SESSION 3: REVIEW EXAM #1

1. The daily cost, $C(x)$, (in dollars) a company incurs for making and selling x travel mugs is given the table below.

   (a) Find the average rate of change of cost on the following intervals: $[97, 100]$, $[98, 100]$, $[99, 100]$. Round your answer to four decimals if necessary.

x	96	97	98	99	100	101	102	103	104
$C(x)$	117.98	118.49	118.99	119.5	120	120.5	121	121.49	121.98

$$[97,100] \Longrightarrow \frac{C(100)-C(97)}{100-97} = \frac{120-118.49}{3} = \frac{1.51}{3} = .5033$$

$$[98,100] \Longrightarrow \frac{C(100)-C(98)}{100-98} = \frac{120-118.99}{2} = \frac{1.01}{2} = .505$$

$$[99,100] \Longrightarrow \frac{C(100)-C(99)}{100-99} = \frac{120-119.5}{1} = \frac{.5}{1} = .5$$

   (b) Use your answer from part (a) and the average rates of change on the intervals given below, to approximate the instantaneous rate of change of cost when 100 travel mugs are made and sold. Round your answer to four decimals if necessary.

- average rate of change on $[100, 104]$ is $\dfrac{121.98-120}{104-100} = \dfrac{1.98}{4} = 0.4950$ dollars per travel mug
- average rate of change on $[100, 103]$ is $\dfrac{121.49-120}{103-100} = \dfrac{1.49}{3} = 0.4967$ dollars per travel mug
- average rate of change on $[100, 102]$ is $\dfrac{121-120}{102-100} = \dfrac{1}{2} = 0.5$ dollars per travel mug
- average rate of change on $[100, 101]$ is $\dfrac{120.5-120}{101-100} = \dfrac{0.5}{1} = 0.5$ dollars per travel mug

instantaneous rate of change $\approx .5$, so when 100 mugs are sold the cost is increasing at a rate of \$0.50 per mug.

2. A company's daily profit when x gift baskets are made and sold is $P(x)$ dollars. Given $\dfrac{P(35+h)-P(35)}{h} = 10-h$,

(a) find and interpret the average rate of change of profit when the number of gift baskets made and sold is between $\underbrace{35 \text{ and } 50}_{}$.

distance between these is h

$h = 50 - 35 = 15$

$$\frac{P(35+15)-P(35)}{15} = 10 - 15 = \boxed{-5}$$

When the number of gift baskets sold increases from 35 to 50, on average, the profit decreases by \$5 per basket.

(b) find and interpret the instantaneous rate of change of profit when 35 gift baskets are made and sold.

$$\lim_{h\to 0} \frac{P(35+h)-P(35)}{h} = \lim_{h \to 0} (10-h) = 10$$

when 35 gift baskets are made and sold, the profit increases at a rate of \$10 per basket.

3. Use algebraic methods to find the equation of the line tangent to $g(x) = \dfrac{2x}{3x+5}$ at $a = -1$

$$g(-1) = \frac{2(-1)}{3(-1)+5} = \frac{-2}{-3+5} = \frac{-2}{2} = -1 \Longrightarrow \underbrace{\overset{x_1 \quad y_1}{(-1,-1)}}_{\text{point of tangency}}$$

$$g'(-1) = \lim_{h\to 0} \frac{g(-1+h) - g(-1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2(-1+h)}{3(-1+h)+5} - \frac{2(-1)}{3(-1)+5}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2+2h}{-3+3h+5} - \frac{-2}{2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2+2h}{3h+2} + 1\left(\frac{3h+2}{3h+2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2+2h+3h+2}{3h+2}}{h}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{3h+2} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{5}{3h+2} = \frac{5}{3(0)+2} = \overset{m_{tan}}{\frac{5}{2}}$$

