶ Eff(5, 12) ≈ 5.1162 %

effective interest rate of B:
▶ Eff(7, 4) ≈ 7.1859 %

effective interest rate of C:
$e^{.06} - 1 = .061837$
$= 6.1837\%$

Account A is smallest.

Pr 2. You would like to have $750,000 in your retirement account when you retire in 30 years. Your retirement account earns 5.6% annual interest, compounded monthly. How much do you need to deposit at the end of each month to meet your retirement goal, if you make an initial deposit of $5000? How much of the $750,000 did you invest over the 30 years?

monthly payments

N = 30 × 12 = 360
I% = 5.6
PV = −5000
PMT = ? −776.88
FV = +750000
P/Y = 12
C/Y = 12

$\boxed{\$776.88}$

776.88 × 360 + 5000
$\boxed{= \$284,676.80}$

Pr 3. You purchased a home five years ago for $240,000. The bank required a 10% down payment, and gave you a 30-year loan with a 4.2% interest rate, compounded monthly.

(a) What is the monthly payment?
(b) What is the current balance on the loan?
(c) You have the opportunity to refinance with a 15-year loan with a 3.6% interest rate. What will be the new monthly payment?
(d) If you refinance, how much will you have saved by the time the house is paid off?

N = 30 × 12 = 360

.90 × 240000 = loan

$I = 4.2$
$PV = +216000$
$PMT = -1056.27$
$FV = 0$
$P/Y = 12$
$C/Y = 12$

$.10 \times 240000 =$ down payment

$$\begin{array}{r} 240000 \\ -24000 \\ \hline 216000 \end{array}$$

a) \$1056.27

b) current balance?
replace N with $N = 5 \times 12 = 60$
\$ 195,990.75

c) what changes?
$I = 3.6$, $N = 15 \times 12$
$PV = 195,990.75$
$FV = 0$
new payment = \$1410.75

d) Total amount to pay off old loan
$= \text{old PMT} \times 25 \times 12$
$- \text{new PMT} \times 15 \times 12$
$= \$62,946$

come back to this

Pr 4. Determine the value of w, x, and y given $\begin{bmatrix} 2 & w-3 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -6 \\ -8 & 12 \end{bmatrix}^T = 2\begin{bmatrix} -1 & 6 \\ 4 & -4 \end{bmatrix}$

$$\begin{bmatrix} 2 & w-3 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -8 \\ -6 & 12 \end{bmatrix} = \begin{bmatrix} 2\cdot(-1) & 2\cdot 6 \\ 2\cdot 4 & 2\cdot(-4) \end{bmatrix}$$

$$\begin{bmatrix} 2-y & w-3-(-8) \\ 2-(-6) & 4x-12 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 8 & -8 \end{bmatrix}$$

$2 - y = -2$ → $-y = -4$ →
$w + 5 = 12$ → $w = 7$
$4x - 12 = -8$ → $4x = 4$ →
$8 = 8$ ✓
$x = 1$
$y = 4$

Pr 5. Compute $\begin{bmatrix} 2 & 3x & 5 \\ 6w & 0 & 2y \end{bmatrix} \begin{bmatrix} -6 & 3m \\ 3n & 4 \\ -p & 0 \end{bmatrix}$.

$$\begin{bmatrix} 2\cdot(-6) + 3x\cdot(3n) + 5(-p) & 2(3m) + 3x(4) + 5\cdot 0 \\ 6w(-6) + 0(3n) + 2y(-p) & 6w(3m) + 0(4) + 2y\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 + 9xn - 5p & 6m + 12x \\ -36w - 2yp & 18wm \end{bmatrix}$$

Pr 6. An automobile purchased for use by the manager of a firm at a price of \$29,490 is to be depreciated using a linear model over ten years. Suppose the value depreciates by 39% after 5 years. When will the car reach its scrap value of \$1000?

$V(t) = mt + b$

$V(0) = 29490 = m \cdot 0 + b \rightarrow b = 29490$

$m = \dfrac{.61 \times 29490 - 29490}{5-0}$

$V(5) = V(0) - .39V(0)$

$= (1-.39)V(0)$

$= 29490\left(\dfrac{.61-1}{5}\right)$

$= .61 \times 29490$

$m = \dfrac{.39}{5} = -.078$

$V(t) = (-.078 \times 29490)t + 29490$

$\dfrac{1000}{29490} = \dfrac{(-.078t+1)\,29490}{29490}$

$\dfrac{1000}{29490} - 1 = -.078t$

$t = \dfrac{1}{-.078}\left(\dfrac{1000}{29490} - 1\right)$

$\boxed{t = 1.23 \text{ years}}$.

