MATH 308: WEEK-IN-REVIEW 6 (3.4-3.6)

1. Use the Method of Reduction of Order to find a second solution to the following differential equation

$$x^2y'' + 3xy' + y = 0, \quad x > 0, \quad y_1(x) = x^{-1}$$

2. Suppose you were to use the Method of Undetermined Coefficients to solve the following differential equations. Write out the assumed form of the particular solution, but do not carry out the calculations of the undetermined coefficients.

   (a) $y'' + 4y = x^2 - 2x + 1$

   (b) $y'' + 4y = x \sin x$

   (c) $y'' + 2y' - 3y = x^2 e^x$

3. Suppose you were to use the Method of Undetermined Coefficients to solve the following differential equations. Write out the assumed form of the particular solution, but do not carry out the calculations of the undetermined coefficients.

   (a) $y'' - 2y' + 5y = xe^x \cos(2x) + e^x \sin(2x)$

   (b) $y'' + 4y = x \sin x + \cos(2x)$

4. Solve the following equations using the Method of Undetermined Coefficients. If any initial value is given, then solve the initial value problem. If no initial value is given, then find the general solution. Find an explicit solution if possible.

   (a) $f'' - 7f' + 12f = 2e^{5t}$, $f(0) = 0$, $f'(0) = -1$.

5.

$$g'' + 2g' + 2g = 2t, \quad g(0) = 0, \quad g'(0) = 1.$$

6.

$$u'' + 2u' + u = 2e^{-t}$$

7. Solve the following equations using the Method of Variation of Parameters. If any initial value is given, then solve the initial value problem. If no initial value is given, then find the general solution. Find an explicit solution if possible.

   (a)
$$u'' + 2u' + u = 2e^{-t}$$

(b) Solve the initial value problem

$$3y'' + 4y' + y = e^{-t}\sin(t), \quad y(0) = 1, \;\; y'(0) = 0.$$

8. Given the complementary solution $y_c(t) = C_1 t + C_2 t^{-1}$, use Variation of Parameters to find a particular of the differential equation

$$t^2 y'' + ty' - 2y = t^2, \ \ t > 0.$$

9. Find two linearly independent solutions of $t^2y'' - 2y = 0$ of the form $y(t) = t^r$. Using these solutions, find the general solution of $t^2y'' - 2y = t^2$.

10. One solution of $4t^2y'' + 4ty' + (16t^2 - 1)y = 0$, $t > 0$ is $y(t) = t^{-1/2}\cos(2t)$. Find the general solution of $4t^2y'' + 4ty' + (16t^2 - 1)y = 16t^{3/2}$.