Math 152 - Spring 2024
Exam 2 Review
"Hands On Grades Up"
Monday, March 18, 6-8 PM
ZACH 442/440

TEXAS A&M UNIVERSITY
Math Learning Center

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We will begin at 6PM. A problem will be displayed on the table monitors. Collaborate with your table on how to solve each problem. If you have a question, raise your hand. At the end of a predetermined number of minutes, the solutions will be displayed on the wall monitors. Feel free to take a picture of the solution, as the solutions are not posted. Be sure you write clearly in the free response questions, and justify each step with well written mathematics to avoid losing partial credit!

In order for us to cover as much content as possible, many of the area/volume questions are only setting up the integral.

(1) Evaluate $\int_0^{\pi/2} \cos^4 x \sin^3 x \, dx$.

(2) Find $\int \tan^8 x \sec^4 x \, dx$.

(3) Evaluate $\int_0^{\pi/6} \cos^2(5x)\,dx$.

(4) Using an appropriate trigonometric substitution, rewrite $\int_1^2 \frac{\sqrt{4-x^2}}{x}\,dx$ as an equivalent integral in terms of θ and $d\theta$ only. Simplify the integrand, but **do not evaluate the integral**.

(5) Evaluate $\displaystyle\int \frac{\sqrt{x^2-1}}{x^3}\,dx$.

(6) Find $\displaystyle\int \frac{2x+12}{x^3+4x^2+4x}\,dx$.

(7) Evaluate $\displaystyle\int_0^2 \frac{x^2+3x+12}{(x+1)(x^2+4)}\,dx.$

(8) Evaluate $\displaystyle\int_1^e \frac{1}{x(\ln x)^2}\,dx$ or show it diverges.

(9) Evaluate $\displaystyle\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$ or show it diverges.

(10) Determine whether the improper integral $\displaystyle\int_1^{\infty} \frac{\sin(6x) + 8}{\sqrt{x} + x^4}\, dx$ converges or diverges using The Comparison Theorem for Improper Integrals.

(11) Find a_9 for the sequence $\{a_n\}_{n=1}^{\infty} = \left\{-\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \frac{32}{81}, \cdots\right\}$.

(12) Consider the sequence $a_n = \left\{\frac{\cos n}{n}\right\}_{n=1}^{\infty}$.

(a) Is a_n bounded?

(b) Is a_n increasing, decreasing, or not monotonic?

(13) Determine whether the sequence converges or diverges. If it converges, what value does it converge to, if it diverges, explain why.

(a) $a_n = \arcsin\left(\dfrac{2n-1}{5-4n}\right)$

(b) $a_n = \ln(3n+1) - 2\ln(n)$

(14) Students often struggle with the concepts addressed in the items that follow. Collaborate with your table with the questions posed here:

(a) What is the difference between a sequence and a series?

(b) What does it mean when we say $\sum_{n=1}^{\infty} a_n$ converges?

(c) If $\lim_{n\to\infty} a_n = 2$, what can we say about $\sum_{n=1}^{\infty} a_n$?

(d) If $\lim_{n\to\infty} a_n = 0$, what can we say about $\sum_{n=1}^{\infty} a_n$?

(15) For the series $\sum_{n=1}^{\infty} a_n$, the n^{th} partial sum is given by $s_n = \cos\left(\frac{\pi}{n}\right)$.

(a) Find a_5

(b) Find $\sum_{n=1}^{\infty} a_n$

(c) What is $\lim_{n\to\infty} a_n$?

(16) Determine if the series $\sum_{n=1}^{\infty}\left[\frac{1}{2^n}-\frac{1}{2^{n+1}}\right]$ converges or diverges. If the series converges, find the sum. If it diverges, explain why.

(17) Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{5 \cdot 3^{2n}}$ converges or diverges. If the series converges, find the sum. If it diverges, explain why.

(18) Which of following series diverge using the Test for Divergence?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{5n+1}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{5}{n\sqrt{\ln n}}$

(c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{-n}$

(19) Prove the series $\displaystyle\sum_{n=3}^{\infty} \frac{5}{n\sqrt{\ln n}}$ diverges.

Remainder Estimate for the Integral Test: If $\sum_{n=1}^{\infty} a_n$ was shown to be convergent by the integral test, where $a_n = f(n)$, and s_n was used to approximate $\sum_{n=1}^{\infty} a_n$, then $R_n \leq \int_n^{\infty} f(x)\,dx$.

(20) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. What is the smallest value of n so that s_n approximates $\sum_{n=1}^{\infty} \frac{1}{n^3}$ to within $\frac{1}{205}$?

(21) Consider $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n^4}}{n^5}$.

a.) Prove the series converges. Fully support your conclusion by naming the test, including the necessary criteria for using the test.

b.) Using the Remainder Estimate for the Integral Test, find an upper bound for the remainder if we use s_8 to approximate the sum of the series.