$$y - y_1 = m(x - x_1)$$

$$y - {}^-1 = \frac{5}{2}(x - {}^-1)$$

$$y + 1 = \frac{5}{2}x + \frac{5}{2}$$

$$\boxed{y = \frac{5}{2}x + \frac{3}{2}}$$

$m_{tan} = \frac{5}{2}$ point of tangency $(-1,-1)$

$$y - y_1 = m_{tan}(x - x_1)$$

$$y - {}^-1 = \frac{5}{2}(x - {}^-1)$$

$$y + 1 = \frac{5}{2}x + \frac{5}{2}$$

$$y = \frac{5}{2}x + \frac{5}{2} - 1$$

$$\boxed{y = \frac{5}{2}x + \frac{3}{2}}$$

4. Find the x−value where the graph of the function $h(x) = 10\sqrt{9-x}$ has a slope of -5. Find $h'(x) = 5$

$$h'(x) = \lim_{h\to 0} \frac{h(x+h) - h(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{10\sqrt{9-(x+h)} - 10\sqrt{9-x}}{h} \cdot \frac{(10\sqrt{9-x-h} + 10\sqrt{9-x})}{(10\sqrt{9-x-h} + 10\sqrt{9-x})} \right)$$

$$= \lim_{h\to 0} \frac{100(9-x-h) - 100(9-x)}{h(10\sqrt{9-x-h} + 10\sqrt{9-x})}$$

$$= \lim_{h\to 0} \frac{\cancel{900} - \cancel{100x} - 100\cancel{h} - \cancel{900} + \cancel{100x}}{\cancel{h}(10\sqrt{9-x-h} + 10\sqrt{9-x})}$$

$$= \lim_{h\to 0} \frac{-100}{10\sqrt{9-x-h} + 10\sqrt{9-x}}$$

$$= \frac{-100}{10\sqrt{9-x-0} + 10\sqrt{9-x}} = \frac{-100}{20\sqrt{9-x}} = \frac{-5}{\sqrt{9-x}}$$

$$h'(x) = 5$$

$$\left(\frac{-5}{\sqrt{9-x}}\right)^2 = (5)^2$$

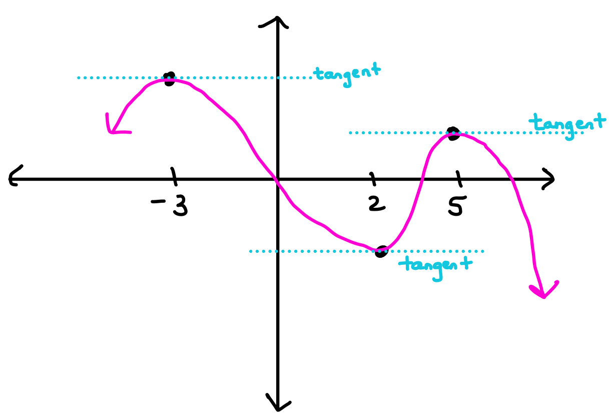
$$\frac{25}{9-x} = 25$$

$$25 = 225 - 25x$$

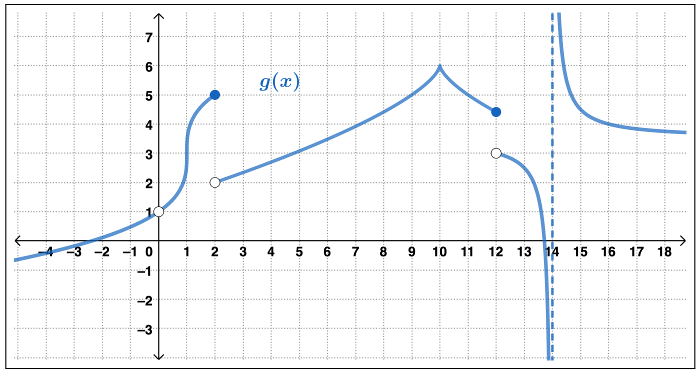
$$\frac{-200}{-25} = x$$

$$\boxed{8 = x}$$

5. Draw a graph of a function that has horizontal tangents at $x = -3,\ 2,$ and 5.



6. Use the graph of $g(x)$ below to find:



(a) the values of c where $\lim_{x\to c} g(x)$ DNE.

$x = 2$

$\lim_{x\to 2^-} g(x) \approx 5$

$\lim_{x\to 2^+} g(x) \approx 2$

$x = 12$

$\lim_{x\to 12^-} g(x) \approx 4.1$

$\lim_{x\to 12^+} g(x) \approx 3$

$x = 14$

$\lim_{x\to 14^-} g(x) \to -\infty$

$\lim_{x\to 14^+} g(x) \to \infty$

(b) the intervals on which $g(x)$ is continuous.

