Math 150 - Week-In-Review 8

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Problem Statements, sections 6.1, 6.2, 7.1

1. Solve the following system of linear equations.

$$\begin{cases} -3x + 5y = 20 \\ \frac{1}{8}x - \frac{5}{24}y = -\frac{5}{6} \end{cases}$$

Addition Method:

$\xrightarrow{\times \frac{1}{8}}$ $-\frac{3}{8}x + \frac{5}{8}y = \frac{20}{8}$

$\xrightarrow{\times 3}$ $\frac{3}{8}x - \frac{15}{24}y = -\frac{15}{6}$

simplify $\begin{cases} -\frac{3}{8}x + \frac{5}{8}y = \frac{5}{2} \\ \frac{3}{8}x - \frac{5}{8}y = -\frac{5}{2} \end{cases}$

add

$0 = 0$ infinitely many solutions (Dependant sys.)

Parametric Solution:

Choose a free variable x

Choose a parameter t

let $x = t$

then $-3(t) + 5y = 20$

$5y = 20 + 3t$

$y = 4 + \frac{3}{5}t$

Solution set:

$(x, y) = (t, 4 + \frac{3}{5}t)$

2. Solve the following system of nonlinear equations.

$$\begin{cases} (x+2)^2 + y^2 = 2 \\ y - \sqrt{x} = 0 \end{cases}$$

$\rightsquigarrow y = \sqrt{x}$

Substitution method:

$(x+2)^2 + (\sqrt{x})^2 = 2$

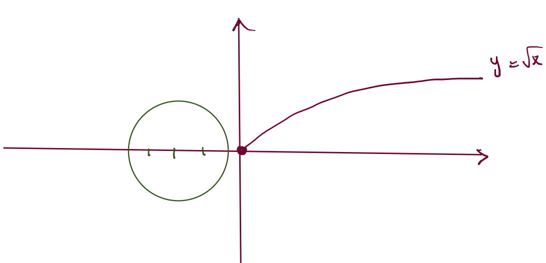
$x^2 + 4x + 4 + x = 2$

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{25 - 4(2)}}{2} = \frac{-5 \pm \sqrt{25 - 8}}{2}$

$x_1 = \frac{-5 + \sqrt{17}}{2} < 0$ Not in domain of second equation

$x_2 = \frac{-5 - \sqrt{17}}{2} < 0$



$\Rightarrow$ No solutions!

3. Find all solutions to the system of equations

$$\begin{cases} x^2 + y^2 = 25 \\ xy = 12 \end{cases}$$

let $y = \frac{12}{x}$

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$x^2 + \frac{144}{x^2} = 25$

(Substitution method)

$x^4 + 144 = 25x^2$

$(x^2)^2 - 25x^2 + 144 = 0$

let $u = x^2$

$u^2 - 25u + 144 = 0$

$(u-9)(u-16) = 0$

$u = 9 \qquad x^2 = 9 \Rightarrow x = \pm 3$

$u = 16 \qquad x^2 = 16 \qquad x = \pm 4$

$y = \frac{12}{x}$

let $x = 4 \rightsquigarrow y = 3$

let $x = -4 \rightsquigarrow y = -3$

let $x = 3 \rightsquigarrow y = 4$

let $x = -3 \rightsquigarrow y = -4$

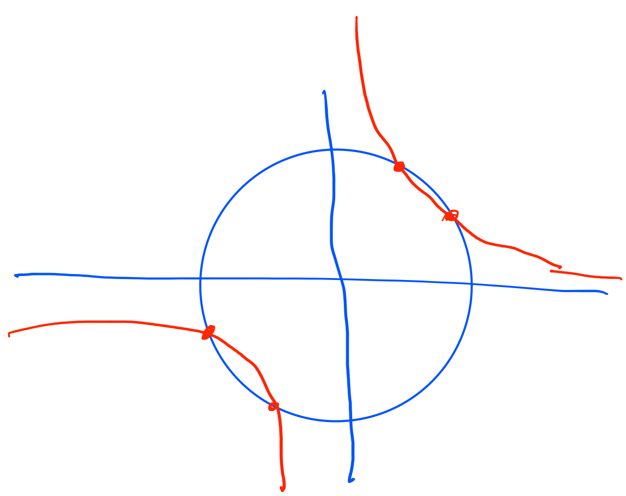
Check your answers by plugging the points back into both equations

check
$(x,y) = (4,3)$
$(4)^2 + (3)^2 = 25$
$16 + 9 = 25$ ✓
$(4)(3) = 12$
$12 = 12$ ✓

check
$(x,y) = (-4,-3)$
$(-4)^2 + (-3)^2 = 25$
$16 + 9 = 25$ ✓
$(-4)(-3) = 12$
$12 = 12$ ✓

check
$(x,y) = (3,4)$
$(3)^2 + (4)^2 = 25$
$9 + 16 = 25$ ✓
$(3)(4) = 12$
$12 = 12$ ✓

