24A_WIR_M142_H9

Antiderivatives:

① $x^n \longrightarrow \frac{x^{n+1}}{n+1} + c$ $\xrightarrow{\text{exception}}$ $\Rightarrow$ ③ $\frac{1}{x} \longrightarrow \ln|x| + c$

ie $x^{-1} \longrightarrow \ln|x| + c$

② $e^x \longrightarrow e^x + c$

Problem 1. Find the following indefinite integrals.

$x = x^1$
$1 = x^0$

(1) $\int (5x^4 - x^3 + 6x - 2)\, dx$ ↪ tells you what your variable is $dx \to x$.

$= \int 5x^4 dx + \int -x^3 dx + \int 6x\, dx + \int -2\, dx$

$= 5\int x^4 dx - 1\int x^3 dx + 6\int x\, dx - 2\int 1\, dx$

$= 5\left(\frac{x^{4+1}}{4+1}\right) - 1\left(\frac{x^{3+1}}{3+1}\right) + 6\left(\frac{x^{1+1}}{1+1}\right) - 2\left(\frac{x^{0+1}}{0+1}\right) + C$

$= \frac{5x^5}{5} - \frac{x^4}{4} + \frac{{}^3\cancel{6}x^2}{\cancel{2}} - \frac{2x}{1} + C$

$= x^5 - \frac{1}{4}x^4 + 3x^2 - 2x + C$

(2) $\int \left(\sqrt{u} + \frac{1}{\sqrt{u}}\right) du$

$= \int (u^{1/2} + u^{-1/2})\, du$

$= \frac{u^{1/2+1}}{1/2+1} + \frac{u^{-1/2+1}}{-1/2+1} + C$

$= \frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2} + C = \frac{2}{3}u^{3/2} + 2u^{1/2} + C$

(3) $\int \left(2e^x + \frac{1}{5x} + \frac{4}{x^3}\right) dx$

$\frac{x}{5/7} = x \div \left(\frac{5}{7}\right) = x \cdot \left(\frac{7}{5}\right) = \frac{7x}{5}$

$= \int \left(2e^x + \left(\frac{1}{5}\right)\left(\frac{1}{x}\right) + 4 \cdot x^{-3}\right) dx$ ($\frac{1}{x} \to x^{-1}$)

$= (2)e^x + \left(\frac{1}{5}\right)\ln|x| + (4)\frac{x^{-3+1}}{-3+1} + C$

$= 2e^x + \frac{1}{5}\ln|x| + \frac{{}^2\cancel{4}x^{-2}}{-\cancel{2}} + C$

$= 2e^x + \frac{1}{5}\ln|x| - 2x^{-2} + C$

$\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}$

$\frac{1}{2} \to \frac{3}{2} \to \frac{5}{2} \to \frac{7}{2}$

$(x^3)^{\frac{1}{2}} = x^{3/2}$ $\qquad \frac{1}{3} \to \frac{4}{3} \to \frac{7}{3} \to \frac{10}{3}$

(4) $\int \left(5\sqrt{x^3} + 6x^{-1}\right)\,dx$

$= \int \left(5(x^3)^{1/2} + 6\cdot\frac{1}{x}\right)dx$

$= 5\cdot\frac{x^{3/2+1}}{3/2+1} + 6\ln|x| + C$

$= 5\frac{x^{5/2}}{5/2} + 6\ln|x| + C \qquad = 2x^{5/2} + 6\ln|x| + C$

$\cancel{5}x^{5/2}\cdot\frac{2}{\cancel{5}}$

(5) $\int (x-2)(2x^2+3)\,dx$ $\longrightarrow$ FOIL IT OUT!

$= \int (2x^3 + 3x - 4x^2 - 6)\,dx$

$= \int (2x^3 - 4x^2 + 3x - 6\cdot 1)\,dx$

$= \cancel{2}\cdot\frac{x^4}{\cancel{4}\,2} - 4\cdot\frac{x^3}{3} + \frac{3x^2}{2} - 6x + C$

$= \boxed{\frac{1}{2}x^4 - \frac{4}{3}x^3 + \frac{3}{2}x^2 - 6x + C}$

$1 = x^0$

$\int 1\,dx = \int x^0\,dx$

$= \frac{x^{0+1}}{0+1} = x$

why not u-sub?