Pr 7. Dave sells organic bath soap at his stand at the local farmers market. He makes the soap for \$1 per bar, and sells them at \$6 per bar. Suppose that it costs him \$ 30 in fixed costs. Determine the break-even point.

break-even: $C(x) = R(x)$

Solve as an ordered pair $(x, R(x))$

$C(x) = 1 \cdot x + 30$

$R(x) = 6x$

$x + 30 = 6x$

$-x \qquad -x$

$\dfrac{30}{5} = \dfrac{5x}{5} \rightarrow x = 6$

$R(6) = 6 \cdot 6 = 36$

$\boxed{\text{break-even point} = (6, 36)}$

Pr 8. Determine the value of k so that the following system of linear equations has infinitely many solutions. → Same slope Same y-intercept

$$-x + ky = 8$$
$$3x - 6y = -24$$

$ky = x + 8$

$-6y = -3x - 24$

$$\left[\begin{array}{cc|c} -1 & k & 8 \\ 3 & -6 & -24 \end{array}\right]$$

$y = \dfrac{1}{k}x + \dfrac{8}{k}$

$y = \dfrac{-3}{-6}x + \dfrac{-24}{-6} \rightarrow y = \dfrac{1}{2}x + 4$

$\dfrac{1}{k} \cancel{\Rightarrow} \dfrac{1}{2}$

$\dfrac{8}{k} = 4$

$2 = k$

$\rightarrow 8 = 4k \rightarrow k = 2$ ✓

$\boxed{k = 2 \text{ only}}$

Pr 9. Set up and solve the following problem as a system of linear equations.

Donald has $15,000 to invest. He decides to invest in three different companies. The Huey company costs $250 per share and pays dividends of $3 per share each year. The Dewey company costs $60 per share and pay dividends of $1.00 per share each year. The Louie company costs $80 per share and pays $2.00 per share per year in dividends. 1) Link wants to have twice as much money in the Dewey company as in the Louie company. 2) Link also wants to earn $200 in dividends per year. How much should Link invest in each company to meet his goals?

h = # of shares of Huey

d = # of shares of Dewey

l = # of shares of Louie

	Share	Div
→ Huey	250	3
→ Dewey	60	1
→ Louie	80	2
Total	15000	200

$$250h + 60d + 80l = 15000$$

$$3h + d + 2l = 200$$

$$d = 2l$$

$l = 1$

$d = 2$

$4 - 1$ X

Pr 10. A local burger truck makes 4 types of burgers. The slim costs $3, has one patty and one slice of cheese. The big cheesy costs $7, has two patties, three slices of cheese, and one strip of bacon. The standard costs $5, has one patty, one slice of cheese, and three pieces of bacon. The bacon-me-crazy costs $7, has one patty, one slice of cheese, and 6 strips of bacon. Suppose that we have 1200 strips of bacon, 1000 burger patties, and 800 slices of cheese. How many of each type of burger should we make in order to maximize the profit? Set up the linear optimization problem, but do not solve it.

	Sale ~~cost~~ price	Pat.	cheese	ba
slim	3	1	1	0
big cheesy	7	2	3	1
Standard	5	1	1	3
bacon-	7	1	1	6
Max.		1000	800	1200

P = total profit

s = # of slims sold

c = # of big cheesies

x = # of standards sold

b = # of bacon-me-crazies

Maximize $P = 3s + 7c + 5x + 7b$

subject to: $s + 2c + x + b \le 1000$ (# patties)

$s + 3c + x + b \le 800$ (slices of cheese)

$c + 3x + 6b \le 1200$ (bacon)

$s \ge 0,\ c \ge 0,\ x \ge 0,\ b \ge 0$

Pr 11. Solve the following linear optimization problem using the method of corners.

