STAT 201 - Week-In-Review 10

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Problem Solutions

1. Suppose scores on a Statistics exam are normally distributed with an unknown mean and a standard deviation of ten points. A random sample of 50 scores is taken and gives a sample mean of 75. Find a 95% confidence interval for the true mean of statistics exam scores.

   Solution: Let the random variable X denote the Statistics exam score of a randomly selected student. Then, according to the problem, $X \sim N(\mu, \sigma)$, with an unknown μ and $\sigma = 10$. Here, we have a simple random sample of size n = 50 drawn from the population distribution of X.

   Need to construct a 95% CI for $\mu = E(X)$, the mean Statistics exam scores.

   A $100(1-\alpha)\%$ CI for μ is given by

   $$\bar{X} \pm \mathrm{MoE}\text{ , where } \mathrm{MoE} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

   $z_{\alpha/2}$ being the $100(1-\frac{\alpha}{2})-$th percentile of the $\mathrm{N}(0,1)$ distribution.

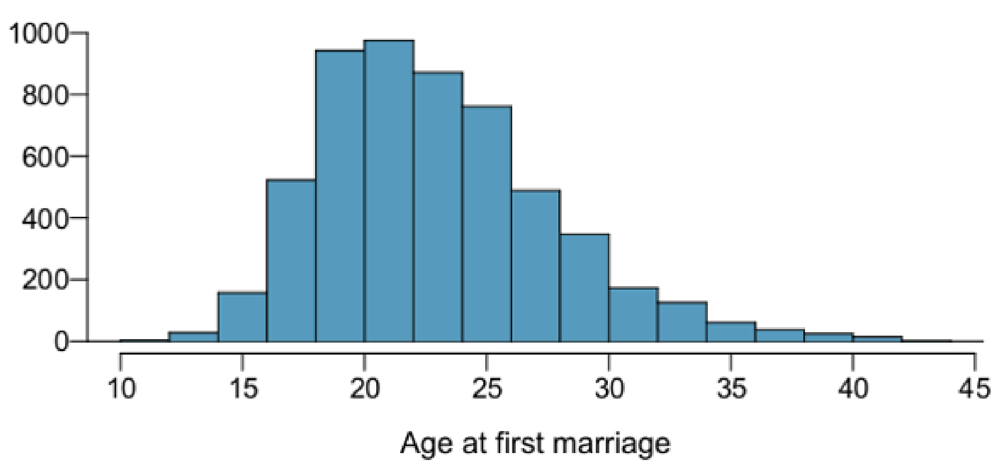
   For a 95% CI, $100(1-\alpha) = 95 \Rightarrow \alpha = 0.05$, and $z_{\frac{\alpha}{2}} = z_{0.025} \approx 1.95996$. Hence,

   $$\Rightarrow \mathrm{MoE} = z_{0.025}\frac{\sigma}{\sqrt{n}} = 1.95996 \times \frac{10}{\sqrt{50}} \approx 2.77.$$

   Since the observed $\bar{X} = 75$, the required estimated 95% CI for μ is given by

   $$[\bar{X} - \mathrm{MoE}\ , \bar{X} + \mathrm{MoE}\ ] \approx [72.23, 77.77].$$

2. The National Survey of Family Growth conducted by the Centers for Disease Control gathers information on family life, marriage and divorce, pregnancy, infertility, use of contraception, and men's and women's health. One of the variables collected in this survey is the age at first marriage. The histogram shows the distribution of ages at first marriage of 5,534 randomly sampled women between 2006 and 2010. The average age at first marriage among these women is 23.44. Consider the population standard deviation of 4.72 (NSFG, 2010). (please round ages to two decimal places).



Estimate the average age at first marriage of women using a 95% confidence interval.

Solution: Let the random variable X denote the age at first marriage of a randomly selected woman drawn from the given population.

Here, the parameter of interest is μ = E(X) which denotes the average age of women in the given population at first marriage. Clearly, μ is unknown. It's also given that σ = SD(X) = 4.72 years.

The histogram above strongly indicates that the population distribution is not normal, and it is heavily skewed to the right. However, since the sample size n = 5534 is sufficiently large, one can construct a 100(1 $-$ α)% confidence interval for μ given by

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}},$$

where $z_{\alpha/2}$ denotes the 100(1 $-$ $\alpha/2$)-th percentile of the N(0, 1) distribution.

Now, for a 95% confidence interval, 100(1 $-$ α) = 95, whence α = 0.05. Therefore, the z-critical constant here is given by

$$z_{\alpha/2} = z_{0.025} = 97.5\text{-th percentile of } N(0,1) = 1.95996.$$

Hence,

$$\mathrm{MoE} = z_{0.025}\frac{\sigma}{\sqrt{n}} = 1.95996\frac{4.72}{\sqrt{5534}} \approx 0.12435.$$

Since, the observed $\bar{X}$ = 23.44, an estimated 95% confidence interval for μ is given by

$$[\bar{X} - \mathrm{MoE}, \bar{X} + \mathrm{MoE}] \approx [23.32, 23.56].$$

3. Suppose a simple random sample of size n = 80 is drawn from a normal population with an unknown mean μ and a known standard deviation σ. Let [2.19, 3.67] be an estimated 95% confidence interval for μ based on the observed sample.