$g(x)$ is not continuous at $x = 0$ (I), $x = 2$ (II), $x = 12$ (II, and $x = 14$ (I)

$(-\infty, 0) \cup (0, 2) \cup (2, 12) \cup (12, 14) \cup (14, \infty)$

(c) the values of x where $g(x)$ is not differentiable.

$x = 0, 2, 12, 14$ (Discontinuous)

$x = 1$ (vertical tangent)

$x = 10$ (cusp)

7. Find the value of k that makes $f(x)$ continuous everywhere.

linear function ✓

$$f(x) = \begin{cases} 2x + 3k - 8 & x < 6 \\ x^2 + 8 + k & x \geq 6 \end{cases}$$

quadratic ✓

$$\lim_{x\to 6^+} f(x) = \lim_{x\to 6^-} f(x) = f(6)$$

$$\lim_{x\to 6^+} (x^2 + 8 + K) = 6^2 + 8 + K = 36 + 8 + K = 44 + K$$

$$f(6) = \lim_{x\to 6^-} (2x + 3K - 8) = 2(6) + 3K - 8 = 12 + 3K - 8 = 4 + 3K$$

$44 + K = 4 + 3K$

$40 = 2K$

$\boxed{20 = K}$

8. Calculate the following limits using algebraic techniques. If the limit has infinite behavior, use limit notation to describe the behavior.

(a) $\lim_{x\to 2} \sqrt[3]{13x+1}$ $= \sqrt[3]{13(2)+1} = \sqrt[3]{26+1} = \sqrt[3]{27} = \boxed{3}$

(b) $\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16}$ $= \lim_{x\to 16}\left(\frac{(\sqrt{x}-4)}{(x-16)}\cdot\frac{(\sqrt{x}+4)}{(\sqrt{x}+4)}\right) = \lim_{x\to 16}\frac{\cancel{(x-16)}\ 1}{\cancel{(x-16)}(\sqrt{x}+4)}$

$\left.\begin{array}{l}\sqrt{16}-4 = 4-4 = 0\\ 16-16 = 0\end{array}\right\}$ Case 3 do algebra

$= \frac{1}{\sqrt{16}+4}$

$= \frac{1}{4+4}$

$= \boxed{\frac{1}{8}}$

(c) $\lim_{x\to 4} \frac{x^2-x-12}{x^2-16}$ $= \lim_{x\to 4}\frac{\cancel{(x-4)}(x+3)}{(x+4)\cancel{(x-4)}} = \frac{4+3}{4+4} = \boxed{\frac{7}{8}}$

$\left.\begin{array}{l}4^2-4-12 = 16-16 = 0\\ 4^2-16 = 16-16 = 0\end{array}\right\}$ case 3

(d) $\lim_{x\to -\infty} \frac{\pi^{10}-6+3x^4+5x^6}{-3x^6+7e^8+2x^3}$ $= \lim_{x\to-\infty}\frac{\frac{\pi^{10}}{x^6}-\frac{6}{x^6}+\frac{3x^4}{x^6}+\frac{5x^6}{x^6}}{\frac{-3x^6}{x^6}+\frac{7e^8}{x^6}+\frac{2x^3}{x^6}}$

x^6 is highest power of x

$= \lim_{x\to-\infty}\frac{\cancelto{0}{\frac{\pi^{10}}{x^6}}-\cancelto{0}{\frac{6}{x^6}}+\cancelto{0}{\frac{3}{x^2}}+5}{-3+\cancelto{0}{\frac{7e^8}{x^6}}+\cancelto{0}{\frac{2}{x^3}}} = \boxed{-\frac{5}{3}}$

9. Given $g(x) = \dfrac{4(x-1)^{\cancel{2}}\cancel{(x+3)}\cancel{(x+1)}}{\cancel{(x-1)}(x+1)^{\cancel{2}}\cancel{(x+3)}}$, find the coordinates of any holes and the equations of any vertical asymptotes.

hole hole V.A.