$R = x - y$

Maximize $x - y$ subject to:

$2x + y \le 8$

$2x - 3y \le 4$

$x \ge 0, y \ge 0$

$2x + y \le 8 \rightarrow y \le -2x + 8$

$2x - 3y \le 4 \rightarrow 2x - 3y = 4$

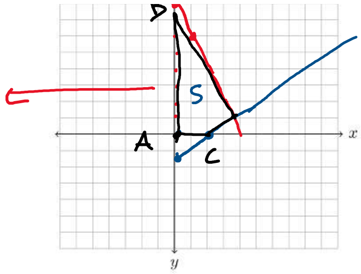
Set $x = 0 \rightarrow 2 \cdot 0 - 3y = 4$

$-3y = 4$

$(0, -4/3) \leftarrow y = -\frac{4}{3}$

Set $y = 0 \rightarrow 2x - 3 \cdot 0 = 4$

S is bounded, so a maximum exists



$2x = 4$
$x = 2$
$(2,0)$

Test point : $(0,0)$

$2\cdot 0 + 0 \le 8 \to 0 \le 8$ ✓

$2\cdot 0 + 3\cdot 0 \le 4 \to 0 \le 4$ ✓

D = intersection
$2x + y = 8$
$2x - 3y = 4$

$$\left[\begin{array}{cc|c} 2 & 1 & 8 \\ 2 & -3 & 4 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 7/2 \\ 0 & 1 & 1 \end{array}\right]$$

Corner points:
$A = (0,0)$
$B = (0,8)$
$C = (2,0)$
$D = (7/2, 1)$

		$R = x - y$
A	$(0,0)$	$0-0=0$
B	$(0,8)$	$0-8=-8$
C	$(2,0)$	$2-0=2$
D	$(\frac{7}{2},1)$	$\frac{7}{2}-1=\frac{5}{2}=2.5$

Maximum is $\frac{5}{2}$ at $(7/2, 1)$.

Pr 12. For the following simplex tableau, identify the basic and non-basic variables. State the solution corresponding to the tableau, and determine if it is an optimal solution. If it is not, identify the pivot row, pivot column, and pivot entry.

(a)
$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & P & \text{constant} \\ 0 & 2 & 1 & 0 & 0 & 8 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 5 \\ \hline 0 & -\frac{1}{2} & 0 & \frac{3}{2} & 1 & 15 \end{array}\right]$$

$8/2 = 4$
$5/\frac{1}{2} = 5 \cdot 2 = 10$

not an optimal solution. pivot on $(1,2)$ with entry 2

basic var's: x, s_1, P

non-basic: $y, s_2 \to y = s_2 = 0$

$s_1 = 8$
$x = 5$
$P = 15$

Solution: 15 @ $(5, 0)$

(b)
$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & P & \text{constant} \\ 1 & 0 & 1 & 0 & 0 & 8 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ \hline -2 & -3 & 0 & 0 & 1 & 0 \end{array}\right]$$

~~8/0~~
$\leftarrow 0/1 = 0$

2nd col

Not an optimal solution. pivot on $(2, 2)$ with entry 1.

2nd col

basic var's: s_1, s_2, P

non-basic: x, y $\quad x = y = 0$

$P = 0$ @ $(0,0)$ $\quad s_1 = 8, \ s_2 = 0$

(c)
$$\left[\begin{array}{cccccc|c} x & y & z & s_1 & s_2 & P & \text{constant} \\ 0 & 2 & 1 & 0 & 0 & 0 & 9 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 2 \\ \hline 0 & \frac{1}{2} & 0 & 2 & \frac{3}{2} & 1 & \boxed{42} \end{array}\right]$$

$\to$ This is an optimal solution

$P = 42$ @ $(2, 0, 9)$

basic : x, z, P $\quad z=9 \quad x=2$

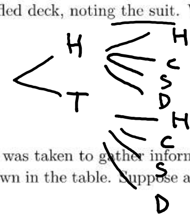
nonbasic: y, s_1, s_2 $\quad y=0$

Pr 13. In an experiment, a fair standard 2-sided coin is flipped, noting which side faces up, and then a card is drawn from a well-shuffled deck, noting the suit. Write the sample space for the experiment.

1 stage $\qquad$ 2nd

\# outcomes = 8

\# total events $= 2^8 = 256$



$S = \{(H,H), (H,C), (H,S), (H,D), (T,H), (T,C), (T,S), (T,D)\}$

Pr 14. A survey of 100 Aggies was taken to gather information on how they commute to campus. A breakdown of those surveyed is shown in the table. Suppose a randomly selected Aggies. What is the probability the person chosen is

	Drive	Bus	Other	Total
Freshmen	~~15~~	10	14	**39**
Sophomore	11	8	12	**31**
Junior	9	5	4	**18**
Senior	6	4	2	**12**
Total	**41**	~~**27**~~	~~**32**~~	**100**

