1 Week 15 HOGU: 6.2, Final Exam Review Part 2

Problem 1. The John Weeks Enterprise is booming! The business needs to save up $\$150,000$ for a planned expansion. They make an initial deposit of $\$25,000$ and plan on depositing $\$500$ at the end of each month in T-bills, which currently yield 5.16% APR. Say that interest in these T-bills is compounded monthly. How many years will it take the John Weeks Enterprise to save up the $\$150,000$ they need?

N = SOLVE ⟶ 147.7138263 ← Remember this is total number compounded, so we need to divide by P/Y or C/Y

I = 5.16

PV = -25000

PMT = -500

must be negative b/c it is coming out of your pocket.

FV = 150000

P/Y = 12

C/Y = 12

PMT: END

$$\frac{147.7138263}{12} \approx 12.31$$

12.31 years

Problem 2. You are building the boat "Sailing for Weeks", which will currently cost $\$44,750$ to craft. You make a down payment of $\$10,000$ and finance the rest with a 10-year loan. The loan charges 7% interest, compounded quarterly.

(a) After making quarterly payments on the boat for 3 years, how much of the principal of your loan is still unpaid?

$N = 10 \cdot 4 = 40$
$I = 7$
$PV = 44750 - 10000 = 34750$
$PMT = \text{SOLVE} \rightarrow 1215.28$
$FV = 0$
$P/Y = 4$
$C/Y = 4$
PMT: END

1 Find Payment First

$N = 3 \cdot 4 = 12$ (made 3 yrs of payments)
$I = 7$
$PV = 34750$
$PMT = -1215.28$
$FV = \text{SOLVE} \rightarrow 26720.31$
$P/Y = 4$
$C/Y = 4$
PMT: END

2 Solve problem using # payments made

$N = 7 \cdot 4 = 28$ (7 yrs left)
$I = 7$
$PV = \text{SOLVE} \rightarrow 26720.31$
$PMT = -1215.28$
$FV = 0$
$P/Y = 4$
$C/Y = 4$
PMT: END

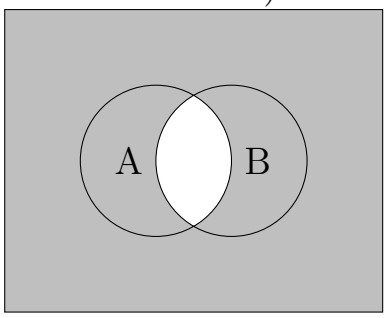
using payments left

$PV = 26720.31$

(b) After 3 years on your old loan above, you find a loan company that will refinance your loan at an APR of 5%, compounded quarterly, with a payment schedule that has you paying off the loan in 5 years! You decide to refinance your purchase and to make quarterly payments on the new loan. How much will your quarterly payments be?

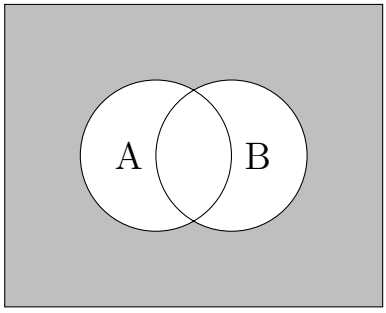
$N = 5 \cdot 4 = 20$
$I = 5$
$PV = 26720.31$
$PMT = \text{SOLVE} \rightarrow -1518.26$
$FV = 0$
$P/Y = 4$
$C/Y = 4$

Problem 3. Using union, intersection, and complements, how would you describe the shaded-in regions of these Venn diagram? (There is more than one correct answer.)



$(A \cap B)^c$ ← Not

The intersection of A + B



Not the union of A + B

$(A \cup B)^c$

Problem 4. The following is a probability distribution with a missing entry:

$U = \{1,2,3,4,5\}$

Outcome	1	2	3	4	5
Probability	$\frac{18}{100}$	$\frac{23}{100}$	$\frac{17}{100}$	$\frac{9}{100}$	$\frac{33}{100}$

These add up to $\frac{83}{100}$ so the missing prob. is $\frac{100-83}{100}$

Let $A = \{1,3,5\}$ and let $B = \{2,4,5\}$. What is $P\left(A \cup B^C\right)$?

$B^c = \{1,2,3,4,5\} = \{1,3\}$

$A \cup B^c$ means everything in A or B^c

$A \cup B^c = \{1,3,5\}$

$P(A \cup B^c) = P(1) + P(3) + P(5)$

$= \frac{18}{100} + \frac{17}{100} + \frac{33}{100}$

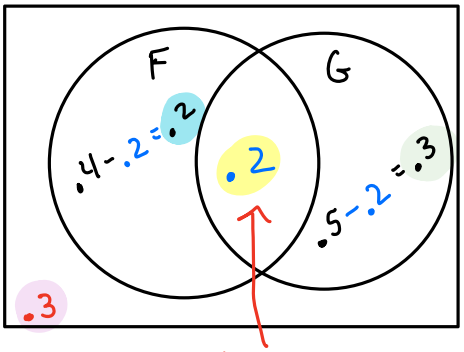
$= \boxed{\frac{68}{100} = \frac{17}{25}}$

Problem 5. Given that F and G are two events of an experiment with $P(F) = 0.4$, $P(G) = 0.5$, and $P(F \cap G) = 0.2$, calculate the following probabilities:

(a) $P(F^C)$ = .3 + .3 = .6

or 1 − .4 = .6

↑ P(F)



do this first

(b) $P(F \cup G)$ = .2 + .2 + .3 = .7

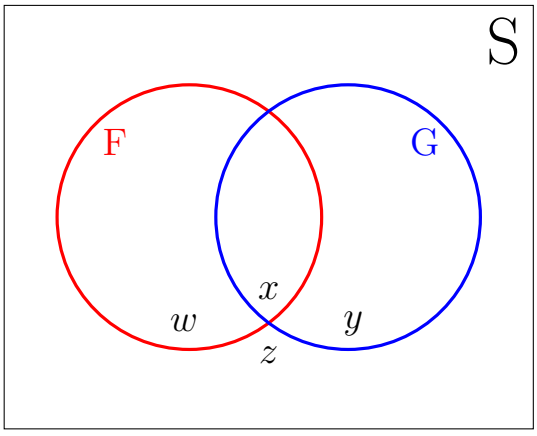
OR P(F) + P(G) − P(F ∩ G)

= .4 + .5 − .2 = .7

.2 + .3 + .2 = .7

1 − .7 = .3

(c) $P(F \cup G^C)$ (You may find the Venn diagram below helpful!)



P(w) = .2

P(x) = .2

P(y) = .3

P(z) = .3

$F \to w\ x$

$G^c \to w\ z$

$F \cup G^c \to w\ x\ z$

$P(F \cup G^c) = P(x) + P(w) + P(z)$

= .2 + .2 + .3

= .7

Problem 6. A local group is sponsoring a game at the Renaissance Fair! A foolish jester asks you to pay $1 to play the game, then flips a fair two-sided coin. As it is in the air, the player calls "heads" or "tails". If the coin lands on the side the player called out, they win $5! Otherwise, the player wins nothing.

(a) Let X be the amount of *net winnings*, in dollars, that a player makes from playing this game. Write a probability distribution for X.

	wins	loses
x	$-1+5=4$	$-1+0=-1$
$P(x)$	.5	.5

(b) What is the expected amount of net winnings for the player? Is this a fair game?

$E(x) = 4(.5) + (-1)(.5) = 1.5$

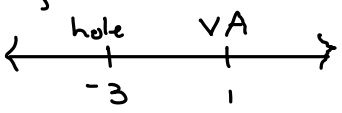
The player is expected to win $1.50. This is not a fair game because $E(x) \neq 0$.

Problem 7. Compute the domain of the following functions. Then state all holes and vertical asymptotes of the function. (Remember that, for holes, you must give an x- and y-coordinate. Write your answer as (x, y).)

(a) $f(x) = \dfrac{(x+3)^2}{(x-1)^2(x+3)}$

Rational Function $\Rightarrow$ denominator can not be zero for domain

$x \neq 1, -3$



Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

hole at $x = -3 \Rightarrow (-3, 0)$

$(x+3)$ divided out completely from denominator

Vertical asymptote: $x = 1$

$f(x) \approx \dfrac{x+3}{x-1}$

$(x-1)$ remains in denominator of simplified function

(b) $f(x) = \dfrac{(x+3)}{(x-1)^2(x+3)^2} \approx \dfrac{1}{(x-1)^2(x+3)}$

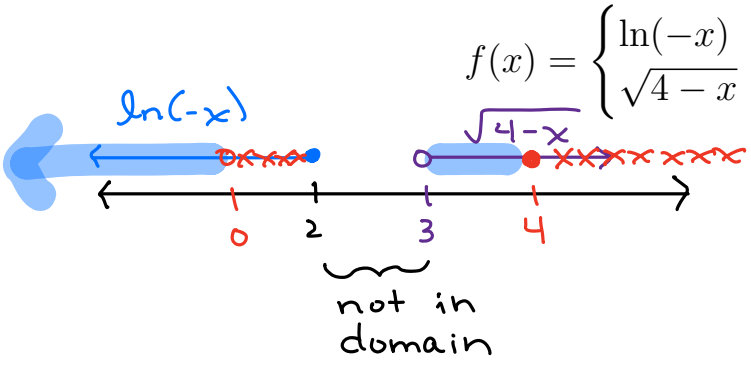
Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

holes: none

Vertical Asymptotes: $x = 1, -3$

Problem 8. Find the domain of the following function:

$$f(x) = \begin{cases} \ln(-x) & \text{if } x \leq 2 \\ \sqrt{4-x} & \text{if } x > 3 \end{cases}$$



$\ln(-x) \Rightarrow -x > 0$

$x < 0$ ← only x values strictly less than zero can be in domain.

$\sqrt{4-x} \Rightarrow 4 - x \geq 0$

$-x \geq -4$

$x \leq 4$ ← x-values less than or equal to 4.

Domain: $(-\infty, 0) \cup (3, 4]$

Problem 9. Solve the equation $e^{4x} = 5e^{7x}$ for x.

$e^{4x} = 5e^{7x}$

$\frac{e^{4x}}{e^{7x}} = 5$ isolate x by dividing by e^{7x}

$e^{4x-7x} = 5$ use laws of exponents

$e^{-3x} = 5$ simplify

$\ln e^{-3x} = \ln 5$ take ln of both sides

$-3x = \ln 5$ divide by -3

$x = -\frac{\ln 5}{3}$

Problem 10. Write the expression $\frac{1}{3}\ln(x) - \ln(x+y) + 4\ln(2z)$ as a single logarithm. Assume all variables represent positive numbers.

$\frac{1}{3}\ln(x) - \ln(x+y) + 4\ln(2z)$

exponents first $= \ln x^{1/3} - \ln(x+y) + \ln(2z)^4$

$= \ln\left(\frac{x^{1/3}}{x+y}\right) + \ln(2z)^4$

$= \ln\left(\frac{x^{1/3}}{x+y} \cdot \frac{2^4 z^4}{1}\right)$

$= \ln\left(\frac{16x^{1/3}z^4}{x+y}\right)$