Math 151 - Week-In-Review 12 (5.5; Final Exam)

Justin Cantu

Disclaimer: This review does not cover every concept covered in MATH151 and should not be used as your sole source of study for the exam. You should also review lecture notes, Week-in-Review problems, HOGU problems, past exams, quizzes, and homework.

1. Evaluate $\int_0^1 \frac{2x}{(3x^2+1)^2}\,dx$.

   (a) $\frac{3}{4}$
   (b) $\frac{1}{4}$
   (c) $-\frac{1}{3}$
   (d) $\frac{21}{64}$
   (e) $\frac{21}{32}$

$u = 3x^2+1$

$du = (3x^2+1)'dx$

$du = 6x\,dx \Rightarrow 2x\,dx = \frac{du}{3}$

$x=0: \quad u(0) = 3\cdot 0^2+1 = 1$

$x=1: \quad u(1) = 3\cdot 1^2+1 = 4$

$= \int_1^4 \frac{\frac{du}{3}}{u^2} = \frac{1}{3}\int_1^4 \frac{du}{u^2}$

$= \frac{1}{3}\left(-\frac{1}{u}\right)\Big|_1^4$

$= -\frac{1}{3}\left(\frac{1}{4}-1\right) = -\frac{1}{\cancel{3}}\cdot\left(-\frac{\cancel{3}}{4}\right) = \boxed{\frac{1}{4}}$

2. After an appropriate u-substitution, which of the following is equivalent to $\int x\sqrt{x+1}\,dx$?

   (a) $\int \sqrt{u^2+u}\,du$
   (b) $\int (u+u^{1/2})\,du$
   (c) $\int (u^{3/2}+u^{1/2})\,du$
   (d) $\int (u^{3/2}-u^{1/2})\,du$
   (e) None of these.

$u = x+1 \Rightarrow x = u-1$

$dx = du$

$= \int \overbrace{(u-1)}^{x}\sqrt{u}\,du$

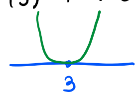
$= \int [u\sqrt{u} - \sqrt{u}]\,du = \int (u^{3/2} - u^{1/2})\,du$

3. For a continuous function f, if $f'(3) = 0$ and $f''(3) = 7$, what can we conclude about the graph of f at $x = 3$?

   (a) There is a local maximum at $x = 3$.
   (b) The absolute maximum is at $x = 3$.
   (c) There is a local minimum at $x = 3$.
   (d) The absolute minimum is at $x = 3$.
   (e) No conclusion can be made.

$f'(3)=0 \rightarrow f$ has a horizontal tangent @ $x=3$

$f''(3)=7>0 \Rightarrow f$ is concave up @ $x=3$



f has a local min @ $x=3$

$24x^{-2}+12x+b$

4. Let b be a real number. Find the x-value(s) where $f(x) = \dfrac{24}{x^2} + 12x + b$ has a local maximum or minimum.

   (a) local min at $x = \sqrt[3]{4}$ only
   (b) local max at $x = 0$, local min at $x = \sqrt[3]{4}$
   (c) local min at $x = 0$, local max at $x = \sqrt[3]{4}$
   (d) local max at $x = \sqrt[3]{4}$ only
   (e) local max at $x = 0$

$f'(x) = -2(24)x^{-3}+12 = 0$, dividing by 12: $\dfrac{0}{12}$

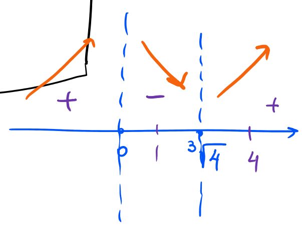
$f'(x) = 12 - \dfrac{48}{x^3} > 0$

$-4x^{-3}+1=0$ or $-\dfrac{4}{x^3}+1=0$

solve for x: $(x^3)\dfrac{4}{x^3} = 1(x^3)$

$x^3=4$ or $\boxed{x=\sqrt[3]{4}}$ ≈ 1.3333

$\boxed{x=0}$



$\boxed{\text{no local max}}$

$\boxed{\text{local min @ } x=\sqrt[3]{4}}$

$f'(4)>0$

$f'(1) = 12-48<0$

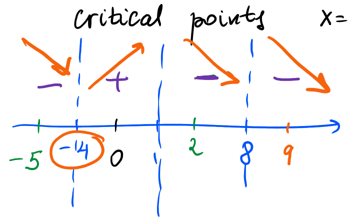
$f'(-1) = 12+48>0$

5. Suppose the function $f(x)$ is defined on all real numbers except $x = 8$ and $f'(x) = \dfrac{-7(x-1)(x+4)}{(x-8)^4}$.

