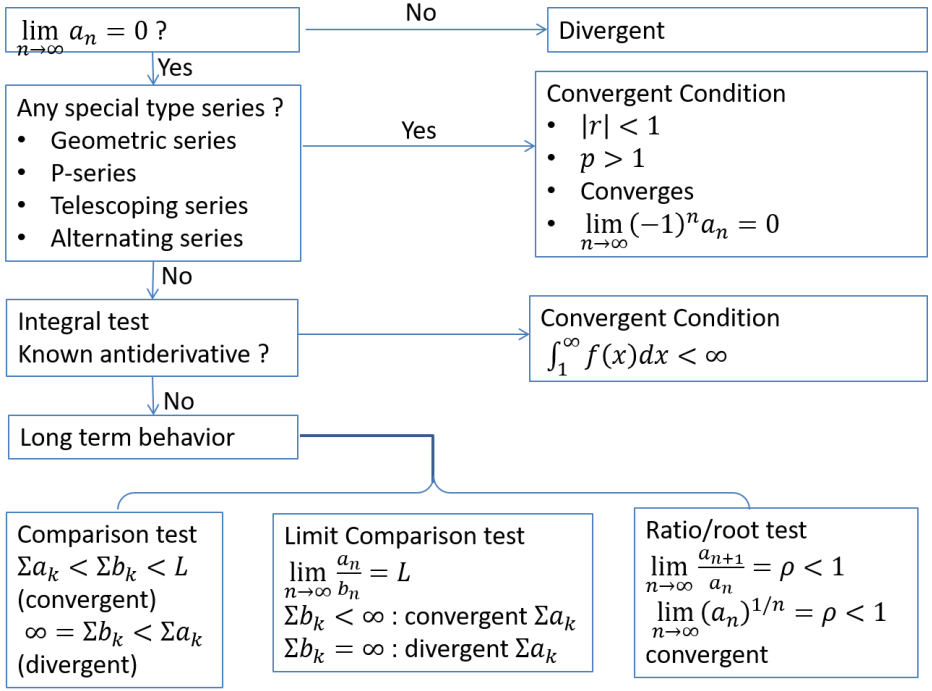
Week in Review
Math 152

Week 10

Alternating series

Absolute Convergence and the Ratio Test

Week 10



+/- terms but Not alternating
⇒ Absolute convergence

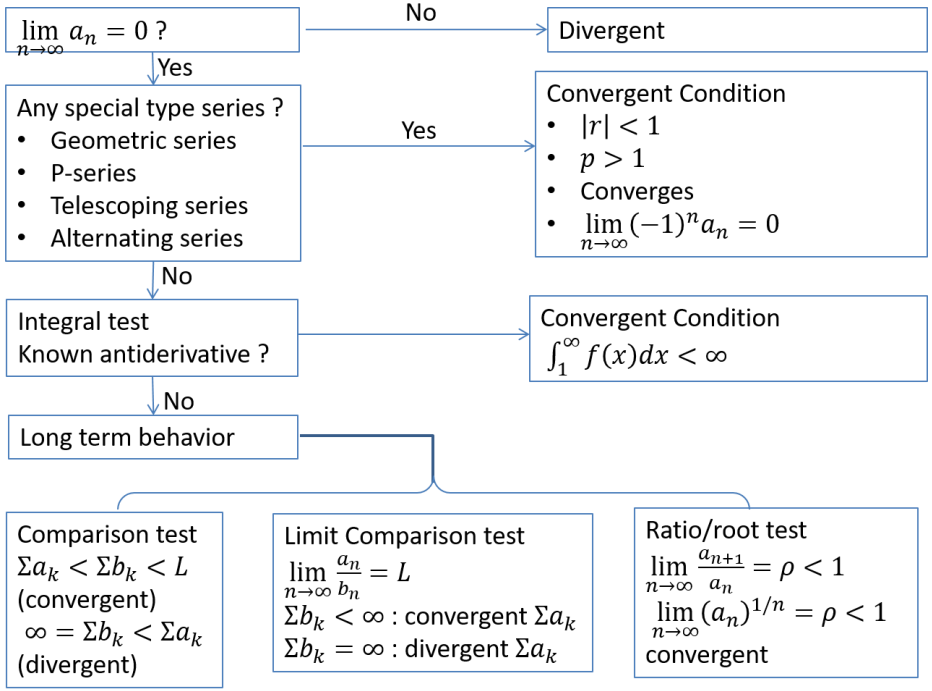
Explain why following two series do NOT converge

1. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

2. $\sum_{n=1}^{\infty} (-1)^n$

3. $\sum_{n=1}^{\infty} \frac{1}{e^{1/n}}$

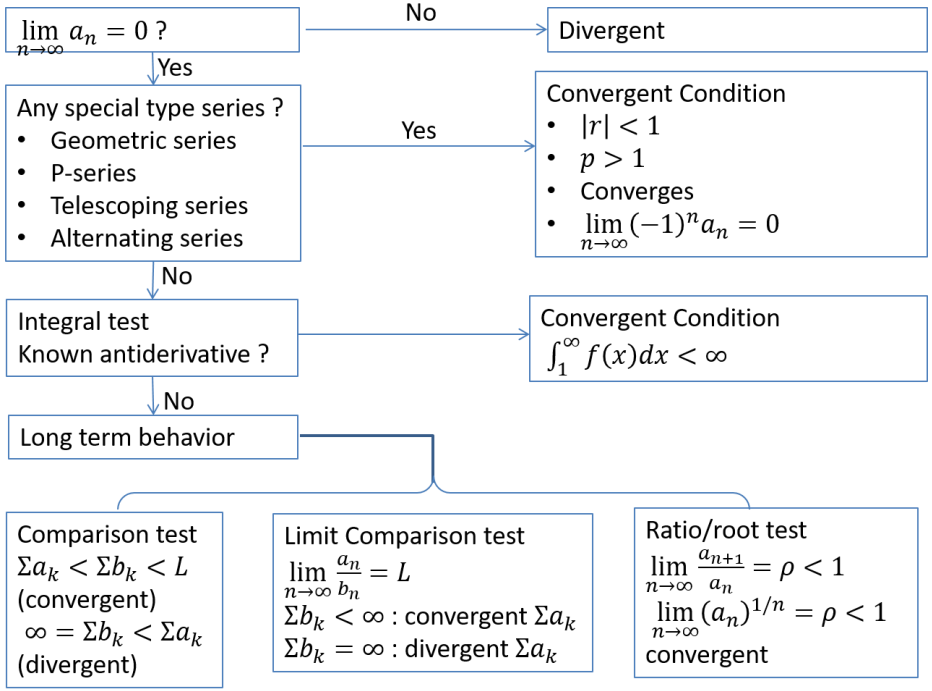
Week 10



+/- terms but Not alternating
⇒ Absolute convergence

Evaluate $\sum_{n=1}^{\infty} \frac{1}{e^n}$

Week 10



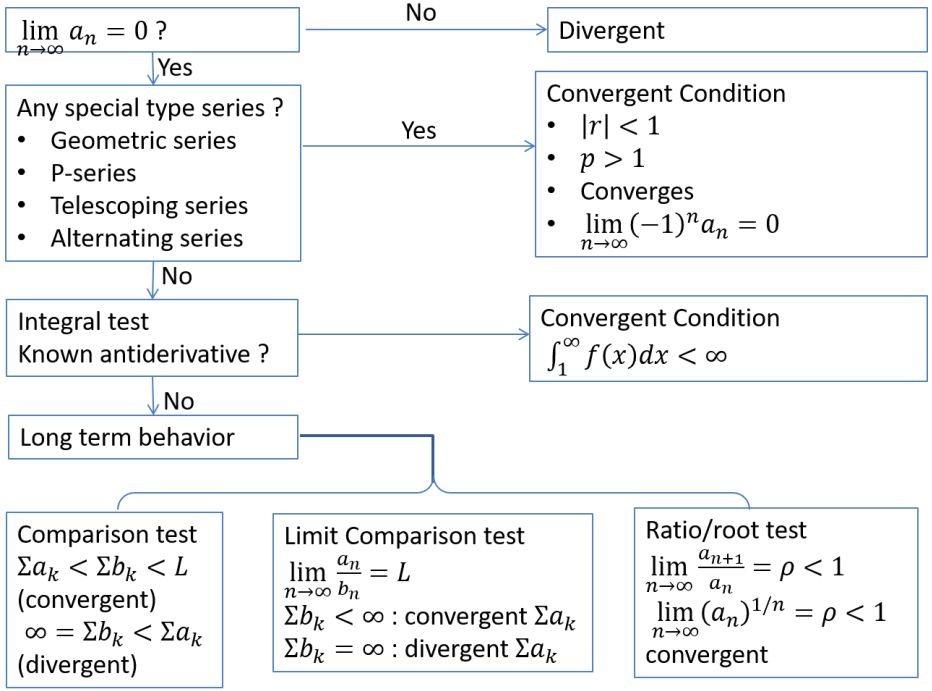
+/- terms but Not alternating
⇒ Absolute convergence

