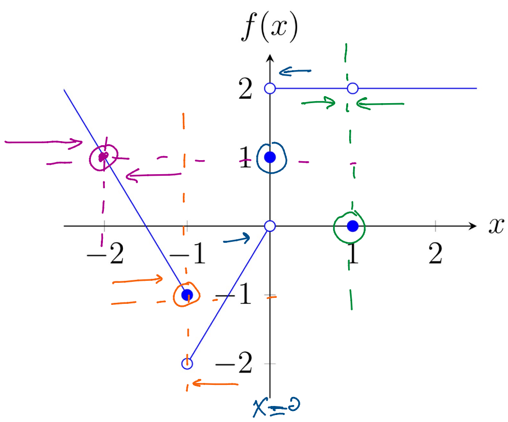
Week-in-Review 1 (1.3, 1.4)

Problem 1. Answer the following questions based on the graph of $f(x)$ below:



(1) Use the definition of continuity to show that f is continuous at $x = -2$.

a) $f(-2) = 1$

b) $\lim_{x \to (-2)} f(x) = LHL = RHL = 1$

c) $f(-2) = \lim_{x \to (-2)} f(x) = 1.$

(2) Why is f not continuous at $x = -1$?

a) $f(-1) = -1$

b) $\lim_{x \to -1} f(x)$ DNE

$-1 \neq -2$

(3) Why is f not continuous at $x = 0$?

a) $f(0) = 1$

b) $\lim_{x \to 0} f(x)$ DNE

$0 \neq 2$

(4) Why is f not continuous at $x = 1$?

a) $f(1) = 0$

b) $\lim_{x \to 1} f(x) = 2$

c) $f(1) \neq \lim_{x \to 1} f(x)$

$0 \neq 2$

allowed values of x for f(x)

Problem 2. Find the Domain of the following functions and use that information to determine where the function is not continuous.

(1) State the Rules of Domains

a) No division by zero

b) If $\sqrt[n]{*}$, n = even, $* \geq 0$

c) If $\log_b(*)$, $* > 0$

Numerator ①

Denominator ②

(2) $f(x) = \dfrac{\sqrt[7]{6x^2 + 11x - 7}}{x^2 - 5x + 6}$ → seventh root

① odd root
all x allowed

② $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x \neq 3$, $x \neq 2$

D: $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

f(x) is NOT continuous @ $x = 2$, $x = 3$

(3) $f(x) = \dfrac{\sqrt{3x - 2}}{6x^2 + 8x - 30}$

① even root

$3x - 2 \geq 0$

$3x \geq 2$

$x \geq \frac{2}{3}$

② $6x^2 + 8x - 30 = 0$

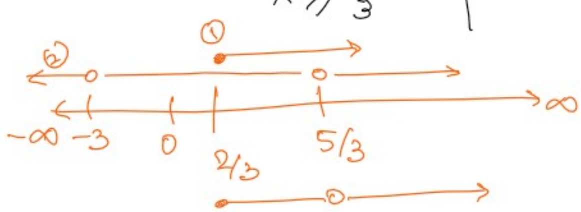
$x^2 + 8x - 180 = 0$

$(x + \frac{18}{6})(x - \frac{10}{6}) = 0$

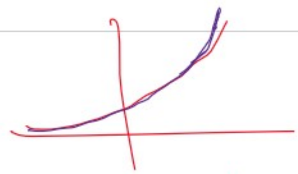
$\boxed{(x+3)}\boxed{(x - \frac{5}{3})} = 0$

$\cancel{x = -3}$, $x = 5/3$

f(x) has a discontinuity @ $x = 5/3$



D: $\left[\frac{2}{3}, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$



① e^{5-x} → all x values allowed

↓

exponential f^n

(4) $f(x) = \dfrac{e^{5-x}}{\sqrt{x-4}}$ → D: $(4, \infty)$

② $\sqrt{x-4}$ → even root

$x - 4 \geq 0$
$x \geq 4$ } Rule #2

BUT $\underbrace{x \neq 4}_{\text{Rule \#1}}$ ie $x > 4$

☆ (5) $f(x) = e^{\left(\frac{x+1}{5x^2 - 10x}\right)}$ → even though exponential f^n

