Math 140: Week-in-Review 7 (Review of Algebra & Chapter 5.1)

1. Evaluate each of the following and simplify completely.

   (a) $\frac{3}{\cancel{4}_{1}} \cdot \cancel{16}^{4} = (3)(4) = \boxed{12}$

   (b) $\left(-\frac{4}{15}\right) \div \frac{2}{5} = \frac{-\cancel{4}^{2}}{\cancel{15}_{3}} \cdot \frac{\cancel{5}^{1}}{\cancel{2}_{1}} = \boxed{-\frac{2}{3}}$

   (c) $-\frac{13}{10} + \frac{7}{15} = -\frac{13}{10} \cdot \left(\frac{3}{3}\right) + \frac{7}{15}\left(\frac{2}{2}\right) = -\frac{39}{30} + \frac{14}{30} = \frac{-39+14}{30} = -\frac{25}{30}$

   $= \boxed{-\frac{5}{6}}$ in lowest terms

   (d) $\frac{3}{5} - \left(\frac{6}{7}\right) = \frac{3}{5} \cdot \left(\frac{7}{7}\right) - \frac{6}{7} \cdot \left(\frac{5}{5}\right) = \frac{21}{35} - \frac{30}{35} = \boxed{\frac{-9}{35}}$

   (e) $\frac{\cancel{8}^{1}}{\cancel{13}_{1}} \cdot \frac{\cancel{26}^{2}}{\cancel{15}_{1}} \cdot \frac{\cancel{45}^{3}}{\cancel{16}_{2}} = \frac{6}{2} = \boxed{3}$

   (f) $\frac{5}{7} + \frac{8}{13} + \left(-\frac{5}{7}\right) = \cancel{\frac{5}{7}} + \frac{8}{13} - \cancel{\frac{5}{7}} = \boxed{\frac{8}{13}}$

2. Evaluate each of the following and simplify completely.

(a) $\frac{1}{2^{-3}} = 2^{-(-3)} = 2^{3} = \boxed{8}$

(b) $\left(\frac{3}{7}\right)^{-2} = \left(\frac{7}{3}\right)^{2} = \frac{7^2}{3^2} = \boxed{\frac{49}{9}}$

(c) $2^3 \cdot 2^2 = 2^{3+2} = 2^{5} = \boxed{32}$

(d) $\frac{3^2}{3^4} = 3^{2-4} = 3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

(e) $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \boxed{\frac{3}{2}}$

(f) $-\sqrt{64} = \boxed{-8}$

(g) $\sqrt{-81}$ Does not exist as a real number

* Domain of $f(x) = \sqrt{x}$ is $[0, \infty)$ *

(h) $\sqrt[3]{-8} = -2$

* check $(-2)^3 = -8 \Leftrightarrow \sqrt[3]{-8} = -2$ *

* domain of $f(x) = \sqrt[3]{x}$ is $(-\infty, \infty)$ *

3. Simplify the following using the order of operations

PEMDAS → order of arithmetic operations

P → parentheses (brackets)
E → exponents (powers)
M → multiplication
D → division
A → addition
S → subtraction

