MATH 140: WEEK-IN-REVIEW 10 (CHAPTERS 5.5 & 5.6)

1. Compute the following values for the function $f(x) = \begin{cases} 5x-3 & \text{if } x < -4, \\ 2x^2-1 & \text{if } -4 \le x \le 4, \\ 6 & \text{if } 4 < x \le 6, \\ \dfrac{3}{x-4} & \text{if } x > 8. \end{cases}$

(a) $f(-5) = 5(-5)-3$
$= -25-3$
$= \boxed{-28}$

(b) $f(-4) = 2(-4)^2-1 = 2\cdot 16-1$
$= 32-1$
$= \boxed{31}$

(c) $f(0) = 2(0)^2-1$
$= \boxed{-1}$

(d) $f(4) = 2(4)^2-1$
$= 2\cdot 16-1$
$= 32-1 = \boxed{31}$

(e) $f(8)$ DNE since $x=8$ is not in domain

(f) $f(9) = \frac{3}{9-4} = \boxed{\frac{3}{5}}$

2. State the domain of the function $g(x) = \begin{cases} 5x+7 & \text{if } x < -4, \quad A \\ \dfrac{1}{3x+7} & \text{if } -3 \leq x < 3, \quad B \\ -5\sqrt{x-2} & \text{if } x \geq 3. \quad C \end{cases}$

In A, $5x+7$ is a polynomial $\Rightarrow$ no restrictions : $(-\infty,-4)$

In B, $\dfrac{1}{3x+7}$ is a rational function $\Rightarrow$ denominator $\neq 0$, $3x+7 \neq 0 \Rightarrow \dfrac{3x}{3} \neq \dfrac{-7}{3}$

$x \neq -\frac{7}{3}$: $[-3,-\frac{7}{3}) \cup (-\frac{7}{3},3)$

In C, $-5\sqrt{x-2}$ is an even root function $\Rightarrow x-2 \geq 0 \Rightarrow x \geq 2$. But in C, $x \geq 3$

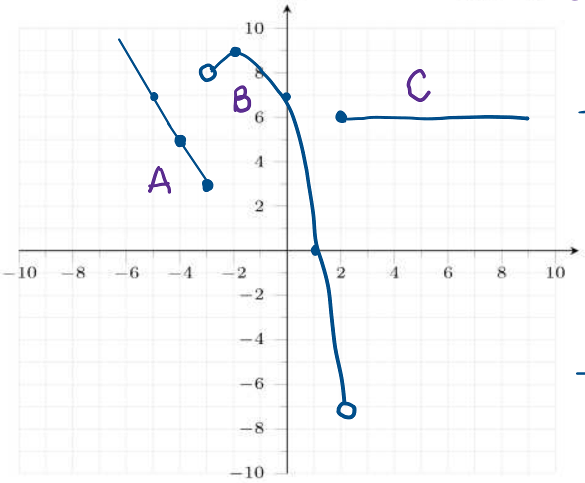
: $[3,\infty)$

$\boxed{\text{Domain of } g(x): (-\infty,-4) \cup [-3,-\tfrac{7}{3}) \cup (-\tfrac{7}{3},\infty)}$

3. Sketch the graph of $h(x) = \begin{cases} -2x-3 & \text{if } x \leq -3, \quad A \\ -x^2-4x+5 & \text{if } -3 < x < 1, \quad B \rightarrow \text{vertex } (-2,9) \\ 6 & \text{if } x \geq 2. \quad C \end{cases}$

A: $-2x-3$ (line)

x	$f(x)$	(x,y)
-3	$-2(-3)-3 = 3$	$(-3,3)$
-4	$-2(-4)-3 = 5$	$(-4,5)$
-5	$-2(-5)-3 = 7$	$(-5,7)$



B: $-x^2-4x+5$ (Quadratic fn)

x	$f(x)$	(x,y)
-3	$-(-3)^2-4(-3)+5 = -9+12+5 = 8$	$(-3,8)$ hole
-2	$-(-2)^2-4(-2)+5 = -4+8+5 = 9$	$(-2,9)$
0	$-0^2-4\cdot 0+5 = 5$	$(0,5)$
2	$-2^2-4\cdot 2+5 = -7$	$(2,-7)$
1	$-1^2-4\cdot 1+5 = 0$	$(1,0)$

4. Rewrite the function $f(x) = |18 - 3x|$ as a piecewise-defined function.

$$|18-3x| = \begin{cases} -1\cdot(18-3x) & \text{if } 18-3x<0 \\ 18-3x & \text{if } 18-3x \geq 0 \end{cases}$$

$$= \begin{cases} -18+3x & \text{if } x>6 \\ 18-3x & \text{if } x \leq 6 \end{cases}$$