   Where does f has a local minimum?

   (a) $x = 8$ only
   (b) $x = 1$ only
   (c) $x = -4$ only
   (d) $x = 1$ and $x = 8$ only
   (e) f does not have a local minimum

critical points $x=8$, $x=1$, $x=-4$



$f'(9) = \dfrac{-7(8)(13)}{1^4} < 0$

$f'(2) = \dfrac{-7(2-1)(2+4)}{(2-8)^4} < 0$

$f'(0) = \dfrac{-7(0-1)(0+4)}{(0-8)^4} > 0$

$f'(-5) = \dfrac{-7(-5-1)(-5+4)}{(-5-8)^4} < 0$

L'Hospital's Rule

6. Evaluate $\lim_{x\to 0^-} \frac{e^{4x} - 5 + 4(x+1)}{x^2}$ $= \left| \frac{0}{0} \right| = \lim_{x\to 0^-} \frac{4e^{4x}+4}{2x} \left| \frac{8}{-0} \right| = -\infty$

(a) 0
(b) 8
(c) ∞
(d) 2
(e) $-\infty$

$f(7) = 7^2 - 1$
$f(5) = 5^2 - 1$
$f(3) = 3^2 - 1$

7. Approximate the area under the curve $f(x) = x^2 - 1$ on the interval $[2, 8]$ using three rectangles of equal width and midpoints.

$\Delta x = \frac{8-2}{3} = \frac{6}{3} = 2$

$x_0 = 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 = x_3$

(a) 106
(b) 226
(c) 80
(d) 304
(e) 160

$A \approx f(3)\Delta x + f(5)\Delta x + f(7)\Delta x$
$= 2\left(f(3) + f(5) + f(7)\right) = 2\left(3^2 - 1 + 5^2 - 1 + 7^2 - 1\right)$
$= 2\left(9 + 25 + 49 - 3\right) = 2(80) = 160$

8. A particle has acceleration (in m/s^2) given by $a(t) = 12t$ on the interval $[0, 10]$. If this particle has an initial velocity of 12 m/s and position of 15 m at $t = 1$, determine its position at $t = 5$.

$v(0) = 12, \quad s(1) = 15, \quad s(5) = ?$

(a) 339 m
(b) 315 m
(c) 325 m
(d) 311 m
(e) 301 m

$a(t) = 12t$

velocity $v(t) = \int a(t)\,dt = \int 12t\,dt = \frac{12t^2}{2} + C$

$\boxed{v(t) = 6t^2 + C}$

$12 = v(0) = C \Rightarrow C = 12$

$\boxed{v(t) = 6t^2 + 12}$

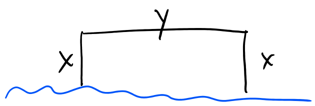
position $s(t) = \int v(t)\,dt = \int (6t^2 + 12)\,dt = \frac{6t^3}{3} + 12t + K = 2t^3 + 12t + K$

$15 = s(1) = 2 + 12 + K$ or $15 = 14 + K \Rightarrow K = 1$

$\boxed{s(t) = 2t^3 + 12t + 1}$ $s(5) = 2 \cdot 5^3 + 12(5) + 1 = 250 + 60 + 1 = \boxed{311}$

9. Rancher John wants to fence a new pasture using a straight river as one side of the boundary. If Rancher John has 1200 yards of fencing materials, what are the dimensions of the largest area of the pasture that Rancher John can enclose?