(A) Find the sample mean.

Solution: Since the underlying population distribution is normal with a known σ, a 100(1 $-$ α)% CI for μ is given by $\bar{X} \pm \mathrm{MoE}$, where $\mathrm{MoE} = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$.

The theoretical length of the CI is $2\mathrm{MoE}$ and its midpoint is $\bar{X}$.

Since, the estimated 95% confidence interval is given to be [2.19, 3.67], its midpoint is $(2.19 + 3.67)/2 = 2.93$. Hence, $\bar{X} = 2.93$.

(B) Find the corresponding margin of error.

Solution: Again, the length of the given CI is 3.67 $-$ 2.19 = 1.48. Hence,

$$2 \times \mathrm{MoE} = 1.48 \Rightarrow \mathrm{MoE} = \frac{1.48}{2} = 0.74.$$

(C) Suppose we wish to construct a 98% confidence interval based on a new sample while keeping the margin of error unaltered. Should we increase, or decrease the sample size?

Solution: If we wish to construct a 98% CI for μ, we are aiming at a higher level of confidence or, coverage probability. This means that we have to pay an additional price through a larger value of the error of estimation or the MoE. Thus, the MoE would tend to increase if we wish to ensure a higher coverage probability.

However, according to our present requirement, the MoE must remain unaltered. Hence, we must pay that additional price through the collection of a larger sample. Therefore, we should increase the sample size n in this context.

(D) Now suppose we wish to construct a new confidence interval based on a new sample of size n = 30 while keeping the margin of error unaltered. Should we increase, or decrease the confidence level?

Solution: If we wish to construct a new CI for μ with a smaller sample size of n = 30, we lose information about μ. Consequently, we need to pay an additional price through an increased value of the error of estimation or, the MoE. Therefore, the MoE would tend to increase if we use a smaller sample size.

However, according to our present requirement, the MoE must remain unaltered. Hence, we must pay that additional price by sacrificing some level of confidence. So, to keep the MoE unaltered, we should decrease the level of confidence in this context.

4. Peggy is interested in the mean height μ of young American women aged between 18 to 24 years in the US, and wishes to construct a 95% CI for μ. Assume that the corresponding population standard deviation σ is known to be 2.5 inches.

What would be the minimum sample size she needs to take so as to ensure that the corresponding error bound does not exceed 0.58 inches?

Solution: We have a simple random sample of size n drawn from a continuous population distribution with an unknown mean μ, and a known standard deviation of σ = 2.5.

Assume that n is sufficiently large (n $\geq$ 30) so that a 100(1 $-$ α)% (approximate) CI for μ is given by

$$\bar{X} \pm \mathrm{MoE} \text{ , where } \mathrm{MoE} = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}},$$

$z_{\alpha/2}$ being the 100(1 $-$ $\frac{\alpha}{2}$)$-$th percentile of the $\mathrm{N}(0, 1)$ distribution.

According to the problem, n should be so large that

$$\mathrm{MoE} = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \leq 0.58, \text{ that is } \sqrt{n} \geq z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{0.58},$$

with α = 0.05. Since $z_{\frac{\alpha}{2}} = z_{0.025} = 1.95996$, and $\sigma = 2.5$, we obtain

$$\Rightarrow \sqrt{n} \geq 1.95996\left(\frac{2.5}{0.58}\right) \Rightarrow n \geq \left[1.95996\left(\frac{2.5}{0.58}\right)\right]^2 \approx 71.3705.$$

Since n is a positive integer, the required minimum value of n must be rounded up to 72.

5. Which of the following are the correct interpretations of a 95% confidence interval? Select all that apply.

   (a) The interval estimator contains the unknown true parameter value with probability 0.95 (before sampling).

   (b) The interval estimator fails to contain the unknown true parameter value with probability 0.05 (before sampling).

   (c) We are 95% confident that the calculated CI based on the present sample would contain the unknown true parameter value.

   (d) If repeated random samples of a fixed size are drawn from the given population distribution under identical conditions a large number of times, **approximately** 95% of the estimated CIs contain the unknown true parameter value.

   (e) If repeated random samples of a fixed size are drawn from the given population distribution under identical conditions a large number of times, **approximately** 5% of the estimated CIs **fail** to contain the unknown true parameter value.

   (f) A future random sample of the same size drawn under identical conditions from the given population distribution will result in a CI that will contain the unknown true parameter value with probability 0.95.

6. Dewey, Lie, and Howei is a regional branch office of the tax preparation conglomerate Hookem, Billem, Cheatem, and Soakemi. One week before the deadline for filing federal income tax returns, the systolic blood pressure of a random sample of 45 employees at Dewey, Lie, and Howe is measured. Based on the observations in this sample, a 95% confidence interval for the mean systolic blood pressure μ of all employees is calculated to be (142, 165).

