Math 308: Week-in-Review 11

Shelvean Kapita

1. Find the general solution of the system and the fundamental matrix. Classify the type of the critical point, and determine whether it is stable or unstable. Sketch the phase portrait.

(a)

$$\mathbf{x}' = \underbrace{\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}}_{A} \mathbf{x}$$

$\text{tr } A = 3 - 2 = 1$

$\det A = (3)(-2) - (2)(-2)$
$= -6 + 4$
$= -2$

negative determinant

(1) Find eigenvalues

$\lambda^2 - \text{tr}(A)\lambda + \det A = 0$

$\Rightarrow \lambda^2 - \lambda - 2 = 0 \Rightarrow (\lambda - 2)(\lambda + 1) = 0$

$\lambda_1 = 2, \ \lambda_2 = -1$ (opposite sign eigenvalues) $\Rightarrow$ saddle point

always unstable

(2) Find eigenvectors

$\lambda_1 = 2$: $(A - 2I)\vec{v}_1 = \vec{0} \Rightarrow \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} x_1 - 2x_2 = 0 \\ 2x_1 - 4x_2 = 0 \end{cases}$

$\Downarrow$

$\vec{v}_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda_2 = -1$: $(A + I)\vec{v}_2 = \vec{0} \Rightarrow \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \{ 2x_1 - x_2 = 0$

(need just one equation)

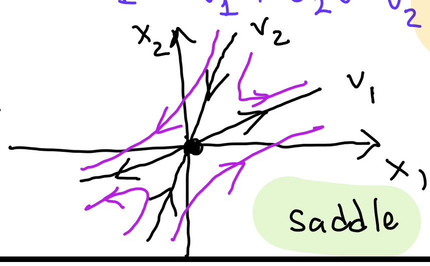
$\Downarrow$

$\vec{v}_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(3) General solution:

$x(t) = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2 = c_1 e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Phase portrait



saddle

(b)

$$\underbrace{\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}}_{A} \mathbf{x}$$

tr $A = 3-1 = 2$

det $A = (3)(-1)-(1)(-4) = 1$

$(\text{tr } A)^2 - 4 \det A = 2^2 - 4\cdot 1 = 0$

$\Downarrow$ unstable node (deficient) ; repeated eigenval

(1) Find eigenvalues

$\lambda^2 - \text{tr}(A)\lambda + \det A = 0$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda-1)^2 = 0 \Rightarrow \lambda = 1$ (repeated)

(2) Find eigenvector(s)

$(A-I)\vec{v} = \vec{0} \Rightarrow \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_1 - 2x_2 = 0 \Rightarrow \vec{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

One solution: $\vec{x}_1(t) = e^t\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find $\vec{x}_2(t)$.

$\vec{x}_2(t) = e^t\left[t\vec{v} + \vec{u}\right]$ where $\vec{u}$ solves $(A-I)\vec{u} = \vec{v}$

$\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow u_1 - 2u_2 = 1$. Set, eg, $u_2 = 0 \Rightarrow u_1 = 1$

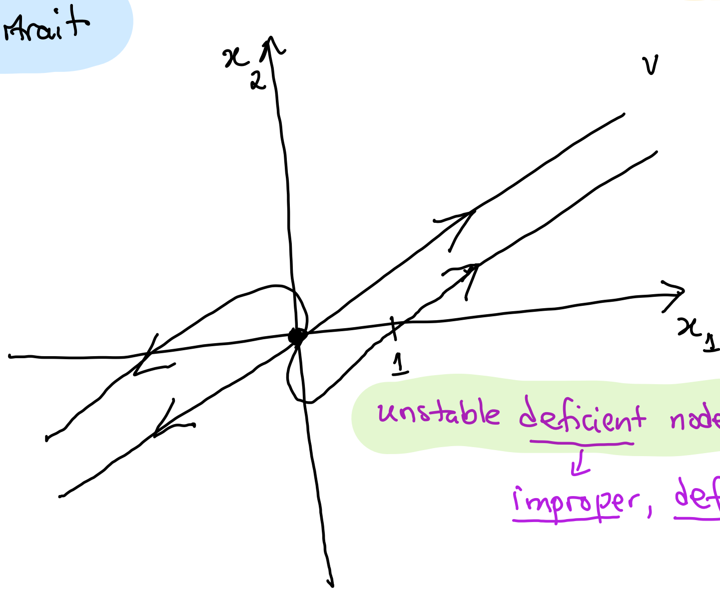
$\Downarrow$

$\vec{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(3) General solution:

$\vec{x}(t) = c_1 e^t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^t\left[t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] = c_1 e^t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^t\begin{pmatrix} 2t+1 \\ t \end{pmatrix}$

Phase portrait



$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

direction vector @ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

unstable deficient node

$\Downarrow$

improper, defective

(c)

$$\mathbf{x}' = \underbrace{\begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix}}_{A} \mathbf{x}$$

$\operatorname{tr} A = 1-3 = -2$

$\det A = (1)(-3)-(5)(-1) = 2$

$(\operatorname{tr} A)^2 - 4\det A = 4 - 4(2) = -4$

$\Downarrow$

stable spiral

(1) Find eigenvalues

$\lambda^2 - \operatorname{tr}(A)\lambda + \det A = 0$

$\lambda^2 + 2\lambda + 2 = 0 \Rightarrow \lambda = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$\lambda_{1,2} = -1 \pm i$

(2) Find eigenvectors

$(A - (-1+i)I)\vec{v} = \vec{0} \Rightarrow \begin{pmatrix} 1-(-1+i) & -1 \\ 5 & -3-(-1+i) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2-i & -1 \\ 5 & -2-i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$(2-i)x_1 = x_2 \Rightarrow \vec{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2-i \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} i$

(3) General solution ; if eigenvalues $\lambda_{1,2} = \alpha + i\beta$ & $\vec{v} = \vec{A} + i\vec{B}$ then

$$\vec{x}(t) = c_1 e^{\alpha t}\left[\vec{A}\cos(\beta t) - \vec{B}\sin(\beta t)\right] + c_2 e^{\alpha t}\left[\vec{B}\cos(\beta t) + \vec{A}\sin(\beta t)\right]$$

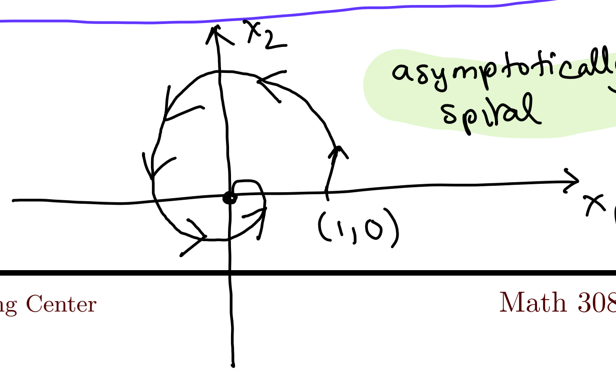
$$x(t) = c_1 e^{-t}\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\cos(t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix}\sin(t)\right] + c_2 e^{-t}\left[\begin{pmatrix} 0 \\ -1 \end{pmatrix}\cos(t) + \begin{pmatrix} 1 \\ 2 \end{pmatrix}\sin(t)\right]$$

$$x(t) = c_1 e^{-t}\begin{bmatrix} \cos(t) \\ 2\cos(t) + \sin(t) \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} \sin t \\ 2\sin(t) - \cos(t) \end{bmatrix}$$

« general solution

$\begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

direction vector @ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$



asymptotically stable spiral

2. Classify the types and stability of the equilibrium point(s) of the system

$$\mathbf{x}' = \underbrace{\begin{pmatrix} \alpha - 1 & \alpha + 1 \\ -2/3 & 1/3 \end{pmatrix}}_{A} x$$

for different values of the parameter α.