$(x-2)(2x^2+3)$

$\uparrow$

$u = 2x^2+3$

$\frac{du}{dx} = 4x$

(6) $\int \left(\frac{4x^3 + x\sqrt{x} + 5x^2}{8x}\right)dx$

$= \int \left(\frac{4x^3}{8x} + \frac{\cancel{x}\sqrt{x}}{8\cancel{x}} + \frac{5x^2}{8x}\right)dx$

$= \int \left(\frac{1}{2}x^2 + \frac{1}{8}\sqrt{x} + \frac{5}{8}\cdot x\right)dx \qquad \sqrt{x} \to x^{1/2}$

$= \frac{1}{2}\cdot\frac{x^3}{3} + \left(\frac{1}{8}\right)\frac{x^{3/2}}{\left(\frac{3}{2}\right)} + \frac{5}{8}\cdot\frac{x^2}{2} + C$

$\frac{1}{8}\cdot\frac{1}{\frac{3}{2}} = \frac{1}{\cancel{8}\,4}\cdot\frac{\cancel{2}}{3} = \frac{1}{12}$

$= \boxed{\frac{1}{6}x^3 + \frac{1}{12}x^{3/2} + \frac{5}{16}\cdot x^2 + C}$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

What is y'? $\quad y = f(x)$

$y' = f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx}$ → derivative.

$y' \xrightarrow{\text{antiderivative}} y$

Problem 2. If $y' = \frac{3}{x} + \frac{1}{x^2}$ and $y(1) = 1$, what is y?

(initial condition)

$x^{-1} \xrightarrow{\text{AD}} \frac{x^{-1+1}}{-1+1} = \frac{x^0}{0}$??

$y' = 3(x^{-1}) + x^{-2}$

$= 3\ln|x| + \frac{x^{-2+1}}{-2+1} + C$

$= 3\ln|x| + \frac{x^{-1}}{-1} + C$

$f(x) = y = 3\ln|x| - \frac{1}{x} + C$

$y(1) = 3\ln|1| - \frac{1}{1} + C$

$= -1 + C$

$\therefore -1 + C = 1 \quad \therefore C = 2$

$y = 3\ln|x| - \frac{1}{x} + 2$

Problem 3. What is the most general antiderivative of $f(x) = 3\sqrt{x} - \frac{1}{x^2} - x^{3/2}$?

$\int f(x)dx = \int\left(3\sqrt{x} - \frac{1}{x^2} - x^{3/2}\right)dx$

$= 3\frac{x^{3/2}}{3/2} - \frac{x^{-1}}{(-1)} - \frac{x^{5/2}}{5/2} + C$

$= 2x^{3/2} + x^{-1} - \frac{2}{5}x^{5/2} + C$

Problem 4. Rewrite the integral $\int (x+4)e^{3x^2+24x}\,dx$ in terms of u after an appropriate u-substitution.

$= \int (x+4)\, e^{(3x^2+24x)}\, dx.$

$(x+4)\,dx = \frac{1}{6}du$

$u = 3x^2 + 24x$

$\frac{du}{dx} = 3\cdot(2x) + 24(1) = 6x + 24 = 6(x+4)$

$du = 6(x+4)\,dx$

$\frac{du}{6(x+4)} = \frac{6(x+4)}{6(x+4)}\cdot dx \Rightarrow dx = \frac{du}{6(x+4)}$

$= \int (x+4)e^u \cdot \frac{du}{6(x+4)}$

$= \int \frac{1}{6}e^u\,du = \frac{1}{6}e^u + C$

$= \frac{1}{6}e^{(3x^2+24x)} + C$

$\int f(x)\,dx$ = antiderivative of $f(x)$

why c?

$f(x) = 3x^2+5$ $f'(x) = 6x$

$f(x) = 3x^2+10$ $f'(x) = 6x$

$u = 8x+3$ $f(x) = 3x^2$ $f'(x) = 6x$

$du = (8)\,dx$.

Problem 5. Find the following indefinite integrals using the appropriate u-substitution.