   Which of the following statements gives a statistically correct, acceptable interpretation of this interval?

   (a) We are 95% confident that the interval (142, 165) contains the population mean systolic blood pressure μ of all employees.

   (b) 95% of the sample of employees have a systolic blood pressure between 142 and 165.

   (c) 95% of the population of employees have a systolic blood pressure between 142 and 165.

   (d) The probability that the population mean systolic blood pressure μ is between 142 and 165 is 0.95.

   (e) If the sampling procedure were repeated many times and a 95% confidence interval calculated from the observations in each sample, 95% of the resulting confidence intervals would contain the population mean systolic blood pressure μ; the interval (142, 165) is possibly one such interval.

(f) We are 95% confident that the population mean systolic blood pressure μ of all employees is in the interval (142, 165).

(g) If the sampling procedure were repeated many times and the sample mean systolic blood pressure from the observations in each sample were calculated, then 95% of the sample means would be between 142 and 165.

(h) We are 95% confident that the systolic blood pressures in this sample have a sample mean that is between 142 and 165.

(i) On 95% of the days the population mean systolic blood pressure μ is between 142 and 165.

7. The 95% confidence interval for the average lifetime μ of light bulbs is (166, 198) hours. Which of the following is the correct interpretation of the above confidence interval for μ?

(a) 95% of all light bulbs will last between 166 hours and 198 hours.

(b) The true average is between 166 and 198 about 95% of the time.

(c) The true average is between 166 and 198 hours with 95% confidence.

(d) We are 95% confident the next light bulb will last between 166 and 198 hours.

(e) If we repeat the process with 1000 samples, about 950 of them would correctly capture the true average μ. The interval $(166, 198)$ is possibly one of them.

8. The pH of rain, measured at a weather station in Michigan, was observed for 39 consecutive rain storms. The sample mean is 4.6982 and the sample variance is 0.39623. Assume the observations to be independent and the population distribution to be approximately normal.

Based on the above information, obtain a 99% confidence interval for the mean pH μ of the population of storms at that location.

Solution: Let the random variable X denote the pH of rain measured on a randomly selected day at the given weather station in Michigan. According to the problem, X $\overset{a}{\sim}$ N(μ, σ), where both μ and σ are unknown.

Parameter of interest: μ = E(X), the mean pH of rain.

Here, we have a simple random sample of size n = 39 drawn from the population distribution of X $\overset{a}{\sim}$ N(μ, σ).

Since the underlying population distribution is approximately normal with an unknown σ, a $100(1-\alpha)$% CI for μ is given by

$$\bar{X} \pm \text{MoE} \text{ , where MoE} = t_{\alpha/2;n-1} \cdot \frac{S}{\sqrt{n}},$$

$t_{\alpha/2;n-1}$ being the $100(1-\frac{\alpha}{2})-$th percentile of a t_{n-1} distribution.

Since, n = 39 and α = 0.01, using the attached Student's t-distribution table, we obtain $t_{\alpha/2;n-1} = t_{0.005;38} = 2.7115$. Also, $S = \sqrt{0.39623}$. Hence,

$$\mathrm{MoE} = t_{\alpha/2;n-1} \cdot \frac{S}{\sqrt{n}} = 2.7115 \times \frac{\sqrt{0.39623}}{\sqrt{39}} \approx 0.2733.$$

Since the observed $\bar{X} = 4.6982$, the required estimated 99% CI for μ is given by

$$[\bar{X} - \mathrm{MoE}, \bar{X} + \mathrm{MoE}] \approx [4.4249, 4.9715].$$

9. A hospital is trying to cut down on emergency room wait times. It is interested in the amount of time patients must wait before being called back to be examined. An investigation committee randomly surveyed 70 patients. The sample mean was 1.5 hours with a sample standard deviation of 0.5 hours.

   Construct a 95% confidence interval for the population mean time spent waiting.

   Solution: Let the random variable X denote the waiting time of a randomly selected patient in the emergency room. Then, the parameter of interest is $\mu = E(X)$, the mean wait time of patients in the emergency room.

   Here, we have a simple random sample of size n = 70 drawn from the population distribution of X, which is continuous with an unknown population mean μ and an unknown population standard deviation σ.

   Although, the underlying population distribution is not necessarily normal with an unknown σ, the sample size n = 70 is sufficiently large (> 30).
   Hence, a $100(1-\alpha)\%$ (approximate) CI for μ is given by

$$\bar{X} \pm \mathrm{MoE} \text{ , where } \mathrm{MoE} = t_{\alpha/2;n-1} \cdot \frac{S}{\sqrt{n}},$$

   $t_{\alpha/2;n-1}$ being the $100(1-\frac{\alpha}{2})-$th percentile of a t_{n-1} distribution.