Evaluate

1. $\sum_{n=1}^{\infty} \frac{1}{e^n}$

2. $\sum_{n=1}^{\infty} \frac{5}{2^{n-1}}$

Week 10



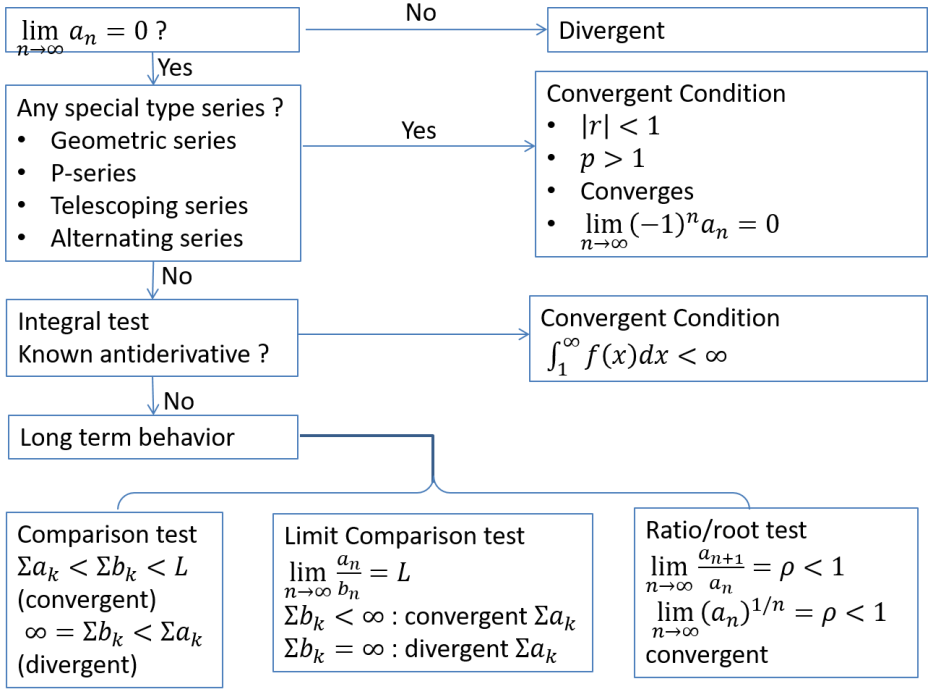
+/- terms but Not alternating
⇒ Absolute convergence

