Note ♯9: Sections 16.4-16.5

Problem 1. Evaluate $\oint_C y^2dx + xdy$ where C is the triangular path from $(1,1)$ to $(3,1)$ to $(2,2)$ then back to $(1,1)$.

Problem 2. Evaluate $\oint_C (\cos(e^x) + y^2)\,dx + \left(x^2 + \sqrt{y^3+y}\right)dy$, where C consists of the line segment from $(-3,0)$ to $(3,0)$ and the top half of $x^2+y^2=9$. Assume counterclockwise orientation.

Problem 3. Evaluate $\oint_C y^2dx + x^2dy$ where C is the boundary of the region bounded by the semicircle $y=\sqrt{4-x^2}$ and the x axis. Assume positive orientation.

Problem 4. Evaluate $\oint_C (y^2+x^{10})\,dx + y^7dy$, where C is the curve that encloses the region bounded by $x=0$, $y=0$, and $y=x^2-1$, traversed counterclockwise.

Problem 5. Suppose a particle travels one revolution counterclockwise around the circle $x^2+y^2=4$ under the force field $\mathbf{F}(x,y) = \langle y^3+\sin(x), e^y - x^3\rangle$. Find the work done by $\mathbf{F}$.

Problem 6. Suppose a particle travels one revolution clockwise around the unit circle under the force field $\mathbf{F}(x,y) = \langle e^x - y^3, \cos(y) + x^3\rangle$. Find the work done by $\mathbf{F}$.

Problem 7. Find the divergence and curl of $\mathbf{F}(x,y,z) = xe^{yz}\mathbf{j} + x^3z\mathbf{k}$.

Problem 8. Find the divergence and curl of $\mathbf{F}(x,y,z) = \langle xyz, 2\sin(xz), y^2z^3\rangle$.

Problem 9. Let f be a scalar field and $\mathbf{F}$ a vector field. Describe if each expression is a scalar field, vector field, or not meaningful.
(a) $\operatorname{curl} f$
(b) $\operatorname{curl}(\operatorname{grad} f)$
(c) $\operatorname{grad}(\operatorname{div}\mathbf{F})$

Problem 10. Is $\mathbf{F}(x,y,z) = \langle 3x^2y+4y^2z, x^3+8xyz, 4xy^2-e^z\rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

Problem 11. (a) Is $\mathbf{F} = \langle x, e^y\sin z, e^y\cos z\rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.
(b) Find $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{r}(t) = \langle t^4, t, 2t^2\rangle$, for $1\le t\le 2$.