   Since, n = 70 and α = 0.05, using the attached Student's t-distribution table, we obtain

$$t_{\alpha/2;n-1} = t_{0.025;69} = 1.9950.$$

   Also, S = 0.5. Hence,

$$\mathrm{MoE} = t_{\alpha/2;n-1} \cdot \frac{S}{\sqrt{n}} = 1.9950 \times \frac{0.5}{\sqrt{70}} \approx 0.1192.$$

   Since, the observed $\bar{X} = 1.5$, the required estimated 95% CI for μ is given by

$$[\bar{X} - \mathrm{MoE}, \bar{X} + \mathrm{MoE}] \approx [1.38, 1.62].$$

10. A New Research Center poll included 1500 randomly selected adults who were asked whether ”global warming is a problem that requires immediate government action”. Results showed that 850 of those surveyed indicated that immediate government action is required.

    Let p be the population proportion of adults who believe that immediate government action is required. Compute a 95% confidence interval for p.

    Solution: Observe that, the underlying variable is categorical and dichotomous in nature. Here, n = 1500 with $\hat{p} = 850/1500$ satisfying

    $$n\hat{p} = 850 > 10, \text{ and } n(1-\hat{p}) = 650 > 10.$$

    Therefore, a $100(1-\alpha)\%$ (approximate) CI for p is

    $$[\hat{p} - \mathrm{MoE}, \hat{p} + \mathrm{MoE}], \text{ where } \mathrm{MoE} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

    $z_{\alpha/2}$ being the $100(1-\frac{\alpha}{2})-$th percentile of the $\mathrm{N}(0,1)$ distribution.

    For a 95% CI, $100(1-\alpha) = 95 \Rightarrow \alpha = 0.05 \Rightarrow z_{\frac{\alpha}{2}} = z_{0.025} = 1.95996$. Hence,

    $$\mathrm{MoE} = z_{0.025} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.95996 \times \sqrt{\frac{\frac{850}{1500}(1-\frac{850}{1500})}{1500}} \approx 0.02508.$$

    Since the observed $\hat{p} = \frac{850}{1500}$, the required estimated 95% CI for p is given by

    $$[\hat{p} - \mathrm{MoE}, \hat{p} + \mathrm{MoE}] \approx [0.5416, 0.5918].$$

11. The university is interested to know whether the students support sport passes to be included in their tuition fees. 250 students are sampled to estimate the proportion of students who support sports passes being included in tuition. Of them, 133 support it and 117 oppose.

    Let p be the population proportion of students who support sport passes to be included in the tuition fees. Compute a 90% confidence interval for p.

    Solution: Observe that, the underlying variable is categorical, and dichotomous.

    Here, n = 250 and $\hat{p} = 133/250 = 0.532$ satisfying

    $$n\hat{p} = 133 > 10, \text{ and } n(1-\hat{p}) = 117 > 10.$$

    Therefore, a $100 \times (1-\alpha)\%$ (approximate) CI for p is

    $$\left[\hat{p} - \mathrm{MoE}, \hat{p} + \mathrm{MoE}\right], \text{ where } \mathrm{MoE} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

    $z_{\alpha/2}$ being the $100(1-\frac{\alpha}{2})-$th percentile of the $\mathrm{N}(0,1)$ distribution.

For 90% CI, $100(1-\alpha) = 90 \Rightarrow \alpha = 0.10$, and $z_{\frac{\alpha}{2}} = z_{0.05} = 1.64485$. Hence

$$\mathrm{MoE} = z_{0.05} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.64485\sqrt{\frac{0.532(1-0.532)}{250}} \approx 0.0519.$$

Since the observed $\hat{p} = 0.532$, the required estimated 90% CI for p is given by

$$[\hat{p} - \mathrm{MoE}, \hat{p} + \mathrm{MoE}] \approx [0.4801, 0.5839].$$

12. The 90% confidence interval for the proportion of patients who experience side effects from the medication Obecalp is $(22.7\%, 27.5\%)$. Which of the following statements is a valid statement about this confidence interval?

    (a) We are 90% confident that all patients will have side effects 22.7% to 27.5% of the time.

    (b) There is a 90% probability that the population proportion of patients who get side effects is between 22.7% and 27.5%.

    (c) The population proportion of patients who will experience side effects is between 22.7% and 27.5% with 90% confidence.

    (d) If we replicate the same experiment under identical conditions based on 1000 repeated samples each having the same size, then in about 900 of the cases, the estimated confidence intervals will cover the population proportion of patients who will experience side effects. The interval $(22.7\%, 27.5\%)$ is possibly one of them.

    (e) Between 22.7% to 27.5% of patients will have side effects 90% of the time.

    (f) The population proportion of patients who experience side effects is between 22.7% and 27.5%.

13. A new method of pre-coating fittings used in oil, brake and other fluid systems in heavy-duty trucks is being studied. What should the minimum sample size n be to estimate the proportion of fittings p that leak to within 0.02 (length = 0.04) with 90% confidence? (No prior information about p is available).

    Solution: Assume that the sample size n is sufficiently large so that we can construct a $100 \times (1-\alpha)\%$ (approximate) CI for p is

$$\left[\hat{p} - \mathrm{MoE}, \hat{p} + \mathrm{MoE}\right], \text{ where } \mathrm{MoE} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

$z_{\alpha/2}$ being the $100(1-\frac{\alpha}{2})$−th percentile of the $\mathrm{N}(0,1)$ distribution.

