Math 308: Week-in-Review 2 (2.2& 2.3)

Section 2.2 (Separation of variables)

1. Use separation of variables to find the general solution to the following differential equations.

   (a) $\dfrac{dy}{dx} = 3y + 2$

   (b) $\dfrac{dg}{dx} = \dfrac{2g}{x+1}$

(c) $\dfrac{d\theta}{dt} = t\sqrt{t^2+1}\sec(\theta), \qquad \theta(0) = \dfrac{\pi}{6}$

(d) $\dfrac{dy}{dt} = \dfrac{2t}{y + yt^2}, \qquad y(2) = 3$. Determine the values of t where the solution is defined.

2. Solve the differential equation $\dfrac{dy}{dt} = (y^2 - 9)\cos(t), \;\; y(0) = 6$. Leave your solution in both implicit and explicit forms.

3. (a) Find the general solution to the differential equation $\frac{dy}{dt} = 2ty^2$.

   (b) Find the specific solution that satisfies the initial condition $y(0) = -1$. Determine the interval where the solution is defined.

   (c) Find the specific solution that satisfies the initial condition $y(0) = 1$. Determine the interval where the solution is defined.

Section 2.3 (Modeling with First-order ODEs)

4. A population of bacteria grows at a rate proportional to its current size. Initially, there are 100 bacteria, and after 2 hours, the population has grown to 500 bacteria.

   (a) Write the differential equation that models the population growth.

   (b) Solve the differential equation to find the population as a function of time.

   (c) Determine the population after 5 hours.

   (d) How long will it take for the population to reach 10,000 bacteria?

5. In deep water, the intensity of light is given by a function, $I = I(x)$, where I is measured in units like W/m^2, and x is the depth in meters. The differential equation describing light intensity is $I' = -kI$, where k depends on properties like water clarity or the presence of particles. If $k = 0.8$ per meter and the intensity of light at the surface of a lake is 1000 W/m^2

   (a) Solve the differential equation that describes the light intensity at depth x.

   (b) Determine the light intensity at a depth of 10 meters below the water surface.

   (c) At what depth is the light intensity 250 W/m^2?

6. A tank initially contains 100 liters of pure water. A brine solution with a concentration of 0.5 kg/L of salt flows into the tank at a rate of 2 L/min. The well-stirred mixture flows out of the tank at the same rate. Find the amount of salt in the tank at any time t.

7. A tank initially contains 200 liters of water with 10 kg of salt dissolved in it. A brine solution with a concentration of 0.2 kg/L flows into the tank at a rate of 3 L/min. The well-stirred mixture flows out at a rate of 2 L/min. Find the amount of salt in the tank at any time t.

8. A car engine is turned off after running at 90°. The ambient temperature is 15°. After 15 minutes, the engine cools to 60°. What is the temperature of the engine after 45 minutes?