$18-3x<0$

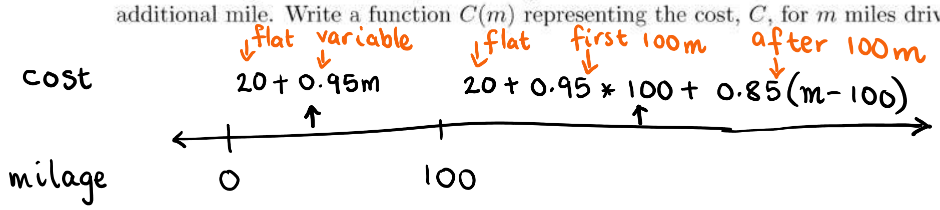
$\frac{18}{3} < \frac{3x}{3}$

$6 < x$

$$= \boxed{\begin{cases} 18-3x & \text{if } x \leq 6 \\ -18+3x & \text{if } x>6 \end{cases}}$$

* in order of interval

5. A truck rental agency charges a flat fee of \$20. If the distance traveled is less than 100 miles, the cost is \$0.95 per mile. For any distance greater than 100 miles, the cost reduces to \$0.85 per each additional mile. Write a function $C(m)$ representing the cost, C, for m miles driven.



$$C(m) = \begin{cases} 20+0.95m & \text{if } 0 \leq m \leq 100 \\ 115+0.85(m-100) & \text{if } m>100 \end{cases}$$

6. Rewrite each exponential expression as a single equivalent expression in the given base.

(a) $7 \cdot 49^{x+2}$ base 7. $49 = 7^2$

$7^1 \cdot (7^2)^{x+2}$

$= 7^1 \cdot 7^{2(x+2)} = 7^1 \cdot 7^{2x+4} = \boxed{7^{2x+5}}$

(b) $\left(\frac{1}{3}\right)^x \cdot \frac{27}{9^x}$ base 3. $27 = 3^3$, $9 = 3^2$

$(3^{-1})^x \cdot \frac{3^3}{(3^2)^x} = 3^{-x} \cdot 3^3 \cdot 3^{-2x} = 3^{-x+3-2x}$

$= 3^{3-3x} = \boxed{3^{3(1-x)}}$

7. Determine if the given function is an exponential function. If it is an exponential function, state whether it represents exponential growth or decay. $\hookrightarrow f(x) = a \cdot b^x$, $b > 0$, $b \neq 1$

* if $0 < b < 1 \Rightarrow$ exp. decay
* if $b > 1 \Rightarrow$ exp. growth

(a) 5^{x+3}

$= 5^x \cdot 5^3$

$= 125 \cdot 5^x$

* exponential function
* $b = 5 > 1 \Rightarrow$ exponential growth

(b) $-4x^{15}$

not an exponential function

* this is a polynomial / power function

(c) $7 \cdot \left(\frac{1}{3}\right)^{-2x}$

$= 7 \cdot (3^{-1})^{-2x}$

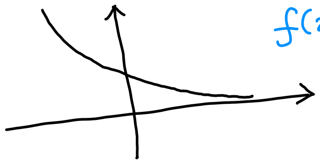
$= 7 \cdot 3^{2x} = 7 \cdot (3^2)^x$

$= 7 \cdot 9^x$

* exponential function
* $b = 9 > 1 \Rightarrow$ exponential growth

8. State the domain, range, end behavior, x and y-intercepts (if any) of each function below.

(a) $f(x) = \left(\frac{3}{4}\right)^{x+2}$



$f(x) \to 0$ as $x \to \infty$
$f(x) \to \infty$ as $x \to -\infty$

$f(x) = \left(\frac{3}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^x$

$= \frac{9}{16} \cdot \left(\frac{3}{4}\right)^x$ * exponential decay ($b = \frac{3}{4} < 1$)

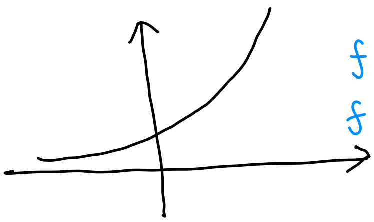
* $a = \frac{9}{16} > 0$

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

y-int: $f(0) = \frac{9}{16} \cdot \left(\frac{3}{4}\right)^0 = \frac{9}{16}$
$(0, \frac{9}{16})$

x-int: NONE

(b) $g(x) = 4^{x-2}$



$f(x) \to \infty$ as $x \to \infty$
$f(x) \to 0$ as $x \to -\infty$

$g(x) = 4^{-2} \cdot 4^x$

$= \frac{1}{4^2} \cdot 4^x$

$= \frac{1}{16} \cdot 4^x$

* exponential growth ($b = 4 > 1$)
* $a = \frac{1}{16} > 0$

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

y-int: $g(0) = \frac{1}{16} \cdot 4^0 = \frac{1}{16}$
$(0, \frac{1}{16})$

x-int: NONE

9. For each funcion below, state the domain using interval notation

(a) $f(x) = 3^{\frac{2x}{x+4}}$

* $f(x) = 3^{\frac{2x}{x+4}}$ is defined when $\frac{2x}{x+4}$ is defined
* $\frac{2x}{x+4}$ is a rational function $\Rightarrow$ denom $\neq 0 \Rightarrow x+4 \neq 0 \Rightarrow x \neq -4$