$$g(x) \approx \frac{4(x-1)}{(x+1)}$$

$x=1$ $\quad \dfrac{4(1-1)}{1+1} = \dfrac{0}{2} = 0 \quad \boxed{(1,0) \text{ hole}}$

$x=-3$ $\quad \dfrac{4(-3-1)}{-3+1} = \dfrac{4(-4)}{-2} = 8 \quad \boxed{(3,8) \text{ hole}}$

$\boxed{x=-1 \quad \text{V. A.}}$

10. Find the horizontal asymptotes for $f(x) = \dfrac{4+e^{-2x}+2e^{7x}}{8e^{7x}-7e^{-4x}}$.

$$\lim_{x\to\infty} \frac{\frac{4}{e^{7x}} + \frac{e^{-2x}}{e^{7x}} + \frac{2e^{7x}}{e^{7x}}}{\frac{8e^{7x}}{e^{7x}} - \frac{7e^{-4x}}{e^{7x}}} = \lim_{x\to\infty} \frac{\cancel{\frac{4}{e^{7x}}}^{0} + \cancel{\frac{1}{e^{9x}}}^{0} + 2}{8 - \cancel{\frac{7}{e^{11x}}}^{0}} = \frac{2}{8} = \frac{1}{4} \quad \boxed{y = \frac{1}{4}}$$

$$\lim_{x\to-\infty} \frac{\frac{4}{e^{-4x}} + \frac{e^{-2x}}{e^{-4x}} + \frac{2e^{7x}}{e^{-4x}}}{\frac{8e^{7x}}{e^{-4x}} - \frac{7e^{-4x}}{e^{-4x}}} = \lim_{x\to-\infty} \frac{\cancel{4e^{4x}}^{0} + \cancel{e^{2x}}^{0} + \cancel{2e^{11x}}^{0}}{\cancel{8e^{11x}}^{0} - 7} = \frac{0}{7} \quad \boxed{y=0}$$

11. Given $h(x) = \begin{cases} \ln(x^2+4) & x < -1 \\ 6e^{x+1} & x > -1 \\ 6 & x = -1 \end{cases}$ find the following limits. Use exact answers and not decimal approximations.

(a) $\lim_{x \to -1^+} h(x) = \lim_{x \to -1^+} 6e^{x+1} = 6e^{-1+1} = 6e^{0} = 6 \cdot 1 = 6$

(b) $\lim_{x \to -1^-} h(x) = \lim_{x \to -1^-} \ln(x^2+4) = \ln((-1)^2+4) = \ln 5$

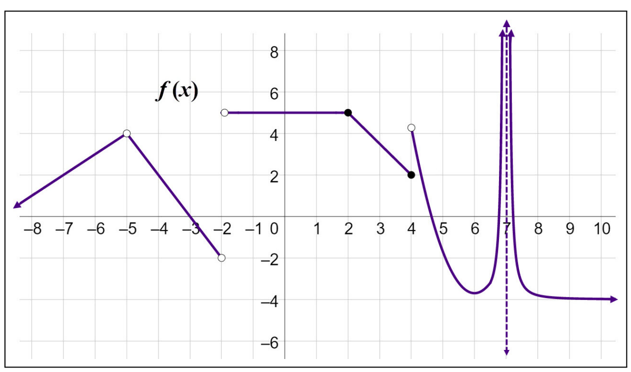
(c) $\lim_{x \to -1} h(x)$ DNE $\ln 5 \neq 6$

12. Given $g(x) = \dfrac{(x+5)(x-2)}{(x+5)}$, and the graph of $f(x)$ below, find $\lim_{x \to -5}\left(2g(x) + \dfrac{x^2}{\sqrt{f(x)}} - 3\right)$.

$\lim_{x \to -5} \dfrac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}}$

$= -5-2$

$= -7$



$2\lim_{x \to -5} g(x) + \dfrac{\left(\lim_{x \to -5} x\right)^2}{\sqrt{\lim_{x \to -5} f(x)}} - \lim_{x \to -5} 3$

$2(-7) + \dfrac{(-5)^2}{\sqrt{4}} - 3 = -14 + \dfrac{25}{2} - 3$

$= -17 + \dfrac{25}{2}$

$\boxed{= -4.5}$