Exponent has

① $x+1$ → no restriction

② $5x^2 - 10x = 0$

$5x(x-2) = 0$

$x = 0$ $x = 2$

D: $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

(6) $f(x) = \dfrac{\log_7(x-12)}{\sqrt{x+5}}$

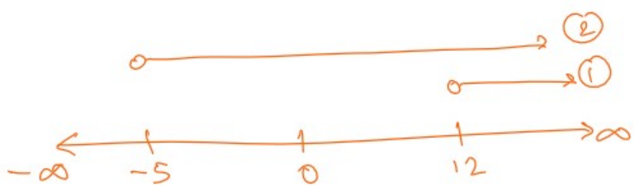
① $x - 12 > 0$

$x > 12$

② $\sqrt{x+5}$ → even root

$x + 5 > 0$ → Rule 1 + Rule 2

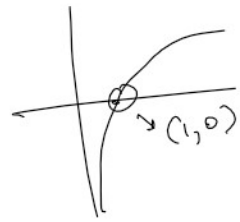
$x > -5$



D: $(12, \infty)$

Ex: $\ln(x) = 5$

$x = e^5$



(7) $f(x) = \dfrac{7x^2 + 11x^3}{\ln(x+8)}$

polynomial

① all x values ok

② $\ln(x+8)$

$x + 8 > 0$

$x > -8$

Rule #3

Rule #1 ? $\ln(x+8) = 0$

we know $\ln(1) = 0$

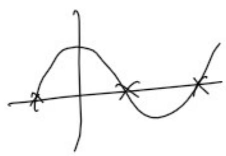
$x + 8 \neq 1$

$x \neq -7$

$\boxed{x + 8 = e^0 = 1}$

-8 -7

D: $(-8, -7) \cup (-7, \infty)$



(8) $f(x) = \dfrac{x^2 + 4x - 21}{x^2 + 10x + 25}$

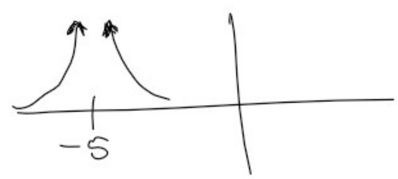
① polynomial → all x ok

② $x^2 + 10x + 25 \neq 0$

$(x+5)(x+5) \neq 0$

$x \neq -5$

D: $(-\infty, -5) \cup (-5, \infty)$



(9) $f(x) = \dfrac{\sqrt[5]{x^3 - 2x} - \sqrt[6]{x + 10}}{3^{4-x^2}}$

odd root

even root

① only look at even root

$x + 10 \geq 0$

$x \geq -10$

② 3^{4-x^2} → exponential fn

$3^{4-x^2} = 0$?

no such x value

D: $[-10, \infty)$

① all x ok.

(10) $f(x) = \dfrac{\boxed{x+2}}{\underbrace{(x+12)(x+2)}}$

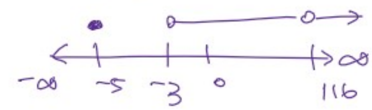
② $(x+12)(x+2)=0$

$x=-12$ $\quad$ $x=-2$

D: $(-\infty,-12) \cup (-12,-2) \cup (-2,\infty)$

(11) $f(x) = \dfrac{\log_2(x+3)}{11+\sqrt{x+5}}$

$-5 < -3$.



① $\log_2(x+3)$

$x+3>0$

$x>-3$

② even root

$x+5 \geq 0$

$x \geq -5$

Rule#1: $11+\sqrt{x+5} \neq 0$

$\sqrt{x+5} \neq -11$

$x+5 \neq (-11)^2 = 121$

$x \neq 116$.

D: $(-3,116) \cup (116,\infty)$

Problem 3. Find any vertical asymptotes and holes for the function

$$f(x) = \frac{5\cancel{(x-a)}(x-b)^2(x-c)}{x^2(x-a)^2\cancel{(x-b)}}$$

$(x-b)$ cancels completely

$x=b$ is a hole.