$\boxed{\text{Domain of } f: (-\infty,-4)\cup(-4,\infty)}$

(b) $h(x) = e^{\sqrt{3-5x}}$

* $h(x) = e^{\sqrt{3-5x}}$ is defined when $\sqrt{3-5x}$ is defined
* $\sqrt{3-5x}$ is an even root function $\Rightarrow 3-5x \geq 0 \Rightarrow \frac{3}{5} \geq \frac{5x}{5} \Rightarrow \frac{3}{5} \geq x$

$\boxed{\text{Domain of } h: \left(-\infty, \frac{3}{5}\right]}$

(c) $g(x) = \dfrac{\sqrt[5]{2x-7}}{2^{x+3}}$

$\sqrt[5]{2x+7}$ is an odd root $\Rightarrow 2x+7$ defined $\Rightarrow$ x any real #

* $g(x) = \dfrac{\sqrt[5]{2x-7}}{2^{x+3}}$ defined when $\rightarrow 2^{x+3}$ exists & $\underbrace{2^{x+3} \neq 0}$

exists for all real #s

always true since it is an exponential function range: $(0,\infty)$

$\boxed{\text{* Domain of } g(x): (-\infty,\infty)}$

10. Algebraically solve each equation for x

(a) $\left(\frac{1}{8}\right)^{2x} = 16^{x-5}$ $\Leftrightarrow (2^{-3})^{2x} = (2^4)^{x-5}$

$2^{-6x} = 2^{4(x-5)} = 2^{4x-20}$ * base = 2

$-6x = 4x - 20$

$\frac{20}{10} = \frac{10x}{10} \Rightarrow \boxed{x = 2}$

(b) $3^{x+2} = 27^{x-1}$ $27 = 3^3$

$3^{x+2} = (3^3)^{x-1} = 3^{3(x-1)} = 3^{3x-3}$ * base 3

$x + 2 = 3x - 3$

$\frac{5}{2} = \frac{2x}{2} \Rightarrow \boxed{x = \frac{5}{2}}$

$32 = 2^5$
$16 = 2^4$
$1 = 2^0$

(c) $\left(\frac{1}{32}\right) \cdot 2^{x^2} \cdot 16^x = 1$

$\frac{1}{2^5} \cdot 2^{x^2} \cdot 2^{4x} = 2^0 \Rightarrow 2^{-5} \cdot 2^{x^2} \cdot 2^{4x} = 2^0$

$2^{-5+x^2+4x} = 2^0$ base 2

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0 \Rightarrow \boxed{x = -5, \ x = 1}$

11. If you invest \$4000 in an account that earns interest at a rate of 3.5% per year, compounded monthly

P APR

(a) how much will be in the account after 10 years?

t

$$A(t) = P\left(1+\frac{r}{m}\right)^{mt}$$

$$= 4000\left(1+\frac{0.035}{12}\right)^{120}$$

$$= \boxed{\$5,673.38}$$

round to the nearest CENT

* P= \$4000 initial principal balance
* r= annual interest rate = 0.035
* m = # of compounding periods per year = 12 (monthly)
* t = time in years = 10 years

(b) If the annual interest is compounded continuously instead of monthly, how much more will be in the account after 10 years compared to your previous answer?

$$A(t) = Pe^{rt}$$

$$= 4000\, e^{0.035 * 10}$$

$$= \boxed{\$5676.27}$$

You earn $\boxed{\$2.89}$ more, after 10 years, if interest is compounded continuously rather than monthly.

12. If $g(x) = -\frac{3}{5}f(x+5) - 9$, write the transformations that would be applied to the graph of $f(x)$ (in the correct order), to produce the graph of $g(x)$.

* Translation to the LEFT 5 units
* Vertical shrinking by a factor of $\frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$
* Reflection across the x-axis
* Translation DOWN by 9 units

13. If the graph of $f(x) = |x|$ is shifted right 2 units, vertically expanded by a factor of 6, reflected over the x-axis, and then shifted 9 units up, what is the equation of the resulting graph?

$f(x) = |x|$

$\downarrow$ SHIFT right 2 units

$|x-2|$

$\downarrow$ Vertically expand by factor of 6

$6|x-2|$

$\rightarrow$ reflection over the x-axis $\rightarrow -6|x-2| \rightarrow$ Shift up by 9 units $\rightarrow -6|x-2|+9$

$$\boxed{g(x) = -6|x-2|+9}$$