Example 1 (16.4). *Use the Green's Theorem to compute the line integral* $\oint_C \mathbf{F} \cdot d\mathbf{r}$, *where* $\mathbf{F}(x,y) = (x^2y^2 + x^2 \sin x)\,\mathbf{i} + (2x^3y + e^y)\,\mathbf{j}$ *and* C *is the boundary of the region bounded by the curves* $y = x^2,\ x = 2,$ *and* $y = 0$.

Example 2 (16.4). *Use the Green's Theorem to compute the line integral* $\oint_C \mathbf{F} \cdot d\mathbf{r}$, *where* $\mathbf{F}(x,y) = \left\langle e^{3x} - 4y, 6x + e^{2y} \right\rangle$ *and* C *is the boundary of the region that has area 4 with counterclockwise orientation.*

Example 3 (16.4)**.** *Use the Green's Theorem to compute* $\int_C (3xy^2 - 2y^3)\,dx + (2x^3 + 3x^2y)\,dy$, *where* C *is the circle* $x^2 + y^2 = 9$ *with positive orientation.*

Example 4 (16.4)**.** *Compute* $\int_C \mathbf{F} \cdot d\mathbf{r}$, *where* $\mathbf{F}(x, y) = \left\langle 2xy^2, 3x^2y - 5 \right\rangle$ *and* C *is the triangle from* $(0, 0)$ *to* $(-2, 2)$ *to* $(1, 2)$ *to* $(0, 0)$.

Example 5 (16.5). *Find the curl and divergence of*

$$\mathbf{F}(x, y, z) = xyz\,\mathbf{i} + (x^2 + yz)\,\mathbf{j} + xz\,\mathbf{k}.$$

Is $\mathbf{F}$ *conservative? Explain.*

Example 6 (16.5). *Let* f *be a scalar function and* $\mathbf{F}$ *and* $\mathbf{G}$ *are vector fields on* $\mathbb{R}^3$. *State whether each expression is meaningful. If so, state whether it;s a vector field or a scalar field.*

(a) $\nabla f \times (\mathbf{F} + \mathbf{G})$

(b) $\nabla f \cdot curl\,\mathbf{F}$

(c) $\nabla f \times div\,\mathbf{F}$

(d) $(curl\,\mathbf{F} \times \mathbf{G}) \cdot \nabla f$

(e) $curl\,(\mathbf{F} \cdot \mathbf{G})$

(f) $div\,(curl\,F \times \nabla f)$

Example 7 (16.5)**.** *Consider the vector field* $\mathbf{F}(x, y, z) = \left\langle 2xy + 3, x^2 + z\cos y, \sin y \right\rangle$.

(a) Determine whether or not $\mathbf{F}$ *is conservative. If it is, find a potential function* f*. That is, find a function* f *such that* $\nabla f = \mathbf{F}$.

(b) Compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, *where* $C: \; r(t) = \left\langle t, 2t, 1 + t^2 \right\rangle; \; 0 \leq t \leq \pi$.

Example 8 (16.6)**.** *Find a parametric representation for the surface.*

(a) *The part of the plane* $2x + z = 8$ *that lies within the cylinder* $x^2 + y^2 = 9$.

(b) *The part of the cylinder* $x^2 + y^2 = 9$ *within the planes* $z = 0$ *and* $z = 3$.

(c) *The part of the cylinder* $y^2 + z^2 = 9$ *within the planes* $x = 0$ *and* $x = 3$.

(d) *The part of the paraboloid* $z = 6 - 2x^2 - 2y^2$ *above the plane* $z = 4$.

Example 9 (16.6). *Find the surface area of the part of the plane* $2x + 3y + z = 8$ *that lies within the cylinder* $x^2 + y^2 = 4$.

Example 10 (16.6)**.** *Find the surface area of* S*, where* S *is the part of the paraboloid* $y = x^2 + z^2$ *that lies within the cylinder* $x^2 + z^2 = 4$*.*

Example 11 (16.6). *Consider that S is the part of the sphere $x^2 + y^2 + z^2 = 36$ that lies within the planes $z = 0$ and $z = 3\sqrt{3}$.*

(a) Find a parametric representation for the surface S.

(b) Find the surface area of the surface S.

Example 12 (16.6). *Find the area of the part of the surface* $z = 1 + 2x^2 + 3y$ *that lies above the region bounded the triangle with vertices* $(0, 0)$, $(2, 0)$, *and* $(2, 4)$.