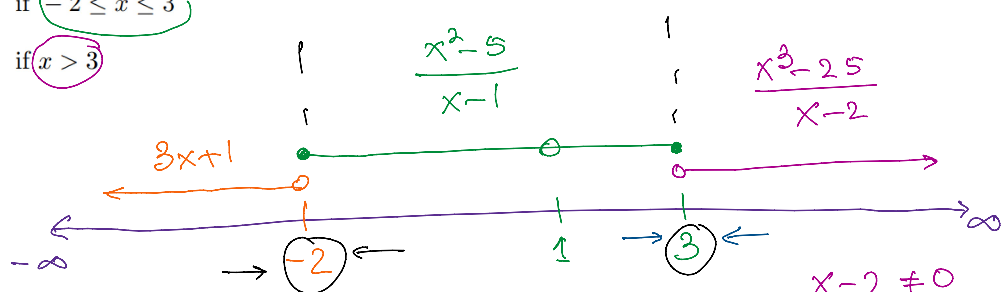
$$f(x) = \frac{5(x-b)(x-c)}{x^2(x-a)}$$

$\underbrace{x=0 \qquad x=a}_{\text{vertical asymptotes.}}$

Problem 4. For what value(s) of x is the piecewise function $f(x)$ given below not continuous?

$$f(x) = \begin{cases} 3x+1 & \text{if } x < -2 \\ \dfrac{x^2-5}{x-1} & \text{if } -2 \le x \le 3 \\ \dfrac{x^3-25}{x-2} & \text{if } x > 3 \end{cases}$$

$x = -2$
$x = 1$



$x - 1 \neq 0$
$x \neq 1$

$x - 2 \neq 0$
$x \neq 2$
less than 3 anyways

① $f(x)$ is not continuous at $x = 1$

② check $x = -2$ ← discontinuity

a) $f(-2) = \dfrac{(-2)^2 - 5}{(-2) - 1} = \dfrac{4-5}{-3} = \dfrac{-1}{-3} = +\dfrac{1}{3}$

b) Left hand limit

$\lim_{x \to (-2)^-} f(x) = 3(-2)+1 = -6+1 = -5$

Right hand limit $\lim_{x \to (-2)^+} f(x) = \left(\dfrac{x^2-5}{x-1}\right) \to +\dfrac{1}{3}$

LHL ≠ RHL
$\lim_{x \to (-2)} f(x)$ DNE

$\boxed{\text{check } x = 3}$ ← $f(x)$ is continuous here

a) $f(3) = \frac{(3)^2-5}{(3)-1} = \frac{9-5}{2} = \frac{4}{2} = 2$ } $f(3) = \lim_{x\to 3} f(x)$

$\lim_{x\to 3} f(x) = 2$

b) LHL $\lim_{x\to 3^-} f(x) = 2$

RHL $\lim_{x\to 3^+} f(x) = \frac{(3)^3-25}{(3)-2} = \frac{27-25}{1} = 2$ } LHL = RHL

Problem 5. Find the value of A so that the piecewise function $f(x)$ given below is continuous.

$$f(x) = \begin{cases} \dfrac{x^2-x-20}{x+4} & x < -4 \\ Ax^3 - 2e^{x+4} + 8 & x \geq -4 \end{cases}$$

$\frac{x^2-x-20}{x+4}$ $\quad$ $Ax^3 - 2e^{x+4} + 8$

$x+4 \neq 0$ $\quad$ $\boxed{-4}$ $\quad$ all x ok

$x \neq -4 \to$ ok

Check $x = -4$

a) $f(-4) = A(-4)^3 - 2e^{(-4)+4} + 8$

$= -64A - 2e^0 + 8$

$= -64A - 2 + 8$

$= -64A + 6 \to$ RHL

b) LHL $\lim_{x\to(-4)^-} f(x) = \frac{(-4)^2-(-4)-20}{(-4)+4} = \frac{16+4-20}{-4+4} = \frac{0}{0}$

$\frac{x^2-x-20}{x+4} = \lim_{x\to -4} \frac{(x-5)\cancel{(x+4)}}{\cancel{(x+4)}} = \lim_{x\to -4}(x-5)$

$= -9$

For $f(x)$ to be continuous

$-64A + 6 = -9$

$-64A = -9-6 = -15$

$A = \frac{-15}{-64} = \frac{15}{64}$

Problem 6. Find any holes and asymptotes for the given functions. Use limit notation to describe infinite and end point behavior.