   (a) 300 yards × 300 yards
   (b) 300 yards × 600 yards
   (c) 250 yards × 700 yards
   (d) 90,000 square yards
   (e) 180,000 square yards



length of the fence is $2x+y=P$

$A = xy \leftarrow$ to be maximized

$2x+y=1200 \Rightarrow y = 1200-2x$

$A = x(1200-2x)$

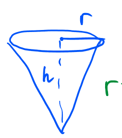
$A(x) = 1200x - 2x^2$

$A'(x) = 1200 - 4x = 0 \Rightarrow \boxed{x = \frac{1200}{4} = 300}$

$y = 1200 - 2x = 1200 - 2(300) = 1200 - 600 = \boxed{600 = y}$

10. Sand is being poured at a rate of 10 ft^3/min onto a cone-shaped pile. If the height of the pile is always twice the base radius, at what rate is the height increasing when the pile is 8 ft high? Recall the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

   (a) $\frac{5}{64\pi}$ ft/min
   (b) $\frac{5}{8\pi}$ ft/min
   (c) $\frac{5}{32\pi}$ ft/min
   (d) $\frac{10}{9\pi}$ ft/min
   (e) $\frac{10}{27\pi}$ ft/min



$\frac{dV}{dt} = 10$

$r = \frac{h}{2} \Leftarrow \boxed{h = 2r}$ Find $\frac{dh}{dt}$, if $h=8$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ or $V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{\cancel{12}\,4}\pi\,\cancel{3}h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4\,\frac{dV}{dt}}{\pi h^2} = \frac{4(10)}{\pi \cdot 8^2}$

$= \frac{40}{64\pi} = \frac{5}{8\pi}$

11. Let f be a differentiable function such that $f(3) = 1$ and $f'(3) = -3$. If $h(x) = \dfrac{2f(x)}{x^2+1}$, find $h'(3)$.

$$h'(x) = \frac{2f'(x)(x^2+1) - 2f(x)(2x)}{(x^2+1)^2} = \frac{2f'(x)(x^2+1) - 4xf(x)}{(x^2+1)^2}$$

$$h'(3) = \frac{2f'(3)(3^2+1) - 4(3)f(3)}{(3^2+1)^2} = \frac{2(10)(-3) - 12(1)}{100} = -\frac{72}{100} = -\frac{18}{25}$$

(a) $-\dfrac{18}{25}$

(b) $-\dfrac{12}{25}$

(c) $\dfrac{18}{25}$

(d) $-\dfrac{26}{5}$

(e) $\dfrac{24}{5}$

12. Find the value c that satisfies the conclusion of the Mean Value Theorem for $f(x) = -3x^2 + 5x + 5$ on the interval $[0, 3]$.

$f'(x) = -6x + 5$

$$\frac{f(3)-f(0)}{3-0} = f'(c)$$

$$\frac{-3\cdot 9 + 5\cdot 3 + 5 - 5}{3} = -6\cdot c + 5$$

$$\frac{-12}{3} = -6c + 5 \quad \text{or} \quad -4 = -6c + 5$$

$$6c = 9 \Rightarrow c = \frac{9}{6} = \frac{3}{2}$$

(a) $\dfrac{17}{6}$

(b) 3

(c) $\dfrac{3}{2}$

(d) 0

(e) $\dfrac{17}{18}$

13. Use logarithmic differentiation to find the derivative of $f(x) = \dfrac{(x^3+2x)^{400}}{(1+x)^{300}}$.

(a) $f'(x) = \left[\dfrac{400(3x^2+2)}{x^3+2x} - \dfrac{300}{1+x}\right]\dfrac{(x^3+2x)^{400}}{(1+x)^{300}}$

(b) $f'(x) = \left[\dfrac{400}{x^3+2x} - \dfrac{300}{1+x}\right]\dfrac{(x^3+2x)^{400}}{(1+x)^{300}}$

(c) $f'(x) = \dfrac{400(3x^2+2)}{x^3+2x} - \dfrac{300}{1+x}$

(d) $f'(x) = \dfrac{400(3x^2+2)(x^3+2x)^{399}}{(1+x)^{300}} - \dfrac{300(x^3+2x)^{400}}{(1+x)^{301}}$