(a) $2(3 + 5 \cdot 4) - 6^2$

$= 2(3+20) - 36$
$= 2 \cdot 23 - 36$
$= 46 - 36 = \boxed{10}$

(b) $4^2 - 15 \div (8 - 3)$

$= 16 - 15 \div 5$
$= 16 - 3$
$= \boxed{13}$

(c) $(x - y)^2$ when $x = 5$ and $y = 7$

$(5-7)^2 = (-2)^2 = \boxed{4}$

4. Simplify each of the following expressions

(a) $2a^2 + 4a + 8 + 6a^2 + 5a - 4$

$= (2+6)a^2 + (4+5)a + (8-4)$

$= \boxed{8a^2 + 9a + 4}$

(b) $(2x)^2(4x)$

$(2x)^2 = 2^2 \cdot x^2 = 4x^2$

$= (4x^2)(4x)$
$= (4)(4)x^2 \cdot x$
$= \boxed{16x^3}$

(c) $\left(\frac{1}{\cancel{5}_{1}}b^6\right)(\cancel{15}^{3}b^3) = 3b^{6+3}$

$= \boxed{3b^9}$

Expand and simplify each of the following expressions

(d) $4s^3(s^2 - 2s + 3)$ $= 4s^3 \cdot s^2 - 4s^3 \cdot 2s + 4s^3 \cdot 3$

$= \boxed{4s^5 - 8s^4 + 12s^3}$

(e) $(2y + 3)(y + 5)$ $= 2y(y+5) + 3(y+5)$

$= 2y^2 + 10y + 3y + 15$

$= \boxed{2y^2 + 13y + 15}$

(f) $(2x + 4)(x^2 + x - 1)$ $= 2x(x^2 + x - 1) + 4(x^2 + x - 1)$

$= 2x^3 + 2x^2 - 2x + 4x^2 + 4x - 4$

$= \boxed{2x^3 + 6x^2 + 2x - 4}$

(g) $(4 + 2x)(4 - 3x)$ $= 4(4 - 3x) + 2x(4 - 3x)$

$= 16 - 12x + 8x - 6x^2$

$= \boxed{-6x^2 - 4x + 16}$

(h) $\dfrac{2x - 3}{x - 2} \cdot \dfrac{2 - x}{x + 1}$ $= \dfrac{(2x-3) \cdot [-(\cancel{x-2})]}{(\cancel{x-2}) \cdot (x+1)} = -\dfrac{(2x-3)}{x+1}$

$= \boxed{\dfrac{-2x+3}{x+1}}$

5. Factor each of the following expressions

(a) $5x^3 - 10x^2 - 15x$ * common factor : $5x$

$= 5x(x^2 - 2x - 3)$ * find a and b such that $ab = -3$ and $a+b = -2$

$= \boxed{5x(x-3)(x+1)}$ * $a = -3$, $b = 1$

(b) $w^2 - 8w + 15$ * find a and b such that $ab = 15$ and $a+b = -8$

$= \boxed{(w-5)(w-3)}$ $a = -5$, $b = -3$

(c) $r^2 - 4r - 12$ * find a and b such that $ab = -12$ and $a+b = -4$

$= \boxed{(r-6)(r+2)}$ $a = -6$, $b = 2$

(d) $6y^2 - y - 15$ * find a and b such that $ab = (6)(-15) = -90$ and $a+b = -1$, then re-write expression $a = -10$, $b = 9$

$= 6y^2 + 9y - 10y - 15$

$= 3y(2y+3) - 5(2y+3)$

$= \boxed{(2y+3)(3y-5)}$

(e) $96p^2 - 76p$ * greatest common divisor of 96 and 76 is 4

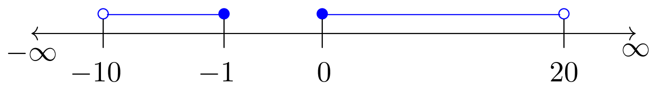
$= \boxed{4p(24p - 19)}$

(f) $6q^2 + 3$

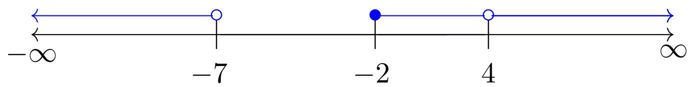
$= \boxed{3(2q^2 + 1)}$

↳ irreducible - cannot be factored further

6. Express each of the following using interval notation

(a) 

$(-10,-1] \cup [0,20)$

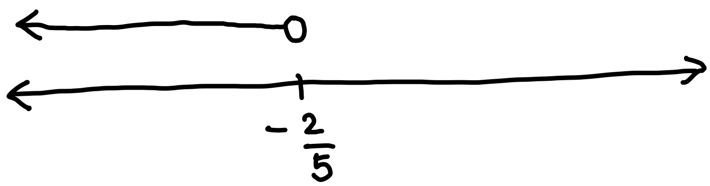
(b) 

$(-\infty,-7) \cup (-2,4) \cup (4,\infty)$

7. For each of the following, draw a number line representing the given information, and then write the equivalent statement using interval notation.