(a) P(rides the bus) $= \boxed{\dfrac{27}{100}}$

(b) P(is a Junior or Senior) $= \dfrac{18+12}{100} = \dfrac{30}{100}$

(c) P(is a Freshman or does not drive) $= \dfrac{27 + 32 + 15}{100} = \dfrac{74}{100}$

$= \dfrac{39 + 27 + 32 - 10 - 14}{100}$

(d) P(is a Senior and rides the bus)

$= \boxed{\dfrac{4}{100}}$

$P((A \cap B)^c) = 1 - P(A \cap B)$

Pr 15. Given $P(A) = 0.4$, $P(B) = 0.7$, and $P(A \cup B) = 0.9$, compute $P\left[(A \cap B)^C\right]$.

Approach 2:

$w + x = P(A) = .4$

$x + y = P(B) = .7$

$w + x + y = P(A \cup B) = .9$

$w + x + y + z = 1$

$y = 1 - x = 1 - .2$

$P((A \cap C)^c) = .8$

Answer $= w + y + z$ (use calc)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.9 = .4 + .7 + x$

$.9 = 1.1 - x$

$x + .9 = 1.1$

$x = 1.1 - .9 = .2$

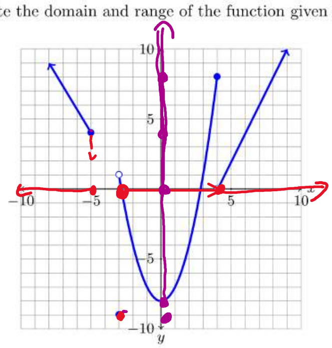
Pr 16. Your insurance company has a policy to insure personal property. Assume your personal property is worth \$2,000, and according to campus statistics there is a 1% chance that your property will be stolen during the next year and a 10% chance that your property is damaged beyond repair through natural causes during the next year. If your property is stolen the policy will give you \$2,000, while if it is damaged beyond repair you receive get \$1,000. What is the insurance company's expected profit on this policy, if the premium for the policy is \$300?

$$E(X) = .01(-2000+300) + .1(-1000+300) + .89(300)$$

event	stolen	broken	good
prob	.01	.1	1 − .1 − .01
X	−2000+300	−1000+300	300

$$= .01(-1700) + .1(-700) + .89 \cdot 300 = \$180$$

Pr 17. State the domain and range of the function given in the graph below, using interval notation.



Domain: $(-\infty, -5] \cup [-3, \infty)$

Range: $[-9, -9] \cup [-8, \infty)$

$\{-9\} \cup [-8, \infty)$

Pr 18. The price-demand function (in dollars) for a particular item is given by $p(x) = -0.05x + 50$, where x is the number of items. The company who produces these items has a production cost of \$2 per item and fixed costs of \$120. What price should the company charge for the item in order to maximize profit?

$R(x) = p \cdot x = (-.05x + 50)x$

$= -.05x^2 + 50x$

$p(x) = -.05x + 50$

$P(x) = R(x) - C(x) = -.05x^2 + 50x - (2x + 120)$

$= -.05x^2 + 48x - 120$

we maximize Profit @ $x = \dfrac{-b}{2a}$

$x = \dfrac{-48}{2(-.05)} = \dfrac{480}{1}$

$= 480$ items

Answer $= p(480) = -.05(480) + 50$

$\boxed{= \$26}$

Pr 19. Compute and simplify the difference quotient of $g(x) = \frac{3x}{2x-3}$.

3 types:

1) $f(x) = ax^2+bx+c \rightarrow$ foil
2) $f(x)\ \sqrt{ax+b} \rightarrow$ conjugate

$$\frac{g(x+h) - g(x)}{h} = \frac{1}{h}\left(g(x+h) - g(x)\right)$$

$$= \frac{1}{h}\left[\frac{3(x+h)}{2(x+h)-3} - \frac{3x}{2x-3}\right]$$

common denominator

$$= \frac{1}{h}\left[\frac{3x+3h}{2x+2h-3} - \frac{3x}{2x-3}\right] =$$

$$= \frac{1}{h}\left[\frac{(3x+3h)}{(2x+2h-3)}\frac{(2x-3)}{(2x-3)} - \frac{(3x)}{(2x-3)}\frac{(2x+2h-3)}{(2x+2h-3)}\right]$$

$$\begin{array}{r} 6x^2 - 9x + 6xh - 9h \\ -6x^2 + 9x - 6xh \\ \hline -9h \end{array}$$