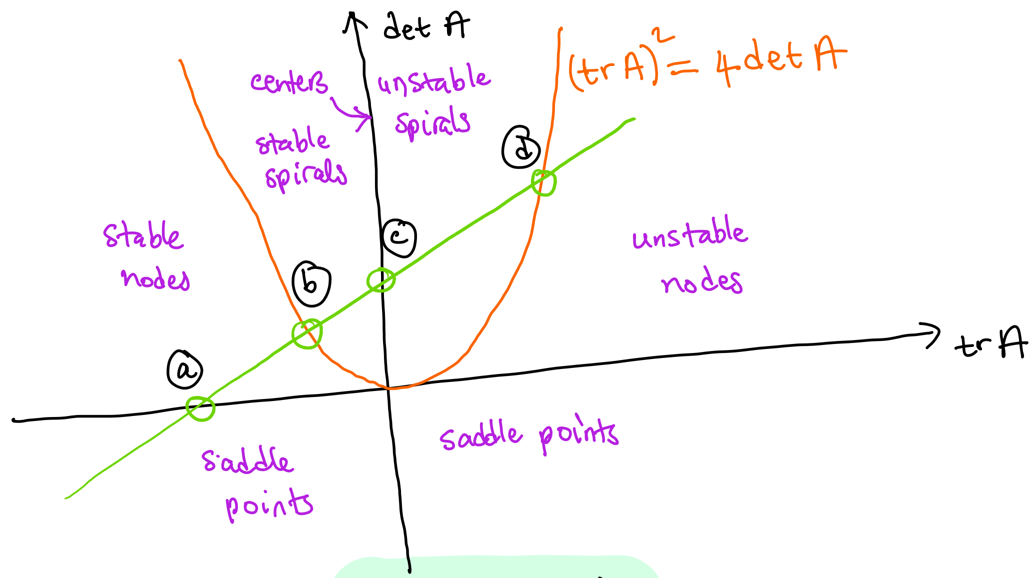
$\operatorname{tr} A = (\alpha - 1) + \frac{1}{3} = \alpha - \frac{2}{3}$

$\det A = \frac{1}{3}(\alpha - 1) + \frac{2}{3}(\alpha + 1)$

$= \alpha + \frac{1}{3}$

$(\operatorname{tr} A, \det A) = (\alpha - 2/3, \alpha + 1/3)$

$\Rightarrow \det A = \operatorname{tr}(A) + 1$



(a) $\det A = \alpha + \frac{1}{3} = 0 \Rightarrow \alpha < -1/3$ (saddle points)

(b),(d) $(\operatorname{tr} A)^2 = 4 \det A \Rightarrow (\operatorname{tr} A)^2 = 4(\operatorname{tr} A + 1) \Rightarrow (\operatorname{tr} A)^2 - 4 \operatorname{tr} A - 4 = 0$

$\Rightarrow \operatorname{tr} A = \frac{4 \pm \sqrt{16 + 16}}{2} = 2 \pm \sqrt{8} = \alpha - 2/3 \Rightarrow \alpha = \frac{8}{3} \pm \sqrt{8}$

(b) $-\frac{1}{3} < \alpha < \frac{8}{3} - \sqrt{8}$: (asymptotically stable nodes)

(c) $\operatorname{tr} A = 0 \Rightarrow \alpha - 2/3 = 0 \Rightarrow \alpha = 2/3$: $\frac{8}{3} - \sqrt{8} < \alpha < 2/3$: (asymptotically stable spirals)

$\alpha = 2/3$ (center)

(d) $2/3 < \alpha < \frac{8}{3} + \sqrt{8}$: (unstable spirals)

(e) $\alpha > \frac{8}{3} + \sqrt{8}$: (unstable nodes)

Copyright © 2024 TAMU Math Learning Center

3. Find the general solution of the non-homogeneous system

$$\mathbf{x}' = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\mathbf{x} + \underbrace{\begin{pmatrix} t^{-1} \\ 2t^{-1}+4 \end{pmatrix}}_{\vec{g}}, \quad t > 0.$$

$\text{tr} A = -5$
$\det A = 0$

* Method of variation of parameters *

$\vec{x}(t) = \vec{x}_c(t) + \vec{x}_p(t) \rightarrow$ particular

$\hookrightarrow$ homogeneous

(1) Homogeneous : $\lambda^2 + \text{tr}(A)\lambda + \det A = 0 \Rightarrow \lambda^2 + 5\lambda = 0 \Rightarrow \lambda(\lambda+5) = 0$

$\lambda_1 = 0$ and $\lambda_2 = -5$

Eigenvectors: $\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 2x_1 - x_2 = 0 \Rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow \vec{x}_1(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_1 + 2x_2 = 0 \Rightarrow \vec{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \Rightarrow \vec{x}_2(t) = e^{-5t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$$\vec{x}_c(t) = c_1\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-5t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$\uparrow$ homogeneous