Check
$(x,y) = (-3,-4)$
$(-3)^2 + (-4)^2 = 25$
$9 + 16 = 25$ ✓
$(-3)(-4) = 12$
$12 = 12$ ✓



Solutions:

$(4,3)$, $(-4,-3)$, $(3,4)$, $(-3,-4)$

4. Determine all solutions to the following system.

$$\begin{cases} \sqrt{y} - x = -1 \\ y = x^2 - 3x - 6 \end{cases}$$

substitution method: $\sqrt{x^2-3x-6} - x = -1$

$\sqrt{x^2-3x-6} = x - 1$

Square both sides $\Rightarrow$ $\cancel{x^2} - 3x - 6 = \cancel{x^2} - 2x + 1$

$-7 = x$

$y = (-7)^2 - 3(-7) - 6$

$= 49 + 21 - 6 = 64$

Check $(x, y) = (-7, 64)$

$\sqrt{64} - (-7) \stackrel{?}{=} -1$

$8 + 7 \neq -1$

extraneous

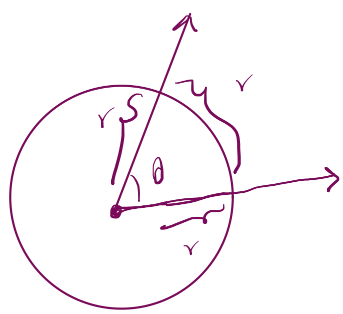
$\Rightarrow$ No solutions.

5. What is an angle?

    a) A measure of how far two points are from each other.
    b) A measure of rotation between two intersecting lines or rays.
    c) The distance between two parallel lines.
    d) The product of two line segments.

6. What is the definition of a radian?

    a) The angle formed by two perpendicular rays.
    b) The angle formed when the arc length is equal to the circle's radius.
    c) A measurement unit used for very small angles.
    d) The angle at the center of a semicircle.



$\theta = 1$ rad.

7. Convert 135° to a fraction of a full circle.

135° is $\dfrac{135^\circ}{360^\circ} \overset{\div 5}{=} \dfrac{27}{72} \overset{\div 9}{=} \dfrac{3}{8}$ th of a full revolution

8. Convert 63° to radians.

$63^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{63\pi}{180} \text{ rad} = \frac{21\pi}{60} = \frac{7\pi}{20} \text{ rad}$

9. Convert $\frac{17\pi}{15}$ to degrees.

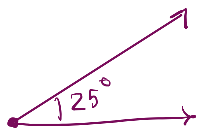
$\frac{17\pi}{15} \text{ rad} \times \frac{180^\circ}{\pi} = \frac{17 \times 180^\circ}{15} = 17 \times 12^\circ = 204^\circ$

10. If an angle measures 210°, what is its radian equivalent?

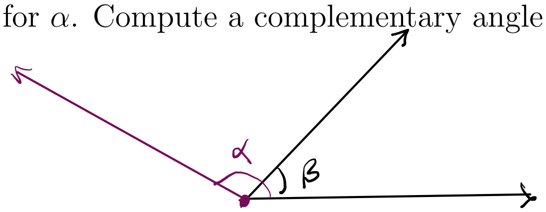
$210^\circ \times \frac{\pi}{180^\circ} = \frac{210\pi}{180} \text{ rad} = \frac{7\pi}{6} \text{ rad}$

11. If an angle measures 25°, what type of angle is it?

acute (i.e. $0 < \theta < 90$)



12. Let $\alpha = 135°$ and $\beta = 55°$. Sketch α and β in standard position. Compute a supplementary angle for α. Compute a complementary angle for β.



θ_1 supplementary angle for α:

$\alpha + \theta_1 = 180°$

$\theta_1 = 180° - 135°$

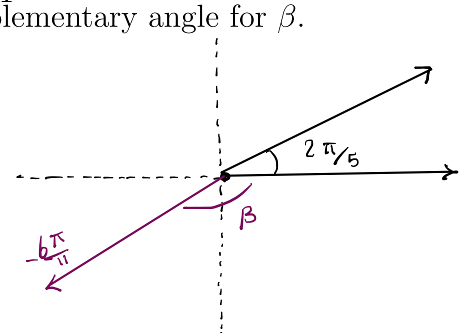
$\theta_1 = 45°$

θ_2 Complementary angle for β:

$\beta + \theta_2 = 90°$

$\theta_2 = 90° - 55° = 35°$

13. Let $\alpha = \dfrac{2\pi}{5}$ and $\beta = \dfrac{-6\pi}{11}$. Sketch α and β in standard position. Compute a supplementary angle for α. Compute a complementary angle for β.