(1) $f(x) = \dfrac{x^2 - 8x + 16}{x^2 - 4x}$ $= \dfrac{(x-4)\cancel{(x-4)}}{x\cancel{(x-4)}}$

$x=4 \rightarrow$ hole

$x=0 \rightarrow$ VA

a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = 1$

b) $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to(-\infty)} \dfrac{x^2}{x^2} = 1$

Horizontal asymptote $y=1$ as $x \to \pm\infty$

(2) $\underbrace{f(x) = ax^2 - bx^5 + cx^3 + dx - 15}_{\text{polynomial}}$ where a, b, c, d are constants and > 0.

polynomial $\rightarrow$ no holes or VA

a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} (-bx^5) \rightarrow -\infty$

b) $\lim\limits_{x\to(-\infty)} f(x) = \lim\limits_{x\to(-\infty)} (-bx^5) \rightarrow (-b)(-\infty)^5$
$\rightarrow +\infty$

$f(x)$ has no HA

Problem 7. Find any horizontal asymptotes for the functions below. If there are none, use limit notation to describe the end point behavior.

(1) $f(x) = \dfrac{2x^3 + x^2 - 1}{5x^3 - 7x + 2}$

a) $\lim_{x\to\infty} f(x) = \dfrac{2x^3}{5x^3} = \dfrac{2}{5}$

HA @ $x \to \infty$ is at $y = 2/5$

b) $\lim_{x\to(-\infty)} f(x) = \dfrac{2x^3}{5x^3} = \dfrac{2}{5}$

HA @ $x \to (-\infty)$ is at $y = 2/5$

(2) $f(x) = \dfrac{4 + 3e^x}{3 + e^{-x}}$

a) $\lim_{x\to\infty} f(x) = \dfrac{4+3e^x}{3+0} = \dfrac{4+\infty}{3} \to \infty$

no HA @ $x \to \infty$

b) $\lim_{x\to(-\infty)} f(x) = \dfrac{4+3(0)}{3+e^{-x}} = \dfrac{4}{3+\infty} = \dfrac{4}{\infty} = 0$

HA @ $x \to -\infty$ is at $y = 0$.

$$\lim_{x\to\infty} e^{x} = e^{\infty} \to \infty$$

$$\lim_{x\to(-\infty)} \boxed{e^{x}} = e^{-\infty} = \frac{1}{e^{\infty}} = \boxed{0}$$

$$\lim_{x\to\infty} \boxed{e^{-x}} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = \boxed{0}$$

$$\lim_{x\to-\infty} e^{-x} = e^{-(-\infty)} = e^{\infty} \to \infty$$

(3) $f(x) = \dfrac{3e^{x} + e^{-x}}{e^{x} - 4e^{-x}}$

a) $\displaystyle\lim_{x\to\infty} f(x) = \frac{3e^{x}+0}{e^{x}-4(0)} = \frac{3e^{x}}{e^{x}} = 3$ } HA is $y=3$ as $x\to\infty$

b) $\displaystyle\lim_{x\to(-\infty)} f(x) = \frac{3(0)+e^{-x}}{0-4e^{-x}} = \frac{\cancel{e^{-x}}}{-4\cancel{e^{-x}}} = -\frac{1}{4}$ } HA is $y = -\frac{1}{4}$ as $x\to(-\infty)$

(4) $f(x) = \dfrac{e^{2x} - 7e^{-3x}}{6e^{3x} - 2e^{-3x}}$

a) $\displaystyle\lim_{\boxed{x\to\infty}} f(x) = \frac{e^{2x}-7(0)}{6e^{3x}-2(0)} = \frac{e^{2x}}{6e^{3x}} = \frac{1}{6}e^{(2x-3x)}$

$\displaystyle = \lim_{x\to\infty} \frac{1}{6}e^{-x} = \frac{1}{6}(0) = 0$

HA @ $x\to\infty$ is $y=0$

b) $\displaystyle\lim_{x\to(-\infty)} f(x) = \frac{0-7e^{-3x}}{6(0)-2e^{-3x}} = \frac{-7\cancel{e^{-3x}}}{-2\cancel{e^{-3x}}} = \frac{7}{2}$

HA @ $x\to(-\infty)$ is $y = 7/2$