Math 151 - Week-In-Review 7

Topics for the week:

3.6 Derivatives of Logarithmic Functions
K.1 Derivatives of Vector Functions
K.2 Slopes and Tangents to Parametric Curves
3.7 Rates of Change in the Natural and Social Sciences
3.8 Exponential Growth and Decay

3.6 Derivatives of Logarithmic Functions

1. Compute the derivative of $f(y) = \log_2(y) + 7^y - 7y^{1/7}$ with respect to y.

2. Compute the first and second derivative of $y = \ln\left(4x^2 + 1\right)$.

3. For $y = \log(x^2)\tan(x^3)$, find $\dfrac{dy}{dx}$.

4. Compute $f'(0)$ for $f(d) = \log_2\left(e^{-d}\cos(\pi d)\right)$

5. Determine $\dfrac{dy}{dx}$ for $y = \sqrt[5]{\dfrac{x^2+9}{x^2-9}}$

6. Determine $\dfrac{dy}{dx}$ for $y = (\ln(x))^{\sin(x)}$.

K.1 Derivatives of Vector Functions

7. Determine the derivative of the vector function: $\mathbf{r}(t) = t\cos\left(t^2\right)\mathbf{i} - t^2\sin(t)\,\mathbf{j}$.

8. Compute the unit tangent vector at the point $t = 0$ for $\vec{r}(t) = \left\langle \ln(5t+1), e^{4t} \right\rangle$ with respect to t.

9. Determine the velocity, acceleration, and speed of the particle with a position function of $\mathbf{r}(t) = \left\langle \dfrac{6t^2}{t-1}, \dfrac{5}{(t+3)^2} \right\rangle$.

K.2 Slopes and Tangents to Parametric Curves

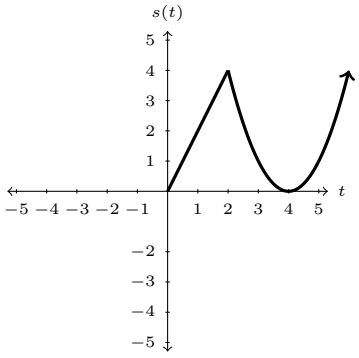
10. Compute $\dfrac{dx}{dt}$, $\dfrac{dy}{dt}$, and $\dfrac{dy}{dx}$ for $x = \log(4t + 64)$ and $y = t \cdot 10^t$.

11. Determine an equation of the tangent to the curve $x = \sec(\theta)\tan(\theta)$ and $y = \cos(2\theta)$ at the point corresponding to $\theta = \dfrac{\pi}{6}$.

12. Compute the points on the curve where the tangent is horizontal or vertical given $x = 12t - 3t^4$ and $y = t^3 - 9t$.

3.7 Rates of Change in the Natural and Social Sciences

13. Given the graph of a position function of a particle is shown below, where t is measured in seconds.



(a) When is the velocity of the particle positive?

(b) When is the particle not moving?

(c) When is the particle moving backwards?

14. A particle moves according to the function $s(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 4$, where t is in seconds and $s(t)$ is in meters.

(a) Compute $v(t)$ and $a(t)$.

(b) When is the particle at rest?

(c) How far did the particle travel in the first 4 seconds?

3.8 Exponential Growth and Decay

15. A chemical has a half-life of 18 days. A sample is obtained and 5 days later there remains 50 grams of the chemical.

    (a) Write a formula that will give the amount of the chemical that remains t days after the sample is obtained.

    (b) What was the initial amount of the sample of this chemical?

    (c) Determine the rate of decay of the chemical after 4 days.