2024 Fall Math 140 Week-In-Review

Week 6: Sections 3.4 and 4.1

Sections 3.4: The Simplex

Some Key Words and Terms: Simplex Method, Standard Maximization Problem, Slack Variables, Initial Tableau, Pivot Column/Row/Element, Basic and Non-Basic Variables, Corner Point, Final Tableau, Solution, Leftovers

Simplex Method:

Standard Maximization Problem:

Slack Variables:

Initial Tableau:

Pivot Column/Row/Element:

Basic and Non-Basic Variables:

Corner Point:

Final Tableau:

Solution:

Leftovers:

Examples:

1. Determine if the following Linear Programming Problems are Standard Maximization Problems.

(a) Objective: **Maximize** $A = 5x + 4y$

Subject to:
$$-2x + 2y \leq 4$$
$$3x + 2y \leq 12$$
$$2 \leq y \leq 5$$
$$x \geq 0$$

(b) Objective: **Minimize** $Z = 12x + 15y + 10z$

Subject to:
$$x + y + z \leq 10$$
$$-5x + 2y + 2z \leq 14$$
$$3y + 6z \leq 24$$
$$x \geq 0,\ y \geq 0,\ z \geq 0$$

(c) Objective: **Maximize** $Z = 12x + 15y + 10z$

Subject to:
$$x + y + z \leq 10$$
$$5x - 2y - 2z \geq -14$$
$$3y + 6z \leq 24$$
$$x \geq 0,\ y \geq 0,\ z \geq 0$$

(d) Objective: **Maximize** $P = 0.12x + 0.05y + 0.18z$

Subject to:
$$-2x + 2y + z \leq 0$$
$$x - 5 \leq y + z$$
$$3z - 5 \geq x + z$$
$$x \geq 0,\ y \geq 0,\ z \geq 0$$

2. Convert the following Standard Maximization Problems into an Initial Tableau.

(a) Objective: **Maximize** $Z = 12x + 15y + 10z$

Subject to:
$$x + y + z \leq 10$$
$$-5x + 2y + 2z \leq 14$$
$$3y + 6z \leq 24$$
$$x \geq 0,\ y \geq 0,\ z \geq 0$$

(b) Objective: **Maximize** $P = 0.15x + 0.09y$

Subject to:
$$-x + 2y \leq 0$$
$$3x + 2y \leq 20$$
$$5x + 3y \leq 30$$
$$-x \geq -12$$
$$x \geq 0,\ y \geq 0$$

3. For each of the following: classify each variable as basic or non-basic, determine the corner point, pivot column/row/element, and pivot on the selected element.

(a)

x	y	s_1	s_2	s_3	R	C
3	5	1	0	0	0	390
6	3	0	1	0	0	450
4	4	0	0	1	0	360
−115	−75	0	0	0	1	0

(b)

x	y	s_1	s_2	s_3	R	C
0	7/2	1	−1/2	0	0	165
1	1/2	0	1/6	0	0	75
0	2	0	−2/3	1	0	60
0	−35/2	0	115/6	0	1	8625

(c)

x	y	s_1	s_2	s_3	P	C
−2/5	0	1	4/5	0	0	66
−3/5	1	0	1/5	0	0	10
8/5	0	0	−1/5	1	0	26
−11/2	0	0	1	0	1	50

4. The following Standard Maximization Problem represents a shoe company making two types of shoes: a running shoe and a walking shoe. Let x represent the number of running shoes produced, y represent the number of walking shoes produced, and R represent the weekly revenue made from selling the shoes. The first constraint represents the number of units of leather used in the production of the shoes for a given week, the second constraint represents the number of units of cloth used in the production of the shoes for a given week, and the third constraint represents the number of units of rubber used in the production of the shoes week. Express the solution of the Standard Maximization Problem in the context of this scenario, including discussing any weekly leftovers the company has.

Maximize: $R = 115x + 75y$

Subject to: $3x + 5y \leq 390$

$6x + 3y \leq 450$

$4x + 4y \leq 360$

$x \geq 0,\ y \geq 0$

x	y	s_1	s_2	s_3	R	C
0	0	1	2/3	−7/4	0	60
1	0	0	1/3	−1/4	0	60
0	1	0	−1/3	1/2	0	30
0	0	0	40/3	35/4	1	9150

Sections 4.1: Mathematical Experiments

Some Key Words and Terms: Sample Space, Outcomes, Event, Tree Diagram, Venn Diagram, Complement, Intersection, Union, Mutually Exclusive, Converting Between Symbolic and Verbal

Sample Space:

Outcomes:

Events:

Tree Diagram:

Venn Diagram:

Complement:

Intersection:

Union:

Mutually Exclusive:

Converting Between Symbolic and Verbal:

Examples:

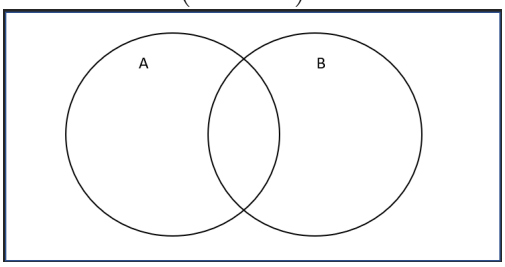
1. For the following experiments, draw a tree diagram to represent the experiment, determine the sample space, the number of simple events, and the total number of outcomes.

   (a) Flipping a fair coin, then rolling a 5 sided die.

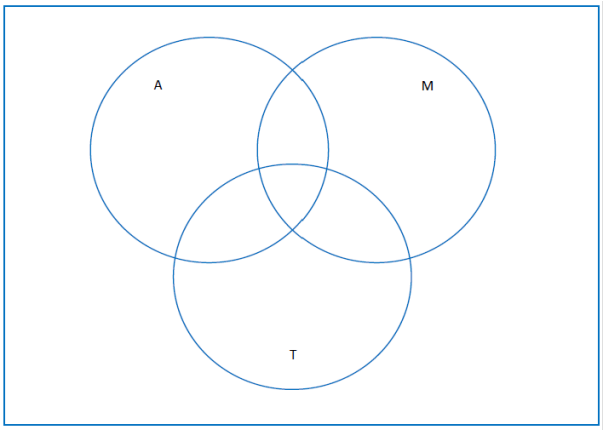
   (b) Spinning a fair spinner showing red, yellow, and blue, then rolling a 4-sided die and recording a 3 or 4 as a "high" result and a 1 or 2 as a "low" result.

2. Shade the following Venn Diagrams for the given event.

$(A^C \cup B)^C$



$(A \cap T^C) \cup M$



3. Let A be "the event that a randomly selected student likes chocolate ice cream", let B be "the event that a randomly selected student is involved in an org", and let C be "the event that a randomly selected students lives on campus". Use these definitions to answer the following.

   (a) Write the event "a randomly selected student is involved in an org or lives on campus, but only likes strawberry ice cream"

   (b) Write the event $A \cap C \cap B^C$ in words using the context given above.