(a) $\left\{x \mid x < -\frac{2}{5}\right\}$

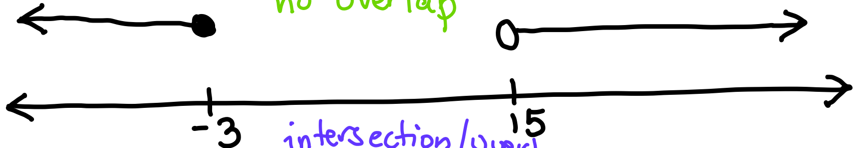
$(-\infty,-\frac{2}{5})$



(b) $\{x \mid x \leq -3$ and $x > 15\}$ $= \{\ \}$

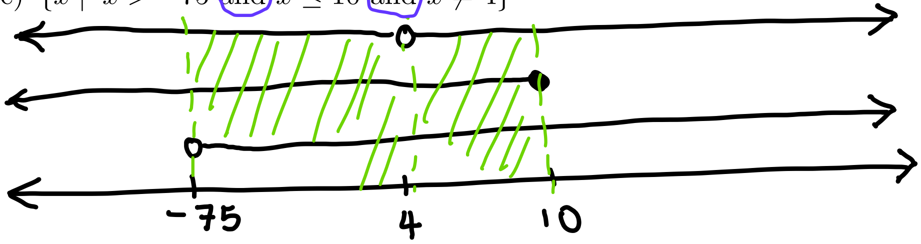
intersection/overlap

no overlap



intersection/overlap

(c) $\{x \mid x > -75$ and $x \leq 10$ and $x \neq 4\}$

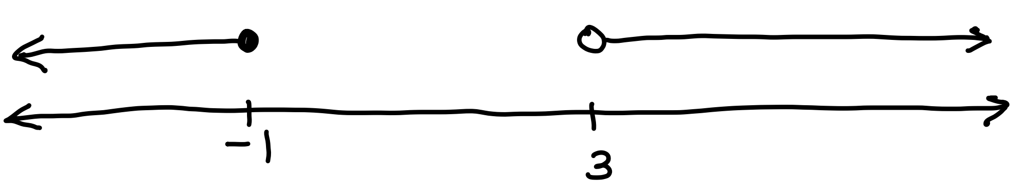


$(-75,4) \cup (4,10]$

(d) $\{x \mid x \leq -1$ or $x > 3\}$

$(-\infty,-1] \cup (3,\infty)$

union, combine



8. State the inputs and outputs of the given relations.

   (a) $R_1 = \{(5,-12),(-4,3),(-10,3),(12,12)\}$

   inputs = $\{-10,-4,5,12\}$ in ascending order

   outputs = $\{-12,3,12\}$

   (b) $R_2 = \{(-20,45),(45,-20),(15,-15),(45,17)\}$

   inputs = $\{-20,15,45\}$

   outputs = $\{-20,-15,17,45\}$

9. Determine if the given relation is a function. If it is a function, state the domain and range of the function.

   (a) $R_3 = \{(5,-12),(-4,3),(-10,3),(12,12)\}$ function

   * each input is mapped to one output

   Domain : $\{-10,-4,5,12\}$

   Range : $\{-12,3,12\}$

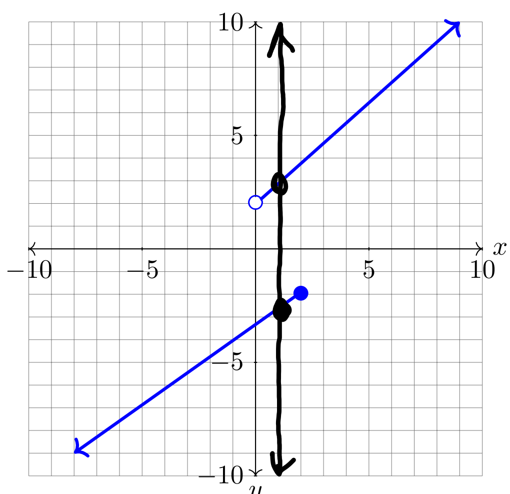
   (b) $6x+7y=42$ function

   * graph of a straight line in standard form. All straight lines, except for vertical lines, pass the Vertical Line Test.