θ_1 supplementary angle for α:

$\alpha + \theta_1 = \pi$

$\theta_1 = \pi - \dfrac{2\pi}{5} = \dfrac{3\pi}{5}$ rad

θ_2 Complementary angle for β:

$\beta + \theta_2 = \dfrac{\pi}{2}$

$\theta_2 = \dfrac{\pi}{2} - \left(-\dfrac{6\pi}{11}\right)$

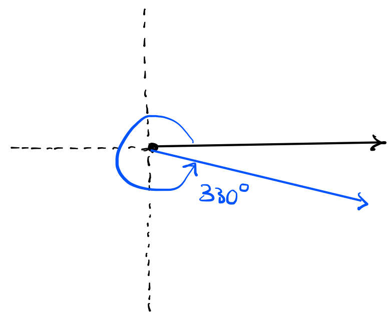
$= \dfrac{11\pi}{22} + \dfrac{12\pi}{22} = \boxed{\dfrac{23\pi}{22}}$ rad

14. Sketch and find two coterminal angles for:

a) $\theta = 330°$

$330° - 360° = -30°$

$330° + 360° = 690°$

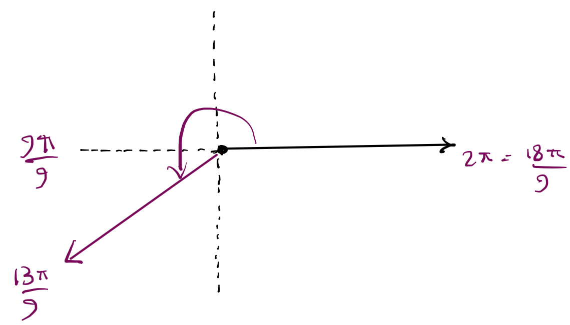


b) $\theta = \dfrac{13\pi}{9}$

Coterminals:

$\dfrac{13\pi}{9} - 2\pi = -\dfrac{5\pi}{9}$

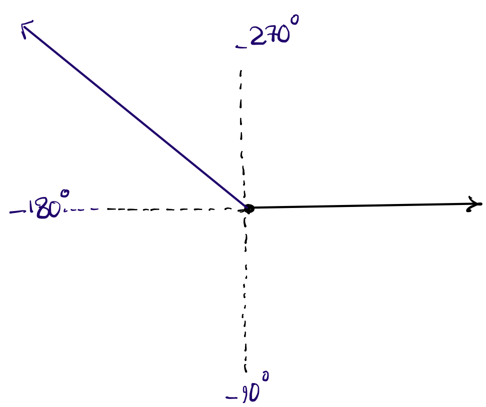
$\dfrac{13\pi}{9} + 2\pi = \dfrac{31\pi}{9}$



c) $\theta = -255°$.

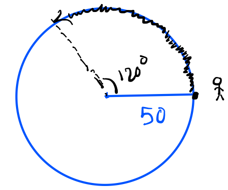
$-225° + 360° = 105°$

$-225° - 360° = -615°$



15. A circular track has a radius of 50 meters. An athlete runs along the track, covering an angle of 120°. How far does the athlete run along the circular path?

angle in arc length should be in radians

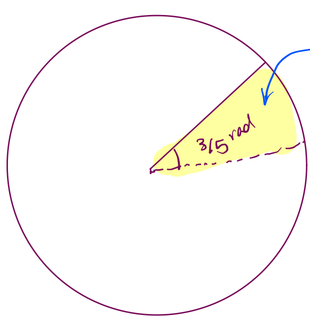


$\theta = 120^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{2\pi}{3}$ rad

arc length $= r \cdot \theta = 50 \times \frac{2\pi}{3} = \frac{100\pi}{3}$ meters

The athlete runs $\frac{100\pi}{3}$ meters along the circular path.

16. A circular sector created by a central angle of $\frac{3}{5}$ radians has an area of 1080 ft^2, determine the radius of the circle. Note: area of the sector is found by $\frac{\theta}{2}r^2$, where θ is measured in radians.



Area of a circle $A = \pi r^2$

Area of a sector with angle θ: $A_{sec} = \frac{r^2 \cdot \theta}{2}$

$1080 = \frac{r^2 \cdot \left(\frac{3}{5}\right)}{2}$

$\frac{5}{3} \times 2160 = r^2$

$5 \times 720 = r^2$

$r = +\sqrt{3600} = +60$ ft

17. A boy rotates a stone in a 3 ft long sling at a rate of 15 revolutions every 10 seconds. Find the angular and linear velocities of the stine.

15 rev 10 sec
x 1
$x = \frac{15}{10}$

$\bar{\omega} = \frac{15}{10}$ rev/sec $\times 2\pi$ rad/rev $= 3\pi$ rad/sec

$\bar{v} = r\bar{\omega} = 3 \times 3\pi = 9\pi$ ft/sec

18. Fill in the following unit circle with the common angles in one full revolution. (i.e angles of the form $\frac{n\pi}{6}, \frac{n\pi}{4}, \frac{n\pi}{3}, n\pi$)

$\frac{\pi}{6}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$