(e) $f'(x) = \dfrac{4(3x^2+2)(x^3+2x)^{399}}{3(1+x)^{299}}$

$$\ln f(x) = \ln \frac{(x^3+2x)^{400}}{(1+x)^{300}}$$

$$\ln f(x) = \ln (x^3+2x)^{400} - \ln(1+x)^{300}$$

$$[\ln f(x)]' = [400 \ln(x^3+2x) - 300 \ln(1+x)]'$$

$$\frac{f'(x)}{f(x)} = 400\frac{(x^3+2x)'}{x^3+2x} - 300\frac{(x+1)'}{x+1}$$

$$\frac{f'(x)}{f(x)} = 400\frac{3x^2+2}{x^3+2x} - 300\frac{1}{x+1}$$

$$f'(x) = \frac{(x^3+2x)^{400}}{(1+x)^{300}}\left[400\frac{3x^2+2}{x^3+2x} - 300\frac{1}{x+1}\right]$$

14. Find the t-value(s) where the tangent line to the curve $\boxed{x = 2t^3 - t^2 + 6}$, $\boxed{y = -t^3 + \frac{9}{2}t^2 - 6t}$ is horizontal or vertical.

(a) horizontal at $t = 1, 2$; vertical at $t = 0, \frac{1}{3}$
(b) horizontal at $t = 0, \frac{1}{3}$; vertical at $t = 1, 2$
(c) horizontal at $t = \frac{2}{3}, 1$; vertical at $t = 0$
(d) horizontal at $t = 0$; vertical at $t = \frac{2}{3}, 1$
(e) horizontal at $t = 1$; no vertical tangents

slope $= \frac{dy}{dx} = \frac{y'(t)}{x'(t)}$

horizontal tangent $\Rightarrow \frac{dy}{dx} = 0$ or $y'(t) = 0$

$y'(t) = -3t^2 + 9t - 6 = 0$

$\frac{3t^2 - 9t + 6}{3} = \frac{0}{3} \Rightarrow t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

H.A. $\boxed{t_1 = 2, t_2 = 1}$

vertical asymptotes $\frac{dy}{dx} = \infty$ or $x'(t) = 0$ or $x'(t) = 6t^2 - 2t = 0$

$2t(3t-1) = 0$ or $\boxed{t_1 = 0, t_2 = \frac{1}{3}}$ V.A.

15. Find a tangent vector of unit length to $\mathbf{r}(t) = \langle \sqrt{10t + 5}, e^{4t-8} \rangle$ at $t = 2$.

(a) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$
(b) $\langle 1, 1 \rangle$
(c) $\langle 1, 0 \rangle$
(d) $\left\langle \frac{1}{5}, \frac{4}{5} \right\rangle$
(e) $\left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$

$\vec{r}'(t) = < \frac{(10)}{2\sqrt{10t+5}}, e^{4t-8}(4) >$

$\vec{r}'(t) = < \frac{5}{\sqrt{10t+5}}, 4e^{4t-8} >$

$\vec{r}'(2) = < \frac{5}{\sqrt{25}}, 4e^{8-8} > = <1, 4>$

$\vec{u} = \frac{\vec{r}'(2)}{|\vec{r}'(2)|} = \frac{<1,4>}{\sqrt{1+4^2}} = \frac{<1,4>}{\sqrt{1+16}} = \frac{<1,4>}{\sqrt{17}}$

16. Determine the values of a and b that make the following function differentiable everywhere:

Both f and f' must be continuous.

$$f(x) = \begin{cases} ax^3 + 16x & \text{if } x < 1 \\ 5x^2 + b & \text{if } x \geq 1 \end{cases}$$

(a) $a = 16$, $b = 5$
(b) $a = -2$, $b = 9$
(c) $a = -\frac{5}{3}$, $b = \frac{28}{3}$
(d) $a = 1$, $b = 0$
(e) There are no such values.