Fundamental Matrix: $\Psi(t) = \begin{pmatrix} 1 & -2e^{-5t} \\ 2 & e^{-5t} \end{pmatrix}$, $\vec{x}_p = \Psi(t)\vec{u}(t)$

where $\vec{u}(t) = \int \Psi^{-1}(t) g(t)\, dt$

$$\Psi^{-1}(t) = \frac{1}{\det \Psi(t)}\begin{bmatrix} e^{-5t} & 2e^{-5t} \\ -2 & 1 \end{bmatrix} = \frac{1}{5e^{-5t}}\begin{bmatrix} e^{-5t} & 2e^{-5t} \\ -2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2e^{5t} & e^{5t} \end{bmatrix}$$

$$\Psi^{-1}(t) g(t) = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2e^{5t} & e^{5t} \end{bmatrix}\begin{bmatrix} t^{-1} \\ 2t^{-1}+4 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} t^{-1} + 4t^{-1} + 8 \\ -2t^{-1}e^{5t} + 2t^{-1}e^{5t} + 4e^{5t} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 5t^{-1} + 8 \\ 4e^{5t} \end{bmatrix}$$

$$\vec{u} = \int \begin{bmatrix} t^{-1} + 8/5 \\ \frac{4}{5}e^{5t} \end{bmatrix} dt = \begin{bmatrix} \ln t + \frac{8}{5}t \\ \frac{4}{25}e^{5t} \end{bmatrix}, \quad \vec{x}_p = \Psi(t)\vec{u} = \begin{pmatrix} 1 & -2e^{-5t} \\ 2 & e^{-5t} \end{pmatrix}\begin{pmatrix} \ln t + \frac{8}{5}t \\ \frac{4}{25}e^{5t} \end{pmatrix}$$

$$= \begin{pmatrix} \ln t + \frac{8}{5}t - \frac{8}{25} \\ 2\ln t + \frac{16}{5}t + \frac{4}{25} \end{pmatrix}$$

$\uparrow$ particular

general solution

$$\vec{x}(t) = c_1\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-5t}\begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} \ln t + \frac{8}{5}t - \frac{8}{25} \\ 2\ln t + \frac{16}{5}t + \frac{4}{25} \end{pmatrix}$$

Copyright © 2024 TAMU Math Learning Center

4. Find a particular solution of

non-constant matrix
can't use eigenvalue methods

$$\mathbf{x}' = \begin{pmatrix} e^{2t} & -1 \\ e^{4t} & e^{2t} \end{pmatrix} \mathbf{x} + \underbrace{\begin{pmatrix} 1 \\ e^{2t} \end{pmatrix}}_{\vec{g}}$$

* Method of variation of parameters *

given the fundamental matrix

$$\Psi(t) = \begin{pmatrix} e^{2t} & -1 \\ e^{4t} & e^{2t} \end{pmatrix}$$ ← homogeneous solutions

$\vec{x}_p = \Psi(t)\vec{u}(t)$ where $\vec{u}(t) = \int \Psi^{-1}(t)\vec{g}(t)\,dt$

$$\Psi^{-1}(t) = \frac{1}{\det \Psi(t)}\begin{bmatrix} e^{2t} & 1 \\ -e^{4t} & e^{2t} \end{bmatrix} = \frac{1}{2e^{4t}}\begin{bmatrix} e^{2t} & 1 \\ -e^{4t} & e^{2t} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} e^{-2t} & e^{-4t} \\ -1 & e^{-2t} \end{bmatrix}$$

$$\Psi^{-1}(t)g(t) = \frac{1}{2}\begin{bmatrix} e^{-2t} & e^{-4t} \\ -1 & e^{-2t} \end{bmatrix}\begin{bmatrix} 1 \\ e^{2t} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2e^{-2t} \\ 0 \end{bmatrix} = \begin{bmatrix} e^{-2t} \\ 0 \end{bmatrix}$$

$$\vec{u}(t) = \int \Psi^{-1}(t)g(t)\,dt = \int \begin{bmatrix} e^{-2t} \\ 0 \end{bmatrix} dt = -\frac{1}{2}\begin{bmatrix} e^{-2t} \\ 0 \end{bmatrix}$$

$$\vec{x}_p = \Psi(t)\vec{u}(t) = -\frac{1}{2}\begin{pmatrix} e^{2t} & -1 \\ e^{4t} & e^{2t} \end{pmatrix}\begin{pmatrix} e^{-2t} \\ 0 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 1 \\ e^{2t} \end{pmatrix}$$

$$\vec{x}_p(t) = -\frac{1}{2}\begin{pmatrix} 1 \\ e^{2t} \end{pmatrix}$$

5. Apply the method of *undetermined coefficients* to find a particular solution of the non-homogeneous system

$$\mathbf{x}' = \begin{pmatrix} 7 & -4 \\ 2 & 3 \end{pmatrix}\mathbf{x} + \underbrace{\begin{pmatrix} 9e^t \\ 25e^{-t} \end{pmatrix}}_{\vec{g}} \qquad \vec{g}(t) = \underbrace{\begin{pmatrix} 9e^t \\ 0 \end{pmatrix}}_{\vec{g}_1} + \underbrace{\begin{pmatrix} 0 \\ 25e^{-t} \end{pmatrix}}_{\vec{g}_2}$$