$$= \frac{1}{h(2x+2h-3)(2x-3)}\left((3x+3h)(2x-3) - 3x(2x+2h-3)\right)$$

$$= \frac{-9h}{h(2x+2h-3)(2x-3)} = \boxed{\frac{-9}{(2x+2h-3)(2x-3)}}$$

rules: · don't divide by 0

· $\sqrt{\text{negatives}}$ are bad

$\log(f(x)) \rightarrow f(x) > 0$

Pr 20. State the domain of $f(x) = \frac{\ln(11-3x)}{e^x\sqrt{2x+13}}$ using interval notation.

1) $\ln(11-3x) \rightarrow 11 - 3x > 0$
$+3x \quad +3x$
$\frac{11}{3} > \frac{3x}{3} \rightarrow \frac{11}{3} > x$ or $x < \frac{11}{3}$

2) $\sqrt{2x+13}$ is denominator $\rightarrow 2x+13 > 0 \rightarrow 2x > -13 \rightarrow x > -\frac{13}{2}$

3) $e^x \neq 0 \rightarrow$ always true

$\boxed{\left(-\frac{13}{2}, \frac{11}{3}\right)}$

Pr 21. Algebraically solve: $8 \cdot 4^{(3x+2)} = 16$. ← common base → 2

$(a^b)^c = a^{bc}$

$a^b a^c = a^{b+c}$

$8 = 2^3, \quad 4 = 2^2, \quad 16 = 2^4$

$2^3 \cdot (2^2)^{(3x+2)} = 2^4$

$2^3 \cdot 2^{2\cdot(3x+2)} = 2^{3+2(3x+2)} = 2^{3+6x+4} = 2^{6x+7}$

$2^{6x+7} = 2^4 \rightarrow 6x + 7 = 4 \rightarrow 6x = -3$
$-7 \quad -7$
$x = -3/6 = -1/2 \quad \boxed{x = -1/2}$

Pr 22. You place \$1000 as an initial deposit in a savings account earning annual interest at a rate of 3.5% and leave it there for 4 years. How long will it take for the savings account to reach \$1200, assuming that the account is compounded continuously?

not used

$A = Pe^{rt}$ solve for t.

$P = 1000$

$A = 1200$

$r = .035\%$

$1200 = 1000\, e^{.035t}$

$$\frac{1200}{1000} = e^{.035t}$$

$$\ln\left(\frac{1200}{1000}\right) = .035t$$

$$t = \frac{1}{.035}\ln\left(\frac{1200}{1000}\right) = \frac{1}{r}\ln\left(\frac{P}{A}\right)$$

$$\boxed{t = 5.21 \text{ years}}$$

Extra problem 1: Expand $\ln\left(\frac{(3x-5)\,x^3}{\sqrt{x}\;e^2}\right)$

$$\ln\left(\frac{(3x-5)\cdot x^3}{\sqrt{x}\cdot e^2}\right) = \ln(3x-5) + \ln(x^3) - \ln(\sqrt{x}) - \ln(e^2)$$

$\sqrt{x} = x^{1/2}$

$\ln(e) = 1$

$$= \ln(3x-5) + 3\ln(x) - \frac{1}{2}\ln(x) - 2\ln(e)$$

$$= \boxed{\ln(3x-5) + 3\ln(x) - 1/2\,\ln(x) - 2}$$

Problem 2: piece wise function: write $|4-x|$ as a piecewise-defined function.

$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$|4-x| = \begin{cases} -(4-x) & 4-x < 0 \quad (x>4) \\ 4-x & 4-x \geq 0 \quad (x \leq 4) \end{cases}$$

$-4+x$

$4 - x < 0$

$4 < x \rightarrow x > 4$

$4 - x \geq 0$

$4 \geq x \rightarrow x \leq 4$

first

$$f(x) = \begin{cases} 4-x & x \leq 4 \\ -4+x & x > 4 \end{cases}$$

Bonus problem 3: $f(x) = x^2 + 5$

$g(x) = x + 2$

Compute $(f \circ g)(2x) = f(g(2x))$

$$= (g(2x))^2 + 5$$

$$= ((2x) + 2)^2 + 5$$

$$= (2x+2)^2 + 5 = 4x^2 + 4x + 4x + 4 + 5$$

$(2x+2)(2x+2)$

$$= \boxed{4x^2 + 8x + 9}$$