In this problem, $\alpha = 0.10 \Rightarrow z_{\alpha/2} = z_{0.05} = \text{the 90-th percentile of N(0,1)} = 1.64485$.

It's important to observe that we do not have any preliminary information about p (other than $0 < p < 1$), nor did we have observe the sample yet. Therefore, using the fact $u(1-u) \leq \frac{1}{4}$ for all $0 \leq u \leq 1$, the maximum error of estimation for all possible samples of size n, can be bounded above as:

$$\mathrm{MoE} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{1}{4n}}.$$

According to the problem, this largest possible value of the MoE can at most be 0.02, that is,

$$z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{1}{4n}} \leq 0.02 \Rightarrow n \geq \frac{z^2_{\alpha/2}}{4(0.02)^2} = \frac{z^2_{0.05}}{4(0.02)^2} = \frac{(1.64485)^2}{4(0.02)^2} = 1690.9572.$$

This must be rounded up to the next positive integer 1691.

Hence, the require minimum sample size n must be 1691.

Important Note: If you report the minimum sample size n as 1690, that would be an incorrect response since n = 1690 violates the condition

$$z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{1}{4n}} \leq 0.02.$$

Statistics - T-Distribution Table

The critical values of t distribution are calculated according to the probabilities of two alpha values and the degrees of freedom. The Alpha (a) values 0.05 one tailed and 0.1 two tailed are the two columns to be compared with the degrees of freedom in the row of the table.

One Tail	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
Two Tails	0.1	0.05	0.02	0.01	0.005	0.002	0.001
df							
1	6.3138	12.7065	31.8193	63.6551	127.3447	318.4930	636.0450
2	2.9200	4.3026	6.9646	9.9247	14.0887	22.3276	31.5989
3	2.3534	3.1824	4.5407	5.8408	7.4534	10.2145	12.9242
4	2.1319	2.7764	3.7470	4.6041	5.5976	7.1732	8.6103
5	2.0150	2.5706	3.3650	4.0322	4.7734	5.8934	6.8688
6	1.9432	2.4469	3.1426	3.7074	4.3168	5.2076	5.9589
7	1.8946	2.3646	2.9980	3.4995	4.0294	4.7852	5.4079
8	1.8595	2.3060	2.8965	3.3554	3.8325	4.5008	5.0414
9	1.8331	2.2621	2.8214	3.2498	3.6896	4.2969	4.7809
10	1.8124	2.2282	2.7638	3.1693	3.5814	4.1437	4.5869
11	1.7959	2.2010	2.7181	3.1058	3.4966	4.0247	4.4369
12	1.7823	2.1788	2.6810	3.0545	3.4284	3.9296	4.3178
13	1.7709	2.1604	2.6503	3.0123	3.3725	3.8520	4.2208
14	1.7613	2.1448	2.6245	2.9768	3.3257	3.7874	4.1404
15	1.7530	2.1314	2.6025	2.9467	3.2860	3.7328	4.0728
16	1.7459	2.1199	2.5835	2.9208	3.2520	3.6861	4.0150
17	1.7396	2.1098	2.5669	2.8983	3.2224	3.6458	3.9651
18	1.7341	2.1009	2.5524	2.8784	3.1966	3.6105	3.9216
19	1.7291	2.0930	2.5395	2.8609	3.1737	3.5794	3.8834
20	1.7247	2.0860	2.5280	2.8454	3.1534	3.5518	3.8495
21	1.7207	2.0796	2.5176	2.8314	3.1352	3.5272	3.8193
22	1.7172	2.0739	2.5083	2.8188	3.1188	3.5050	3.7921
23	1.7139	2.0686	2.4998	2.8073	3.1040	3.4850	3.7676
24	1.7109	2.0639	2.4922	2.7970	3.0905	3.4668	3.7454
25	1.7081	2.0596	2.4851	2.7874	3.0782	3.4502	3.7251
26	1.7056	2.0555	2.4786	2.7787	3.0669	3.4350	3.7067
27	1.7033	2.0518	2.4727	2.7707	3.0565	3.4211	3.68[illegible]
28	1.7011	2.0484	2.4671	2.7633	3.0469	3.4082	3.6739