$f'(x) = \begin{cases} 3ax^2 + 16, & x < 1 \\ 10x, & x \geq 1 \end{cases}$

$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

$\boxed{3a + 16 = 10} \Rightarrow 3a = -6 \Rightarrow \boxed{a = -2}$

$a(1)^3 + 16(1) = 5(1)^2 + b$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$a + 16 = 5 + b$

$\boxed{b = a + 11} \Rightarrow \boxed{b = 11 - 2 = 9}$

17. Suppose $\int_5^9 g(x)\,dx = 4$. Evaluate $\int_5^9 [3 - 4g(x)]\,dx$. $= \int_5^9 3\,dx - 4\int_5^9 g(x)dx$

(a) 39
(b) -13
(c) 19
(d) -4
(e) -36

$= 3x\Big|_5^9 - 4(4)$

$= 3(9-5) - 16 = 12 - 16 = -4$

18. Evaluate $\int_1^2 \left(\frac{9}{x^5} - \frac{2}{x}\right) dx$. $= \int_1^2 \left(9x^{-5} - \frac{2}{x}\right)dx$

(a) $\frac{135}{64} - 2\ln(2)$
(b) $-\frac{135}{64} - \ln(2)$
(c) $\frac{135}{64} + 2\ln(2)$
(d) $-\frac{135}{64} - 2\ln(2)$
(e) $\frac{135}{64} - 2\ln(2)$
(f) $\frac{135}{64} + \ln(2)$

$= \left[9\,\frac{x^{-5+1}}{-5+1} - 2\ln|x|\right]_1^2$

$= \left[-\frac{9}{4x^4} - 2\ln|x|\right]_1^2$

$= -\frac{9}{4(16)} - 2\ln 2 + \frac{9}{4} + 2\cancel{\ln 1}^{\,0}$

$= +\frac{9}{4}\left(1 - \frac{1}{16}\right) - 2\ln 2 = \frac{9}{4}\cdot\frac{15}{16} - 2\ln 2$

$= \frac{135}{64} - 2\ln 2$

19. Evaluate $\int \left(3x^2 - 10 + \frac{3}{x^2+1}\right) dx$. $= \frac{3x^3}{3} - 10x + 3\arctan x + C$

(a) $\frac{1}{3}x^3 - 10x + 3\arctan x + C$
(b) $\frac{1}{3}x^3 - 10x + 3\arcsin x + C$
(c) $x^3 - 10x + 3\arctan x + C$
(d) $x^3 - 10x + 3\arcsin x + C$
(e) $\frac{1}{3}x^3 - 10x + 3\tan x + C$

$= x^3 - 10x + 3\arctan x + C$

20. Given the points $A(1,0)$, $B(0,2)$, and $C(3,4)$, find $\angle BAC$.

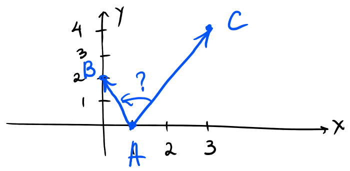
(a) $\arccos\left(\frac{3}{5}\right)$

(b) $\arccos\left(-\frac{1}{\sqrt{65}}\right)$

(c) $180°$

(d) $\arccos\left(\frac{1}{\sqrt{65}}\right)$

(e) $\arccos\left(\frac{3}{\sqrt{17}}\right)$



$\vec{AB} = \langle 0-1, 2-0 \rangle = \langle -1, 2 \rangle$

$\vec{AC} = \langle 3-1, 4-0 \rangle = \langle 2, 4 \rangle$

$\cos(\angle BAC) = \dfrac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|}$

$= \dfrac{\langle -1,2 \rangle \cdot \langle 2,4 \rangle}{\sqrt{1+4} \cdot \sqrt{4+16}} = \dfrac{-2+8}{\sqrt{5} \cdot \sqrt{20}} = \dfrac{6}{\sqrt{5} \cdot 2\sqrt{5}} = \dfrac{3}{5}$

$\angle BAC = \arccos\left(\frac{3}{5}\right)$

21. Find the slope of the tangent line to the graph of $\boxed{x^2y^2 - 3y = 0}$ at the point $(1, -3)$.

(a) $-\frac{5}{2}$

(b) $\frac{5}{2}$

(c) -8

(d) 2

(e) -2

Implicit differentiation.