(1) $\int 7(8x+3)^{10}\,dx$

$u = 8x+3$

$\frac{du}{dx} = 8 \Rightarrow du = 8\,dx \Rightarrow \frac{du}{8} = dx$

$$= 7\int u^{10}\cdot\frac{du}{8} = \frac{7}{8}\cdot\frac{u^{11}}{11} + C = \frac{7}{88}(8x+3)^{11} + C$$

(2) $\int 2x^2\sqrt[4]{x^3+2}\,dx$

x^3+2 & $2x^2$

$u = x^3+2$

$\frac{du}{dx} = 3x^2 \Rightarrow du = 3x^2dx \Rightarrow \frac{du}{3x^2} = dx$

$$= 2\int x^2\cdot\sqrt[4]{u}\cdot\frac{du}{3x^2}$$

$$= \frac{2}{3}\int u^{1/4}\,du$$

$$= \frac{2}{3}\cdot\frac{u^{1/4+1}}{1/4+1} = \frac{2}{3}\frac{u^{5/4}}{\left(\frac{5}{4}\right)} = \frac{2}{3}\cdot\frac{4}{5}\cdot u^{5/4} + c = \frac{8}{15}(x^3+2)^{5/4} + C$$

Ans.

(3) $\int \frac{3(x^3+1)}{(3x^4+12x)^7}\,dx$

$$= \int 3(x^3+1)(3x^4+12x)^{-7}\,dx$$

$u = 3x^4+12x$

$\frac{du}{dx} = 12x^3+12 = 12(x^3+1)$

$du = [12(x^3+1)]dx$

$\frac{du}{12(x^3+1)} = dx$

$$= 3\int (x^3+1)\,u^{-7}\cdot\frac{du}{12(x^3+1)}$$

$$= \frac{3}{12}\int u^{-7}\,du$$

$$= \frac{1}{4}\cdot\frac{u^{-6}}{(-6)} + C = -\frac{1}{24}(3x^4+12x)^{-6} + C$$

(4) $\int \frac{12x}{3x^2+5}\,dx$

$u = 3x^2+5$

$\frac{du}{dx} = 3\cdot(2x)+0 = 6x$

$du = 6x\,dx$

$\frac{du}{6x} = dx$

$= \int \frac{12x}{u}\cdot\frac{du}{6x}$

$= 2\int \frac{1}{u}\,du$

$= 2\ln|u| + c$

$= 2\ln|3x^2+5| + c$ Ans.

(5) $\int x^6 e^{x^7-1}\,dx$

$u = x^7-1$

$\frac{du}{dx} = 7x^6 \Rightarrow du = 7x^6\,dx$

$\frac{du}{7x^6} = dx$

$= \int (x^6)\cdot e^u \cdot \frac{du}{7x^6}$

$= \frac{1}{7}\int e^u\,du = \frac{1}{7}e^u + c$

$= \frac{1}{7}e^{x^7-1} + c$ Ans.

(6) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

$u = e^x + e^{-x}$

$\frac{du}{dx} = e^x + e^{-x}\cdot(-1) = e^x - e^{-x}$

$du = (e^x - e^{-x})\,dx$

$\frac{du}{e^x - e^{-x}} = dx$

$= \int \frac{(e^x - e^{-x})}{u}\cdot\frac{du}{(e^x-e^{-x})}$

$= \int \frac{du}{u} = \ln|u| + c$

$= \ln|e^x + e^{-x}| + c$ Ans.

6

choices are $\frac{1}{x}$ and $\frac{1}{x^2}$

(7) $\int \frac{e^{1/x}}{x^2}\, dx$

$= \int (e^{\frac{1}{x}}) \cdot \left(\frac{1}{x^2}\right) dx$

$= \int e^u \left(\frac{1}{x^2}\right)(-x^2\, du)$

$= \int -e^u\, du = -\int e^u\, du = -e^u + C$

$= -e^{1/x} + C$ Ans.

$u = \frac{1}{x} = x^{-1}$

$\frac{du}{dx} = (-1)x^{-2} = -\frac{1}{x^2}$

$du = \left(-\frac{1}{x^2}\right) dx$

$\frac{du}{\left(-\frac{1}{x^2}\right)} = dx$

$-x^2\, du = dx$

(8) $\int \frac{\ln(5x)}{2x}\, dx$

$= \int \ln(5x) \cdot \left(\frac{1}{2x}\right) dx$

$= \int u \cdot \left(\frac{1}{2x}\right)(x\, du)$

$= \frac{1}{2}\int u\, du = \frac{1}{2} \cdot \frac{u^2}{2} + C$

$= \frac{1}{4}[\ln(5x)]^2 + C$ Ans.