29	1.6991	2.0452	2.4620	2.7564	3.0380	3.3962	3.6594
30	1.6973	2.0423	2.4572	2.7500	3.0298	3.3852	3.6459
31	1.6955	2.0395	2.4528	2.7440	3.0221	3.3749	3.6334
32	1.6939	2.0369	2.4487	2.7385	3.0150	3.3653	3.6218
33	1.6924	2.0345	2.4448	2.7333	3.0082	3.3563	3.6109
34	1.6909	2.0322	2.4411	2.7284	3.0019	3.3479	3.6008
35	1.6896	2.0301	2.4377	2.7238	2.9961	3.3400	3.5912
36	1.6883	2.0281	2.4345	2.7195	2.9905	3.3326	3.5822
37	1.6871	2.0262	2.4315	2.7154	2.9853	3.3256	3.5737
38	1.6859	2.0244	2.4286	2.7115	2.9803	3.3190	3.5657
39	1.6849	2.0227	2.4258	2.7079	2.9756	3.3128	3.5581
40	1.6839	2.0211	2.4233	2.7045	2.9712	3.3069	3.5510
41	1.6829	2.0196	2.4208	2.7012	2.9670	3.3013	3.5442
42	1.6820	2.0181	2.4185	2.6981	2.9630	3.2959	3.5378
43	1.6811	2.0167	2.4162	2.6951	2.9591	3.2909	3.5316
44	1.6802	2.0154	2.4142	2.6923	2.9555	3.2861	3.5258
45	1.6794	2.0141	2.4121	2.6896	2.9521	3.2815	3.5202
46	1.6787	2.0129	2.4102	2.6870	2.9488	3.2771	3.5149
47	1.6779	2.0117	2.4083	2.6846	2.9456	3.2729	3.5099
48	1.6772	2.0106	2.4066	2.6822	2.9426	3.2689	3.5051
49	1.6766	2.0096	2.4049	2.6800	2.9397	3.2651	3.5004
50	1.6759	2.0086	2.4033	2.6778	2.9370	3.2614	3.4960
51	1.6753	2.0076	2.4017	2.6757	2.9343	3.2579	3.4917
52	1.6747	2.0066	2.4002	2.6737	2.9318	3.2545	3.4877
53	1.6741	2.0057	2.3988	2.6718	2.9293	3.2513	3.4838
54	1.6736	2.0049	2.3974	2.6700	2.9270	3.2482	3.4800
55	1.6730	2.0041	2.3961	2.6682	2.9247	3.2451	3.4764
56	1.6725	2.0032	2.3948	2.6665	2.9225	3.2423	3.4730
57	1.6720	2.0025	2.3936	2.6649	2.9204	3.2394	3.4696
58	1.6715	2.0017	2.3924	2.6633	2.9184	3.2368	3.4663
59	1.6711	2.0010	2.3912	2.6618	2.9164	3.2342	3.4632
60	1.6706	2.0003	2.3901	2.6603	2.9146	3.2317	3.4602
61	1.6702	1.9996	2.3890	2.6589	2.9127	3.2293	3.4573
62	1.6698	1.9990	2.3880	2.6575	2.9110	3.2269	3.4545
63	1.6694	1.9983	2.3870	2.6561	2.9092	3.2247	3.4518
64	1.6690	1.9977	2.3860	2.6549	2.9076	3.2225	3.4491
65	1.6686	1.9971	2.3851	2.6536	2.9060	3.2204	3.4466
66	1.6683	1.9966	2.3842	2.6524	2.9045	3.2184	3.4441
67	1.6679	1.9960	2.3833	2.6512	2.9030	3.2164	3.4417
68	1.6676	1.9955	2.3824	2.6501	2.9015	3.2144	3.4395
69	1.6673	1.9950	2.3816	2.6490	2.9001	3.2126	3.4372
70	1.6669	1.9944	2.3808	2.6479	2.8987	3.2108	3.4350

71	1.6666	1.9939	2.3800	2.6468	2.8974	3.2090	3.4329
72	1.6663	1.9935	2.3793	2.6459	2.8961	3.2073	3.4308
73	1.6660	1.9930	2.3785	2.6449	2.8948	3.2056	3.4288
74	1.6657	1.9925	2.3778	2.6439	2.8936	3.2040	3.4269
75	1.6654	1.9921	2.3771	2.6430	2.8925	3.2025	3.4250
76	1.6652	1.9917	2.3764	2.6421	2.8913	3.2010	3.4232
77	1.6649	1.9913	2.3758	2.6412	2.8902	3.1995	3.4214
78	1.6646	1.9909	2.3751	2.6404	2.8891	3.1980	3.4197
79	1.6644	1.9904	2.3745	2.6395	2.8880	3.1966	3.4180
80	1.6641	1.9901	2.3739	2.6387	2.8870	3.1953	3.4164
81	1.6639	1.9897	2.3733	2.6379	2.8859	3.1939	3.4147
82	1.6636	1.9893	2.3727	2.6371	2.8850	3.1926	3.4132
83	1.6634	1.9889	2.3721	2.6364	2.8840	3.1913	3.4117
84	1.6632	1.9886	2.3716	2.6356	2.8831	3.1901	3.4101
85	1.6630	1.9883	2.3710	2.6349	2.8821	3.1889	3.4087
86	1.6628	1.9879	2.3705	2.6342	2.8813	3.1877	3.4073
87	1.6626	1.9876	2.3700	2.6335	2.8804	3.1866	3.4059
88	1.6623	1.9873	2.3695	2.6328	2.8795	3.1854	3.4046
89	1.6622	1.9870	2.3690	2.6322	2.8787	3.1844	3.4032
90	1.6620	1.9867	2.3685	2.6316	2.8779	3.1833	3.4020
91	1.6618	1.9864	2.3680	2.6309	2.8771	3.1822	3.4006
92	1.6616	1.9861	2.3676	2.6303	2.8763	3.1812	3.3995
93	1.6614	1.9858	2.3671	2.6297	2.8755	3.1802	3.3982
94	1.6612	1.9855	2.3667	2.6292	2.8748	3.1792	3.3970
95	1.6610	1.9852	2.3662	2.6286	2.8741	3.1782	3.3959
96	1.6609	1.9850	2.3658	2.6280	2.8734	3.1773	3.3947
97	1.6607	1.9847	2.3654	2.6275	2.8727	3.1764	3.3936
98	1.6606	1.9845	2.3650	2.6269	2.8720	3.1755	3.3926
99	1.6604	1.9842	2.3646	2.6264	2.8713	3.1746	3.3915
100	1.6602	1.9840	2.3642	2.6259	2.8706	3.1738	3.3905
101	1.6601	1.9837	2.3638	2.6254	2.8700	3.1729	3.3894
102	1.6599	1.9835	2.3635	2.6249	2.8694	3.1720	3.3885
103	1.6598	1.9833	2.3631	2.6244	2.8687	3.1712	3.3875
104	1.6596	1.9830	2.3627	2.6240	2.8682	3.1704	3.3866
105	1.6595	1.9828	2.3624	2.6235	2.8675	3.1697	3.3856
106	1.6593	1.9826	2.3620	2.6230	2.8670	3.1689	3.3847
107	1.6592	1.9824	2.3617	2.6225	2.8664	3.1681	3.3838
108	1.6591	1.9822	2.3614	2.6221	2.8658	3.1674	3.3829
109	1.6589	1.9820	2.3611	2.6217	2.8653	3.1667	3.3820
110	1.6588	1.9818	2.3607	2.6212	2.8647	3.1660	3.3812
111	1.6587	1.9816	2.3604	2.6208	2.8642	3.1653	3.3803
112	1.6586	1.9814	2.3601	2.6204	2.8637	3.1646	3.3795