   Domain $(-\infty,\infty)$

   Range $(-\infty,\infty)$

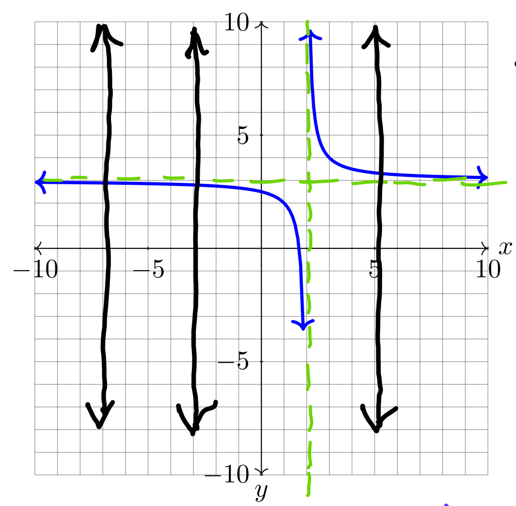
   (c)



   not a function

   * fails the vertical line test

   (d)

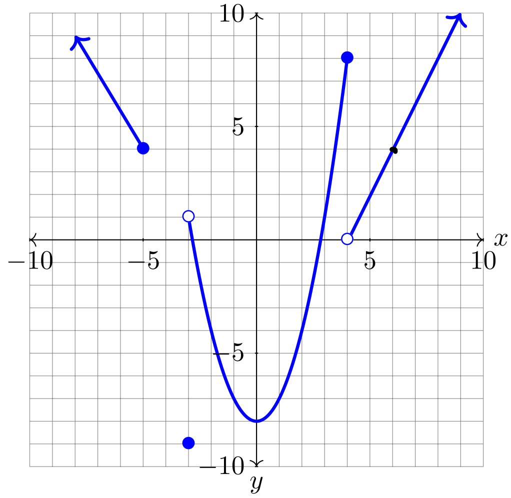


   function: passes the vertical line test

   Domain: $(-\infty,2)\cup(2,\infty)$

   Range: $(-\infty,3)\cup(3,\infty)$

10. Use the graph of $f(x)$ below to answer each of the following.



(a) $f(-4)$ DNE since $x = -4$ is not in the domain

(b) $f(2)$ $= -4$

(c) $f(4)$ $= 8$

(d) $f(-5)$ $= 4$

(e) Determine all values of x such that $f(x) = 4$ $-5, 3.5, 6$

(f) Determine all values of x such that $f(x) = -8$ 0

(g) Determine all values of x such that $f(x) = -9$ -3

(h) State the domain of $f(x)$. $(-\infty, -5] \cup [-3, \infty)$

(i) State the range of $f(x)$. $\{-9\} \cup [-8, \infty)$

11. Given the function $g(x) = 4 - 3x$, determine each of the following. Simplify your solutions as much as possible.

(a) $g(6)$

$$g(6) = 4 - 3(6)$$
$$= 4 - 18$$
$$= \boxed{-14}$$

(b) $g(2s)$

$$g(2s) = 4 - 3(2s) = \boxed{4 - 6s}$$

(c) $g(x-5)$

$$g(x-5) = 4 - 3(x-5)$$
$$= 4 - 3x + 15$$
$$= \boxed{19 - 3x}$$

(d) $g(x) - g(4)$

$$g(x) - g(4) = (4 - 3x) - (4 - 3(4))$$
$$= \cancel{4} - 3x - \cancel{4} + 12$$
$$= \boxed{-3x + 12}$$

12. Given the function $f(x) = 3x^2 - 2x$, determine each of the following. Simplify your solutions as much as possible.

(a) $f(-2)$

$$f(-2) = 3(-2)^2 - 2(-2)$$
$$= (3)(4) + 4$$
$$= \boxed{16}$$

(b) $f(x+h)$

$$f(x+h) = 3(x+h)^2 - 2(x+h)$$
$$= 3(x^2 + 2xh + h^2) - 2x - 2h$$
$$= \boxed{3x^2 + 6xh + 3h^2 - 2x - 2h}$$

(c) $f(x+h) - f(x)$

$$f(x+h) - f(x) = (\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h) - (\cancel{3x^2} - \cancel{2x})$$
$$= \boxed{6xh + 3h^2 - 2h}$$