Evaluate

1. $\sum_{n=1}^{\infty} \frac{1}{n}$

2. $\sum_{n=1}^{\infty} \frac{5}{n^{1.1}}$

Week 10



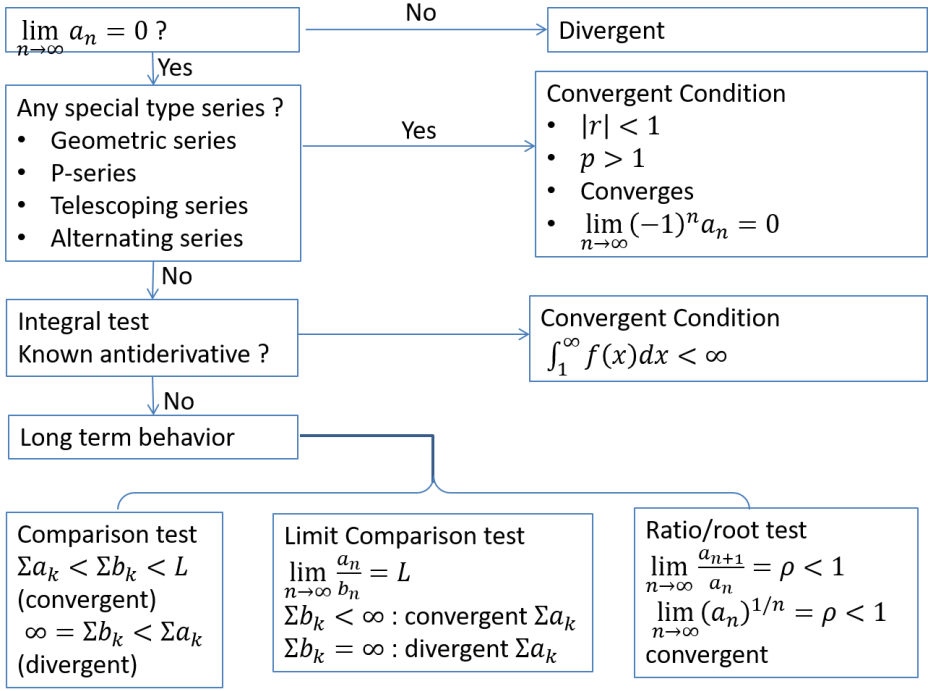
+/- terms but Not alternating
⇒ Absolute convergence

Evaluate

1. $\sum_{n=1}^{\infty} \left[\cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right)\right]$

2. $\sum_{n=1}^{\infty} \left[\ln\left(\frac{1}{n}\right) - \ln\left(\frac{1}{n+1}\right)\right]$

Week 10



+/- terms but Not alternating
⇒ Absolute convergence

Evaluate

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

Definition: Alternating series

An infinite series of the form of

(a) $\sum_{k=1}^{\infty}(-1)^k\, a_k$ (b) $\sum_{k=1}^{\infty}(-1)^{k+1}\, a_k$

where $a_k > 0$

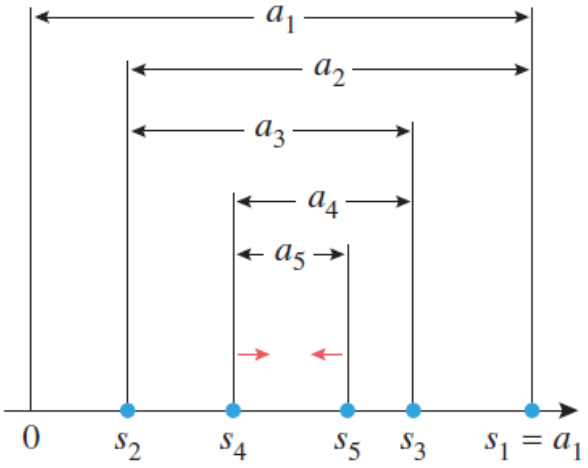
Theorem (*Alternating Series Test*): Alternating series with diminishing oscillation converges

An alternating series converges if the following two conditions are satisfied:

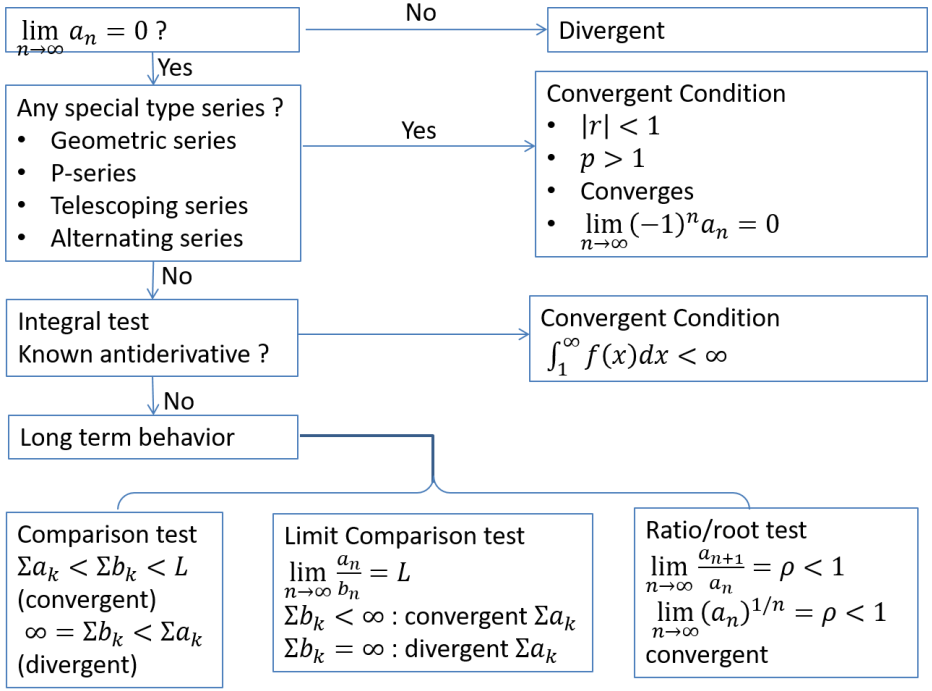
(a) $a_K > a_{K+1} > a_{K+2} > a_{K+3} > \cdots$

(b) $\lim_{k\to\infty} a_k = 0$

- $s_1, s_3, s_5, \cdots, s_{2n-1}, \cdots : \{s_{2n-1}\}$ is decreasing sequence bounded below by 0.
- $s_2, s_4, s_6, \cdots, s_{2n}, \cdots : \{s_{2n}\}$ is increasing sequence bounded above by a_1.
- Since bounded monotone sequences converge both $\{s_{2n-1}\}$ and $\{s_{2n}\}$ converge.
- $\lim_{n\to\infty}(s_{2n} - s_{2n-1}) = \lim_{n\to\infty} a_{2n} = 0$
- $\lim_{n\to\infty} s_{2n} = \lim_{n\to\infty} s_{2n-1} \Rightarrow \lim_{n\to\infty} s_n$ converges



Week 10



+/- terms but Not alternating
⇒ Absolute convergence

Week 10

Definition: Absolute convergence for general mixed sign series

- A series $\sum u_k$ (u_k be positive or negative) is said to ***converge absolutely*** if $\sum|u_k|$ converges
- A series $\sum u_k$ (u_k be positive or negative) is said to ***converge conditionally*** if $\sum u_k$ converges but $\sum|u_k|$ diverges

Absolute convergence Theorem

If $\sum|u_k|$ converges then $\sum u_k$ converges

- If $\sum u_k$ diverges, then $\sum|u_k|$ diverges
- If $\sum|u_k|$ converges then $\lim_{k\to\infty} u_k = 0$