$u = \ln(5x)$

$\frac{du}{dx} = \frac{1}{5x}(5) = \frac{1}{x}$

$du = \left(\frac{1}{x}\right) dx$

$\frac{du}{\left(\frac{1}{x}\right)} = dx$

$x\, du = dx$

(9) $\int \frac{1}{2x\ln(5x)}\, dx$

$= \int \left(\frac{1}{2}\right) \cdot \left(\frac{1}{x}\right) \cdot \left(\frac{1}{\ln(5x)}\right) dx$

$= \frac{1}{2}\int \left(\frac{1}{x}\right) \cdot \frac{1}{u} \cdot x\, du$

$= \frac{1}{2}\int \frac{1}{u} \cdot du = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|\ln(5x)| + C$ Ans.

$u = \ln(5x)$

$\frac{du}{dx} = \frac{1}{5x}(5) = \frac{1}{x}$

$x\, du = dx$

$S(0) = 1800$
$t = 0.$

Problem 6. Yearly sales of a particular item are expected to decrease at a rate of $s(t) = -24t^{2/3}$ items per year, where t is time, in years. If yearly sales now are 1800 items, find a function $S(t)$, which will represent the number of items sold each year.

$$S(t) = \int s(t)\,dt = \int -24t^{2/3}\,dt = -24\int t^{2/3}\,dt$$

$$= -24 \cdot \frac{t^{2/3+1}}{2/3+1} = (-24) \cdot \frac{t^{5/3}}{\frac{5}{3}} + C$$

$$S(t) = (-24)\left(\frac{3}{5}\right) t^{5/3} + C$$

plug in $t=0$.

$$S(0) = (-24)\left(\frac{3}{5}\right) 0^{5/3} + C = 1800 \Rightarrow C = 1800$$

Ans: $S(t) = (-24)\left(\frac{3}{5}\right) \cdot t^{5/3} + 1800$

Problem 7. The marginal revenue function for a company that sells barbecue grills is given by $R'(x) = -0.08x + 350$ dollars per grill sold. Find the company's revenue function, in dollars, when x grills are sold.

$R(x)$

$$R'(x) = -0.08x + 350$$

Initial condition
⇓
$R(0) = 0$

$$R(x) = \int R'(x)\,dx$$
$$= \int (-0.08x + 350)\,dx$$

$$\int 350\,dx = 350\int 1\,dx = 350 \cdot x$$

$$R(x) = (-0.08)\frac{x^2}{2} + 350 \cdot x + C$$

($-0.08/2 = -0.04$)

@ $x=0$, $R(x) = 0 + 0 + C = 0 \Rightarrow C = 0.$

Ans: $R(x) = -0.04x^2 + 350x$

$\frac{d}{dx}(x) = 1$

1 $\xrightarrow{\text{antiderivative}}$ x

8

Problem 8. If the marginal cost function for a company is given by $f(x) = 0.12e^{0.04x}$ dollars per item, and if the company has fixed costs of \$3000, where x represents the number of items produced, find the company's total cost when 150 items are produced.

need to find cost function first.

fixed cost is cost when x=0 items are produced.

$$C(x) = \int f(x)dx = \int 0.12e^{0.04x}\,dx$$

$$= (0.12)\int e^{0.04x}\,dx$$

$u = 0.04x$

$\frac{du}{dx} = 0.04$

$dx = \frac{du}{0.04}$

$$= (0.12)\int e^u \cdot \frac{du}{0.04}$$

$$= \frac{0.12}{0.04}\int e^u\,du$$

$$C(x) = 3e^u + C = 3e^{0.04x} + C$$

use x=0, $C(0) = 3 \cdot e^0 + C$

$= 3 + C = 3000$

$\therefore C = 3000 - 3 = 2997$

$$\therefore C(x) = 3e^{0.04x} + 2997$$

$$C(150) = 3e^{0.04(150)} + 2997$$

$= \$4207.29$