$\frac{d}{dx}(x^2y^2 - 3y) = 0$

$2xy^2 + x^2(2y)y' - 3y' = 0$

$2xy^2 + y'(2x^2y - 3) = 0$

$y' = -\dfrac{2xy^2}{2x^2y - 3} = \dfrac{2xy^2}{3 - 2x^2y}$

$y'(1,-3) = \dfrac{2(1)(-3)^2}{3 - 2(1)^2(-3)} = \dfrac{18}{9} = 2$

22. Calculate $\lim\limits_{x \to 3^+} \dfrac{x^2 - 2x - 7}{(x-3)(x-2)}$. $= \dfrac{9-6-7}{0 \cdot (3-2)} = -\dfrac{4}{0} = -\infty$

(a) $-\infty$

(b) ∞

(c) 1

(d) -4

(e) $-\frac{7}{6}$

23. Use a linear approximation at $a = 27$ to estimate $\sqrt[3]{27.2}$.

(a) $\frac{801}{270}$

(b) $\frac{1}{135}$

(c) $\frac{406}{135}$

(d) $\frac{402}{135}$

(e) $\frac{803}{270}$

$f(a+\Delta x) = f(x) \approx f(a) + f'(a)\,\Delta x$

$f(x) = \sqrt[3]{x} = x^{1/3}$, $a = 27$, $\Delta x = 27.2 - 27 = 0.2$

$f(x) = \sqrt[3]{x}$ | $f(27) = \sqrt[3]{27} = 3$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$ | $f'(27) = \frac{1}{3(27)^{2/3}} = \frac{1}{3\cdot 9} = \frac{1}{27}$

$\sqrt[3]{27.2} \approx f(27) + f'(27)(0.2)$

$= 3 + \frac{1}{27}(0.2) = 3 + \frac{1}{5(27)} = 3 + \frac{1}{135} = \frac{405+1}{135}$

$= \frac{406}{135}$

24. Compute $\lim_{x\to-\infty} \frac{5e^{2x} - 8e^{-3x}}{3e^{2x} + 2e^{-3x}}$.

(a) 0

(b) $\frac{5}{3}$

(c) -4

(d) $-\infty$

(e) ∞

$= \lim_{x\to-\infty} \frac{-8\cancel{e^{-3x}}}{2\cancel{e^{-3x}}} = -4$

$\lim_{x\to-\infty} e^{ax} = \begin{cases} 0, & a > 0 \\ \infty, & a < 0 \end{cases}$

$$\boxed{\lim_{x\to\infty} \frac{5e^{2x} - \cancel{8e^{-3x}}^{\,0}}{3e^{2x} + \cancel{2e^{-3x}}^{\,0}} = \lim_{x\to\infty} \frac{5e^{2x}}{3e^{2x}} = \frac{5}{3}}$$

25. Calculate $\lim_{x\to\infty} [\ln(1+2x) - \ln(2+x)]$.

(a) 0

(b) 1

(c) $\ln(2)$

(d) ∞

(e) $-\infty$

$(\infty - \infty) = \lim_{x\to\infty} \ln \frac{1+2x}{2+x}$

$= \ln\left(\lim_{x\to\infty} \frac{1+2x}{2+x}\right) = \ln\left(\lim_{x\to\infty} \frac{\cancel{x}(\cancel{1/x}+2)}{\cancel{x}(\cancel{2/x}+1)}\right)$

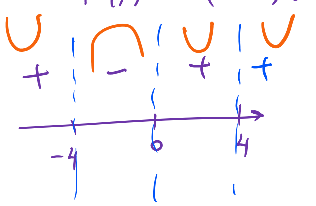
$= \ln 2$

26. Suppose the function f has domain of all real numbers and $f''(x) = 3x(x^2 - 16)(x - 4)$. Determine the x-coordinate(s) of any inflection points.

$f''(x) = 3x\underbrace{(x-4)(x+4)}_{x^2-16}(x-4)$

(a) $x = 0, 4, -4$ only
(b) $x = 0, -4$ only
(c) $x = 4, -4$ only
(d) $x = 0, 4$ only
(e) f has no inflection points

$f''(x) = 3x(x+4)(x-4)^2$



$f''(5) = 3(5)(5+4)(5-4)^2 > 0$

$f''(2) = 3(2)(2+4)(2-4)^2 > 0$

$f''(-2) = 3(-2)(-2+4)(-2+4)^2 < 0$

$f(-5) = 3(-5)(-5+4)(-5-4)^2 > 0$

$x = -4$ and $x = 0$

27. The position (in meters) of an object is given by $\boxed{s(t) = \cos(t) + \frac{1}{4}t^2}$. Find the time(s) when the acceleration of the object is 0 m/s^2 for $0 \le t \le 2\pi$.