113	1.6585	1.9812	2.3598	2.6200	2.8632	3.1640	3.3787
114	1.6583	1.9810	2.3595	2.6196	2.8627	3.1633	3.3779
115	1.6582	1.9808	2.3592	2.6192	2.8622	3.1626	3.3771
116	1.6581	1.9806	2.3589	2.6189	2.8617	3.1620	3.3764
117	1.6580	1.9805	2.3586	2.6185	2.8612	3.1614	3.3756
118	1.6579	1.9803	2.3583	2.6181	2.8608	3.1607	3.3749
119	1.6578	1.9801	2.3581	2.6178	2.8603	3.1601	3.3741
120	1.6577	1.9799	2.3578	2.6174	2.8599	3.1595	3.3735
121	1.6575	1.9798	2.3576	2.6171	2.8594	3.1589	3.3727
122	1.6574	1.9796	2.3573	2.6168	2.8590	3.1584	3.3721
123	1.6573	1.9794	2.3571	2.6164	2.8585	3.1578	3.3714
124	1.6572	1.9793	2.3568	2.6161	2.8582	3.1573	3.3707
125	1.6571	1.9791	2.3565	2.6158	2.8577	3.1567	3.3700
126	1.6570	1.9790	2.3563	2.6154	2.8573	3.1562	3.3694
127	1.6570	1.9788	2.3561	2.6151	2.8569	3.1556	3.3688
128	1.6568	1.9787	2.3559	2.6148	2.8565	3.1551	3.3682
129	1.6568	1.9785	2.3556	2.6145	2.8561	3.1546	3.3676
130	1.6567	1.9784	2.3554	2.6142	2.8557	3.1541	3.3669
131	1.6566	1.9782	2.3552	2.6139	2.8554	3.1536	3.3663
132	1.6565	1.9781	2.3549	2.6136	2.8550	3.1531	3.3658
133	1.6564	1.9779	2.3547	2.6133	2.8546	3.1526	3.3652
134	1.6563	1.9778	2.3545	2.6130	2.8542	3.1522	3.3646
135	1.6562	1.9777	2.3543	2.6127	2.8539	3.1517	3.3641
136	1.6561	1.9776	2.3541	2.6125	2.8536	3.1512	3.3635
137	1.6561	1.9774	2.3539	2.6122	2.8532	3.1508	3.3630
138	1.6560	1.9773	2.3537	2.6119	2.8529	3.1503	3.3624
139	1.6559	1.9772	2.3535	2.6117	2.8525	3.1499	3.3619
140	1.6558	1.9771	2.3533	2.6114	2.8522	3.1495	3.3614
141	1.6557	1.9769	2.3531	2.6112	2.8519	3.1491	3.3609
142	1.6557	1.9768	2.3529	2.6109	2.8516	3.1486	3.3604
143	1.6556	1.9767	2.3527	2.6106	2.8512	3.1482	3.3599
144	1.6555	1.9766	2.3525	2.6104	2.8510	3.1478	3.3594
145	1.6554	1.9765	2.3523	2.6102	2.8506	3.1474	3.3589
146	1.6554	1.9764	2.3522	2.6099	2.8503	3.1470	3.3584
147	1.6553	1.9762	2.3520	2.6097	2.8500	3.1466	3.3579
148	1.6552	1.9761	2.3518	2.6094	2.8497	3.1462	3.3575
149	1.6551	1.9760	2.3516	2.6092	2.8494	3.1458	3.3570
150	1.6551	1.9759	2.3515	2.6090	2.8491	3.1455	3.3565
151	1.6550	1.9758	2.3513	2.6088	2.8489	3.1451	3.3561
152	1.6549	1.9757	2.3511	2.6085	2.8486	3.1447	3.3557
153	1.6549	1.9756	2.3510	2.6083	2.8483	3.1443	3.3552
154	1.6548	1.9755	2.3508	2.6081	2.8481	3.1440	3.3548