General solution:

$\vec{x}(t) = \vec{x}_c(t) + \vec{x}_p = \vec{x}_c + \vec{x}_p^{(1)} + \vec{x}_p^{(2)}$

where $\vec{x}_p^{(1)}(t)$ is the particular solution of

$x' = \begin{pmatrix} 7 & -4 \\ 2 & 3 \end{pmatrix}x + \begin{pmatrix} 9e^t \\ 0 \end{pmatrix}$ and $\vec{x}_p^{(2)}(t)$ is the particular

solution of $x' = \begin{pmatrix} 7 & -4 \\ 2 & 3 \end{pmatrix}x + \begin{pmatrix} 0 \\ 25e^{-t} \end{pmatrix}$

Homogeneous Solution:

$$\lambda^2 - 10\lambda + 29 = 0 \Rightarrow \lambda = \frac{10 \pm \sqrt{100 - 116}}{2} = 5 \pm 2i$$

$$\begin{pmatrix} 7-(5+2i) & -4 \\ 2 & 3-(5+2i) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 2-2i & -4 \\ 2 & -2-2i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1 = (1+i)x_2 \Rightarrow \vec{v} = \begin{pmatrix} 1+i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}i$$

$$\vec{x}_c = c_1 e^{5t}\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}\cos(2t) - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\sin(2t)\right] + c_2 e^{5t}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\cos(2t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix}\sin(2t)\right]$$

$$= c_1 e^{5t}\begin{bmatrix} \cos(2t) - \sin(2t) \\ \cos(2t) \end{bmatrix} + c_2 e^{5t}\begin{bmatrix} \cos(2t) + \sin(2t) \\ \sin(2t) \end{bmatrix}$$

Particular solution(s)

$$x_p^{(1)} = \vec{a}e^t = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}e^t \Rightarrow x_p^{(1)\prime}(t) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}e^t$$

* Method of undetermined coefficients *

$$x_p^{(1)\prime} = Ax_p^{(1)} + g_1 \Rightarrow \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}e^t = \begin{pmatrix} 7a_1 - 4a_2 + 9 \\ 2a_1 + 3a_2 \end{pmatrix}e^t$$

$$\begin{bmatrix} -6 & 4 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix} \Leftarrow \begin{matrix} -6a_1 + 4a_2 = 9 \\ -2a_1 - 2a_2 = 0 \end{matrix} \Leftarrow \begin{cases} a_1 = 7a_1 - 4a_2 + 9 \\ a_2 = 2a_1 + 3a_2 \end{cases}$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} 9 \\ 0 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} -18 \\ 18 \end{bmatrix} = \frac{9}{10}\begin{bmatrix} -1 \\ 1 \end{bmatrix} \Rightarrow \vec{a} = \frac{9}{10}\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^t$$

Copyright © 2024 TAMU Math Learning Center

$$x_p^{(2)}(t) = \vec{b}e^{-t} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} e^{-t} \quad , \quad x_p^{(2)\prime}(t) = \begin{pmatrix} -b_1 \\ -b_2 \end{pmatrix} e^{-t}$$

$$x_p^{(2)\prime}(t) = A x_p^{(2)} + \vec{g}_2$$

$$\begin{pmatrix} -b_1 \\ -b_2 \end{pmatrix} e^{-t} = \begin{pmatrix} 7b_1 - 4b_2 \\ 2b_1 + 3b_2 + 25 \end{pmatrix} e^{-t}$$

$$\begin{cases} -b_1 = 7b_1 - 4b_2 \\ -b_2 = 2b_1 + 3b_2 + 25 \end{cases} \Rightarrow \begin{cases} -8b_1 + 4b_2 = 0 \\ -2b_1 - 4b_2 = 25 \end{cases}$$

$$\Downarrow$$

$$\begin{bmatrix} -8 & 4 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} -4 & -4 \\ 2 & -8 \end{bmatrix} \begin{bmatrix} 0 \\ 25 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} -100 \\ -200 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} -5/2 \\ -5 \end{bmatrix}$$

$$\vec{x}_p = \frac{9}{10} \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{t} - \frac{5}{2} \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}$$