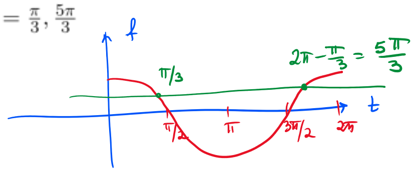
(a) $t = \frac{\pi}{3}, \frac{2\pi}{3}$
(b) $t = \frac{\pi}{6}, \frac{5\pi}{6}$
(c) $t = \frac{4\pi}{3}, \frac{5\pi}{3}$
(d) $t = \frac{7\pi}{6}, \frac{11\pi}{6}$
(e) $t = \frac{\pi}{3}, \frac{5\pi}{3}$

velocity $v(t) = s'(t) = -\sin t + \frac{2}{4}t$

acceleration $a(t) = v'(t) = -\cos t + \frac{1}{2} = 0$

$\cos t = +\frac{1}{2}$

$t = \frac{\pi}{3}, \frac{5\pi}{3}$



28. Differentiate $f(x) = \arcsin(e^{4x})$.

$$\boxed{(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}\, u'}$$

$$f'(x) = \frac{1}{\sqrt{1-(e^{4x})^2}}\,(e^{4x})'$$

$$= \frac{4e^{4x}}{\sqrt{1-e^{8x}}}$$

(a) $f'(x) = \dfrac{4e^{4x}}{\sqrt{1-e^{8x}}}$

(b) $f'(x) = -\dfrac{4e^{4x}}{\sqrt{1-e^{8x}}}$

(c) $f'(x) = \dfrac{4e^{4x}}{1+e^{8x}}$

(d) $f'(x) = -\dfrac{4e^{4x}}{1+e^{8x}}$

(e) $f'(x) = \dfrac{4e^{4x}}{1-e^{8x}}$

29. The velocity (in m/s) of an object is given by $\boxed{v(t) = 3t - 7}$. What is the displacement of the particle over the interval $0 \leq t \leq 4$?

(a) 4 m
(b) -32 m
(c) 32 m
(d) 12 m
(e) -4 m

$$\text{displacement} = \int_0^4 (3t-7)\,dt$$
$$= \left[\frac{3t^2}{2} - 7t\right]_0^4$$
$$= \frac{3(16)}{2} - 7(4) = 24 - 28 = -4$$

30. A horizontal force of 20 pounds is acting on a box as it is pushed up a ramp that is 5 feet long and inclined at an angle of 60° above the horizontal. Find the work done on the box.

(a) 50 ft·lb
(b) $50\sqrt{3}$ ft·lb
(c) $50\sqrt{2}$ ft·lb
(d) 100 ft·lb
(e) 10 ft·lb

$$W = \vec{F}\cdot\vec{D} = |\vec{F}|\cdot|\vec{D}|\cos\theta = 20(5)\cos 60°$$
$$|\vec{F}| = 20 \qquad \theta = 60° \qquad = 100\cdot\frac{1}{2} = 50$$
$$|\vec{D}| = 5$$

31. A plane is flying at 850 mph at N45°E. The wind is blowing at 30 mph at S60°E. Find the true direction of the plane.

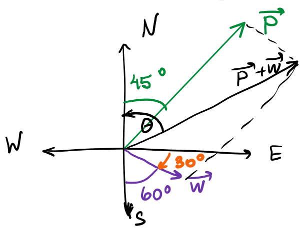
(a) $\theta = \arctan\left(\dfrac{425\sqrt{2}+15\sqrt{3}}{425\sqrt{2}-15}\right)$

(b) $\theta = \arctan\left(\dfrac{425\sqrt{2}+15\sqrt{3}}{425\sqrt{2}+15}\right)$

(c) $\theta = \arctan\left(\dfrac{15-425\sqrt{2}}{15\sqrt{3}+425\sqrt{2}}\right)$

(d) $\theta = \arctan\left(\dfrac{425\sqrt{2}-15}{425\sqrt{2}+15\sqrt{3}}\right)$