155	1.6547	1.9754	2.3507	2.6079	2.8478	3.1436	3.3544
156	1.6547	1.9753	2.3505	2.6077	2.8475	3.1433	3.3540
157	1.6546	1.9752	2.3503	2.6075	2.8472	3.1430	3.3536
158	1.6546	1.9751	2.3502	2.6073	2.8470	3.1426	3.3531
159	1.6545	1.9750	2.3500	2.6071	2.8467	3.1423	3.3528
160	1.6544	1.9749	2.3499	2.6069	2.8465	3.1419	3.3523
161	1.6544	1.9748	2.3497	2.6067	2.8463	3.1417	3.3520
162	1.6543	1.9747	2.3496	2.6065	2.8460	3.1413	3.3516
163	1.6543	1.9746	2.3495	2.6063	2.8458	3.1410	3.3512
164	1.6542	1.9745	2.3493	2.6062	2.8455	3.1407	3.3508
165	1.6542	1.9744	2.3492	2.6060	2.8452	3.1403	3.3505
166	1.6541	1.9744	2.3490	2.6058	2.8450	3.1400	3.3501
167	1.6540	1.9743	2.3489	2.6056	2.8448	3.1398	3.3497
168	1.6540	1.9742	2.3487	2.6054	2.8446	3.1394	3.3494
169	1.6539	1.9741	2.3486	2.6052	2.8443	3.1392	3.3490
170	1.6539	1.9740	2.3485	2.6051	2.8441	3.1388	3.3487
171	1.6538	1.9739	2.3484	2.6049	2.8439	3.1386	3.3483
172	1.6537	1.9739	2.3482	2.6047	2.8437	3.1383	3.3480
173	1.6537	1.9738	2.3481	2.6046	2.8435	3.1380	3.3477
174	1.6537	1.9737	2.3480	2.6044	2.8433	3.1377	3.3473
175	1.6536	1.9736	2.3478	2.6042	2.8430	3.1375	3.3470
176	1.6536	1.9735	2.3477	2.6041	2.8429	3.1372	3.3466
177	1.6535	1.9735	2.3476	2.6039	2.8427	3.1369	3.3464
178	1.6535	1.9734	2.3475	2.6037	2.8424	3.1366	3.3460
179	1.6534	1.9733	2.3474	2.6036	2.8423	3.1364	3.3457
180	1.6534	1.9732	2.3472	2.6034	2.8420	3.1361	3.3454
181	1.6533	1.9731	2.3471	2.6033	2.8419	3.1358	3.3451
182	1.6533	1.9731	2.3470	2.6031	2.8416	3.1356	3.3448
183	1.6532	1.9730	2.3469	2.6030	2.8415	3.1354	3.3445
184	1.6532	1.9729	2.3468	2.6028	2.8413	3.1351	3.3442
185	1.6531	1.9729	2.3467	2.6027	2.8411	3.1349	3.3439
186	1.6531	1.9728	2.3466	2.6025	2.8409	3.1346	3.3436
187	1.6531	1.9727	2.3465	2.6024	2.8407	3.1344	3.3433
188	1.6530	1.9727	2.3463	2.6022	2.8406	3.1341	3.3430
189	1.6529	1.9726	2.3463	2.6021	2.8403	3.1339	3.3428
190	1.6529	1.9725	2.3461	2.6019	2.8402	3.1337	3.3425
191	1.6529	1.9725	2.3460	2.6018	2.8400	3.1334	3.3422
192	1.6528	1.9724	2.3459	2.6017	2.8398	3.1332	3.3419
193	1.6528	1.9723	2.3458	2.6015	2.8397	3.1330	3.3417
194	1.6528	1.9723	2.3457	2.6014	2.8395	3.1328	3.3414
195	1.6527	1.9722	2.3456	2.6013	2.8393	3.1326	3.3411
196	1.6527	1.9721	2.3455	2.6012	2.8392	3.1323	3.3409

197	1.6526	1.9721	2.3454	2.6010	2.8390	3.1321	3.3406
198	1.6526	1.9720	2.3453	2.6009	2.8388	3.1319	3.3403
199	1.6525	1.9720	2.3452	2.6008	2.8387	3.1317	3.3401
200	1.6525	1.9719	2.3451	2.6007	2.8385	3.1315	3.3398