
Problem 1

Define A = event exactly 9 people say yes.

1. The complement of A is the event that:
   - **a)** anything other than 9 people say yes
   - **b)** anything other than 9 people say no
   - **c)** exactly 6 people say yes
   - **d)** exactly 6 people say no
   - **e)** more than 6 people say no

Problem 2

At a large university, the probability that a student takes calculus and statistics in the same semester is 0.0125. The probability that a student takes statistics is 0.125; the probability that a student takes calculus is 0.3

2. Find the probability that a student is taking calculus, given that he or she is taking statistics.
   - **a)** 0.1
   - **b)** 0.1125
   - **c)** 0.0016
   - **d)** 0.1375
   - **e)** 0.4800

   P(calc | stats) = P(calc AND stats) / P(stats) = .0125 / .125 = .1

3. Is the event of taking calculus independent of the event of taking statistics? Justify your answer numerically.

   Using the answer in question two, P(C|S)=.1 and it is not equal to P(C)=.3. Therefore the first condition for independence is not satisfied.

Problem 3

4. If you flip a coin three times, the possible outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. What is the probability of getting at least one head?
   a. ½
   b. ⅞
   c. ¼
   d. ¾
5. When a quarter is tossed four times, 16 outcomes are possible.

1 Duke, Çetinkaya-Rundel

HHHH	HHHT	HHTH	HHTT
HTHH	HTHT	HTTH	HTTT
THHH	THHT	THTH	THTT
TTHH	TTHT	TTTH	TTTT

Here, for example, HTTH represents the outcome that the first toss is heads, the next two tosses are tails, and the fourth toss is heads. The events A and B are defined as follows:

Event A = the first two tosses are heads
Event B = the first and last tosses are the same

Are events A and B mutually exclusive?

a) Yes
b) No

A and B are not mutually exclusive because there are outcomes that are both in A and B like {HHHH, HHTH}.

Problem 4

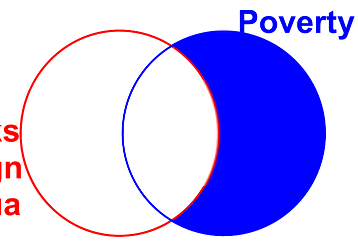
The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English (foreign language) at home, and 4.2% fall into both categories.

Consider P(Poverty) = 0.146;
P(Foreign) = 0.207;
P(Poverty ∩ Foreign) = 0.042

6. Are living below the poverty line and speaking a foreign language at home disjoint?
   No, because there are Americans that live below poverty and speak a foreign language.

7. What percent of Americans live below the poverty line and only speak English at home?
   0.146 – 0.042 = 0.104

1 Duke, Çetinkaya-Rundel



8. What percent of Americans live below the poverty line or speak a foreign language at home?
   P(Poverty ∪ Foreign) = P(Poverty) + P(Foreign) - P(Poverty ∩ Foreign) = 0.146 + 0.207 – 0.042 = 0.311

9. What percent of Americans live above the poverty line and only speak a foreign language at home?
   0.207 - 0.042

10. Is the event that someone lives below the poverty line independent of the event that the person speaks a foreign language at home?
    They are not independent because
    P(Poverty ∩ Foreign) = 0.042 ≠ P(poverty) × P(foreign) = 0.146 x 0.207 = 0.0302

Problem 5

Consider P(A) = 0.3, P(B) = 0.7

11. Can you compute P(A and B) if you only know P(A) and P(B)?
    If A and B are independent then $P(A \cap B) = P(A) \times P(B)$

12. Assuming that events A and B arise from independent random processes,
    a. What is P(A and B)?
       $P(A \cap B) = 0.3 \times 0.7 = .21$
    b. What is P(A or B)?
       $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.7 - 0.21 = 0.79$
    c. What is P(A|B)?
       $P(B) = \frac{P(A \cap B)}{P(B)} = \frac{0.21}{0.7} = 0.3$

1 Duke, Çetinkaya-Rundel

13. If we are given that P(A and B) = 0.1, are the random variables giving rise to events A and B independent?
    A and B are not independent because $P(A \cap B) = 0.1 \neq P(A) \times P(B) = 0.21$

14. If we are given that P(A and B) = 0.1, what is P(A|B)?

$$P(B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.7} = 0.1428$$

Problem 6

15. A quiz consists of 3 multiple choice questions with 4 possible answers each. If a student guesses the answer to each question, what is the probability that they get all the answers correct?

$$P(C_1 \; and \; C_2 \; and \; C_3) = P(C1) * P(C2) * P(C3) = 0.25 \times 0.25 \times 0.25$$

Problem 7 (contingency table, conditional probability, joint probability, marginal probability)

The family college data set contains a sample of 792 cases with two variables, teen and parents, and is summarized in the following Table:

		parents		
		degree	not	Total
teen	college	231	214	445
	not	49	298	347
	Total	280	512	792

The teen variable is either college or not, where the college label means the teen went to college immediately after high school. The parent's variable takes the value degree if at least one parent of the teenager completed a college degree.

1 Duke, Çetinkaya-Rundel

16. If at least one parent of a teenager completed a college degree, what is the chance the teenager attended college right after high school?
    P(C|D) = 231 / 280

17. Probability of a random teenager from the study went to college right after highschool:
    P(C) = 445 / 792

Problem 8 (general multiplication rule)

A smallpox data set provides a sample of 6,224 individuals from the year 1721 who were exposed to smallpox in Boston. Doctors at the time believed that inoculation, which involves exposing a person to the disease in a controlled form, could reduce the likelihood of death.

18. Suppose we are given only two pieces of information: 96.08% of residents were not inoculated, and 85.88% of the residents who were not inoculated ended up surviving. How could we compute the probability that a resident was not inoculated and lived?
    P (Not inoculated and survived) = P (survived | not inoculated) x P (not inoculated)
    = .8588 x .9608

Problem 9

A and B are mutually exclusive, with P(A) = .3 and P(B) = .5.

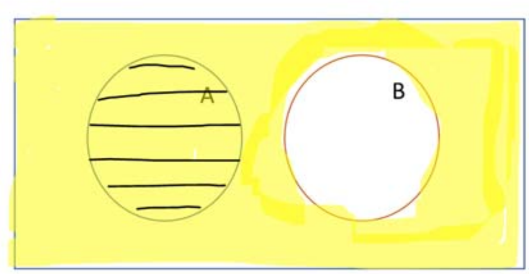
19. What is the probability that either A or B occurs?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = 0.3 + 0.5 - 0 = 0.8$$

20. A occurs but not B?

$P(A \cap B^c) = P(A) = 0.3$

$A \cap B^c$ is the region colored yellow and with black lines.



21. both A and B occur?

1 Duke, Çetinkaya-Rundel

$P(A \cap B) = 0$ because A and B are disjoint.

Of campus professors 60% are male, and of these, 15% work for College of Humanities. Find the following probabilities:

$$\boldsymbol{P(male) = 0.6}$$
$$\boldsymbol{P(Humanities|male) = 0.15}$$

22. Randomly selected professor is a male and works for College of Humanities.

$$\boldsymbol{P(Humanities|male) = \frac{P(humanities \cap male)}{P(male)}}$$

$$\boldsymbol{P(Humanities|male)P(male) = P(humanities \cap male)}$$

$$\boldsymbol{P(humanities \cap male) = 0.15 \times .6 = 0.09}$$

1 Duke, Çetinkaya-Rundel