(e) $\theta = \arctan\left(\dfrac{425\sqrt{2}+15\sqrt{3}}{15-425\sqrt{2}}\right)$



$$|\vec{P}| = 850$$
$$|\vec{W}| = 30$$

$$\vec{P} = 850\langle \cos 45°, \sin 45°\rangle$$
$$= 850\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$$
$$\boxed{\vec{P} = \langle 425\sqrt{2}, 425\sqrt{2}\rangle}$$

$$\vec{W} = 30\langle \cos 30°, -\sin 30°\rangle$$
$$= 30\left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2}\right\rangle$$
$$\boxed{\vec{W} = \langle 15\sqrt{3}, -15\rangle}$$

$$\vec{P}+\vec{W} = \langle 425\sqrt{2}+15\sqrt{3}, 425\sqrt{2}-15\rangle$$

$$\tan\theta = \frac{425\sqrt{2}-15}{425\sqrt{2}+15\sqrt{3}}, \quad \boxed{\theta = \arctan\frac{425\sqrt{2}-15}{425\sqrt{2}+15\sqrt{3}}}$$

Evaluate the integral

(a) $\int \frac{\arctan x}{1+x^2}\,dx$ $\quad\left|\; u = \arctan x,\quad du = \frac{1}{1+x^2}\,dx \;\right|\quad = \int u\,du = \frac{u^2}{2} + C = \frac{\arctan^2 x}{2} + C$

(b) $\int x^3\sqrt{x^2+1}\,dx = \left|\; x^2 = u-1,\quad u = x^2+1,\quad du = 2x\,dx \;\right| = \frac{1}{2}\int \underbrace{2x \cdot x^2 \sqrt{x^2+1}\,dx}_{du} = \frac{1}{2}\int (u-1)\sqrt{u}\,du$

$$= \frac{1}{2}\int \left(u^{3/2} - u^{1/2}\right) du \qquad = \frac{1}{2}\left[\frac{u^{3/2+1}}{\frac{3}{2}+1} - \frac{u^{1/2+1}}{1/2+1}\right] + C$$

$$= \frac{1}{2}\left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + C$$

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \boxed{\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C}$$

(c) $\int \frac{\ln x}{x}\,dx$ $\quad\left|\; u = \ln x,\quad du = \frac{dx}{x} \;\right| = \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{\ln^2 x}{2} + C}$

(d) $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$

$u = 1+2x$

$x = \frac{u-1}{2}$

$dx = \frac{du}{2}$

limits:

$x=0 \longrightarrow u(0) = 1+2\cdot 0 = 1$

$x=4 \longrightarrow u(4) = 1+2\cdot 4 = 9$

$$= \int_1^9 \frac{\frac{u-1}{2}}{\sqrt{u}}\,\frac{du}{2}$$

$$= \frac{1}{4}\int_1^9 \frac{u-1}{\sqrt{u}}\,du$$

$$= \frac{1}{4}\int_1^9 (u-1)\,u^{-1/2}\,du$$

$$= \frac{1}{4}\int_1^9 \left(u^{1/2} - u^{-1/2}\right) du$$

$$= \frac{1}{4}\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right]_1^9$$

$$= \frac{1}{\cancel{4}\,2}\left[\frac{\cancel{2}}{3}u^{3/2} - \cancel{2}\,u^{1/2}\right]_1^9$$

$$= \frac{1}{2}\left(\frac{1}{3}9^{3/2} - 9^{1/2} - \frac{1}{3}(1) + 1\right]$$

$$= \frac{1}{2}\left(\frac{27}{3} - 3 - \frac{1}{3} + 1\right)$$

$\frac{1}{2}\left(\frac{2\,\ldots}{\ldots}\right) \quad \frac{1}{\ldots} \quad \frac{20}{\ldots} \quad \boxed{\frac{10}{\ldots}}$

$$= \frac{\;}{2}\left(\frac{\;}{3} - \cdots - \frac{\;}{3} + 1 \right)$$

$$= \frac{1}{2}\left(\frac{26}{3} - 2 \right) = \frac{1}{2} \cdot \frac{20}{3} = \boxed{\frac{10}{3}}$$