Write $\sum_{k=1}^{\infty} u_k = \sum_{k=1}^{\infty}[(u_k + |u_k|) - |u_k|]$

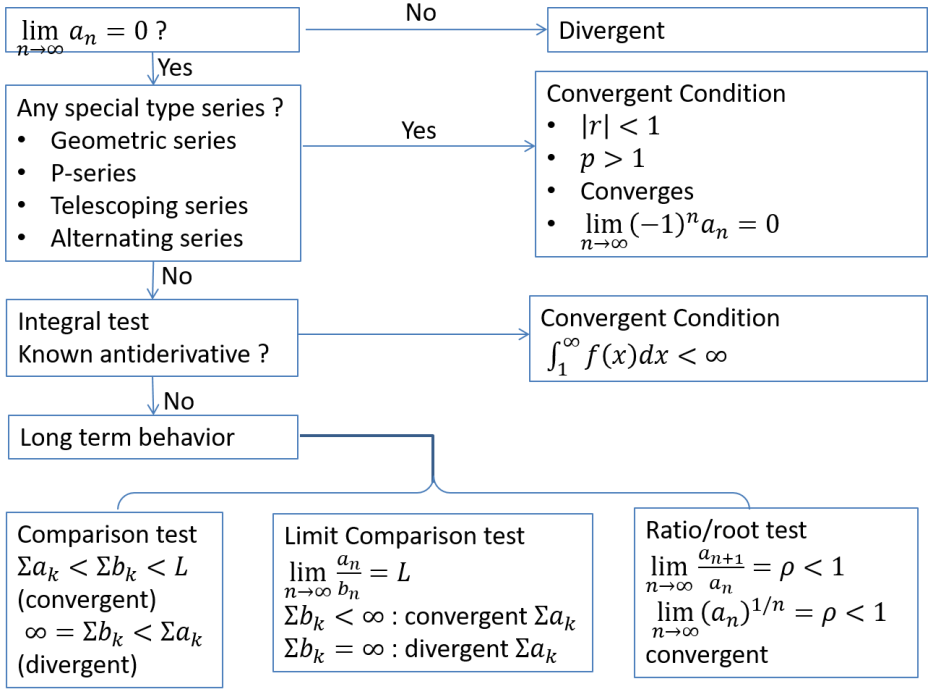
- Since $0 \le u_k + |u_k| \le 2|u_k|$ and $\sum_{k=1}^{\infty} 2|u_k| = 2\sum_{k=1}^{\infty}|u_k|$ (converges), by comparison test $\sum_{k=1}^{\infty}(u_k + |u_k|)$ converges
- Now that $\sum_{k=1}^{\infty}(u_k + |u_k|)$ converge, and $\sum_{k=1}^{\infty}|u_k|$ converges, by the limit theorem of series, $\sum_{k=1}^{\infty} u_k = \sum_{k=1}^{\infty}[(u_k + |u_k|) - |u_k|]$ converges
  - $\sum a_k = S_a$ and $\sum b_k = S_b \Rightarrow \sum(a_k + b_k) = S_a + S_b$

Week 10

Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ converge?

If so, how?

Week 10



+/- terms but Not alternating
⇒ Absolute convergence

Does $\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2}$ converge?
If so, how?

Does $\sum_{n=1}^{\infty} \frac{2+\sin n}{n}$ converge?
If so, how?

Does $\sum_{n=2}^{\infty} \frac{2+\sin n}{n \ln n}$ converge?
If so, how?

Since
$\sum_{n=1}^{\infty} \frac{1}{n^2} \le \sum_{n=1}^{\infty} \frac{2+\sin n}{n^2} \le \sum_{n=1}^{\infty} \frac{3}{n^2}$
and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=1}^{\infty} \frac{3}{n^2} < \infty$,
$\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2} < \infty$

Since $\sum_{n=2}^{\infty} \frac{k}{n} = \infty$, $\sum_{n=1}^{\infty} \frac{2+\sin n}{n} = \infty$

Since $\sum_{n=2}^{\infty} \frac{k}{n \ln n} = \infty$, $\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2} = \infty$

Week 10

Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$. Circle the true statement(s):

If $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

If $\sum_{n=1}^{\infty} b_n$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} b_n$ is divergent.

If $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

There is not enough information.

Week 10

Does $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$ converge?

If so, how?

Week 10

Which of the following statements is true for the following series?

$$\text{(I)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \qquad \text{(II)} \sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^3} \qquad \text{(III)} \sum_{n=1}^{\infty} \frac{e^n}{(-1)^n n}$$

(a) I and III converge conditionally, and II diverges.

(b) I converges conditionally, II converges absolutely, and III diverges.

(c) I and II converge conditionally, and III diverges.

(d) I, II, and III converge conditionally.

(e) I, II, and III converge absolutely.

Week 10

Does $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3n+1)}{n!}$ converge?

If so, how?

Week 10

Does $\sum_{n=1}^{\infty} \frac{2^n}{3^n+n^2}$ converge?

If so, how?

Week 10

(10 points) Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Show all work, as illustrated in class, by naming the test(s), applying the test(s), and drawing the correct conclusion(s).

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$$

Week 10

Determine whether the series converges or diverges. Justify.

$$\sum_{n=1}^{\infty} n^3 \sin\left(\frac{1}{n^3}\right)$$

Determine whether the series converges or diverges. Justify.

$$\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n^3}\right)$$