Sections 2.6, 2.7, 2.8

1. Find the limit.

(a) $\lim\limits_{x\to\infty} \dfrac{x^2-5x+1}{3x+7}$

$$= \lim_{x\to\infty} \frac{x^2\,\frac{x^2-5x+1}{x^2}}{x\,\frac{3x+7}{x}} = \lim_{x\to\infty} \frac{x\left(\frac{x^2}{x^2}-\frac{5x}{x^2}+\frac{1}{x}\right)}{\frac{3x}{x}+\frac{7}{x}}$$

$$= \lim_{x\to\infty} \frac{x\left(1-\frac{5}{x}\xrightarrow{}0+\frac{1}{x}\xrightarrow{}0\right)}{3+\frac{7}{x}\xrightarrow{}0} = \lim_{x\to\infty} \frac{x}{3} = \boxed{\infty}$$

(b) $\lim\limits_{x\to\infty} \dfrac{x^2+x-4}{x^3-2x+1}$

$$= \lim_{x\to\infty} \frac{x^2\,\frac{x^2+x-4}{x^2}}{x^3\,\frac{x^3-2x+1}{x^3}} = \lim_{x\to\infty} \frac{x^2\left(\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{4}{x^2}\right)}{x^3\left(\frac{x^3}{x^3}-\frac{2x}{x^3}+\frac{1}{x^3}\right)} = \lim_{x\to\infty} \frac{x^2\left(1+\frac{1}{x}\xrightarrow{}0-\frac{4}{x^2}\xrightarrow{}0\right)}{x^3\left(1-\frac{2}{x^2}\xrightarrow{}0+\frac{1}{x^3}\xrightarrow{}0\right)} = \lim_{x\to\infty}\frac{1}{x} = 0$$

(c) $\lim\limits_{x\to-\infty} \dfrac{2x^3+3x^2-3x+7}{x^3-16x+5}$

$$= \lim_{x\to-\infty} \frac{\cancel{x^3}\,\frac{2x^3+3x^2-3x+7}{x^3}}{\cancel{x^3}\,\frac{x^3-16x+5}{x^3}} = \lim_{x\to-\infty} \frac{2+\frac{3}{x}\xrightarrow{}0-\frac{3}{x^2}\xrightarrow{}0+\frac{7}{x^3}\xrightarrow{}0}{1-\frac{16}{x^2}\xrightarrow{}0+\frac{5}{x^3}\xrightarrow{}0} = \boxed{2}$$

(d) $\lim\limits_{x\to\infty} \left(\sqrt{x^2+x-1}-\sqrt{x^2-x}\right)\dfrac{\left(\sqrt{x^2+x-1}+\sqrt{x^2-x}\right)}{\sqrt{x^2+x-1}+\sqrt{x^2-x}}$

$a^2-b^2=(a-b)(a+b)$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+x-1}\right)^2-\left(\sqrt{x^2-x}\right)^2}{\sqrt{x^2+x-1}+\sqrt{x^2-x}} = \lim_{x\to\infty} \frac{x^2+x-1-(x^2-x)}{\sqrt{x^2\cdot\frac{x^2+x-1}{x^2}}+\sqrt{x^2\,\frac{x^2-x}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{2x-1}{\sqrt{x^2\left(\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}\right)}+\sqrt{x^2\left(\frac{x^2}{x^2}-\frac{x}{x^2}\right)}} = \lim_{x\to\infty} \frac{2x+1}{\sqrt{x^2\left(1+\frac{1}{x}\xrightarrow{}0-\frac{1}{x^2}\xrightarrow{}0\right)}+\sqrt{x^2\left(1-\frac{1}{x}\xrightarrow{}0\right)}}$$

$$= \lim_{x\to\infty} \frac{2x+1}{\sqrt{x^2}+\sqrt{x^2}} = \lim_{x\to\infty} \frac{2x+1}{2\sqrt{x^2}} \quad \left|\ \begin{array}{l} x>0 \\ \sqrt{x^2}=x \end{array}\ \right| \quad = \lim_{x\to\infty} \frac{2x+1}{2x} = \lim_{x\to\infty} \frac{\cancel{x}\left(\frac{2x+1}{x}\right)}{2\cancel{x}} = \lim_{x\to\infty} \frac{2+\frac{1}{x}\xrightarrow{}0}{2} = \frac{2}{2} = \boxed{1}$$

(e) $\lim\limits_{x\to-\infty} \left(x+\sqrt{x^2+2x}\right)\dfrac{\left(x-\sqrt{x^2+2x}\right)}{x-\sqrt{x^2+2x}}$

$$= \lim_{x\to-\infty} \frac{x^2-\left(\sqrt{x^2+2x}\right)^2}{x-\sqrt{x^2\,\frac{x^2+2x}{x^2}}} = \lim_{x\to-\infty} \frac{x^2-(x^2+2x)}{x-\sqrt{x^2\left(\frac{x^2}{x^2}+\frac{2x}{x^2}\right)}}$$

$$= \lim_{x\to-\infty} \frac{-2x}{x-\sqrt{x^2\left(1+\frac{2}{x}\xrightarrow{}0\right)}} = -\lim_{x\to-\infty} \frac{2x}{x-\sqrt{x^2}} \quad \left|\ \begin{array}{l} x<0 \\ \sqrt{x^2}=-x \end{array}\ \right| \quad = -\lim_{x\to-\infty} \frac{2x}{x+x} = -\lim_{x\to-\infty} \frac{\cancel{2x}}{\cancel{2x}} = \boxed{-1}$$

2. Find the vertical and horizontal asymptotes (if any) for the function $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - x - 6}$.

horizontal

$$\lim_{x\to\infty} \frac{x^2-2x-8}{x^2-x-6} = \lim_{x\to\infty} \frac{\cancel{x^2}\,\dfrac{x^2-2x-8}{x^2}}{\cancel{x^2}\,\dfrac{x^2-x-6}{x^2}} = \lim_{x\to\infty} \frac{\dfrac{x^2}{x^2}-\dfrac{2x}{x^2}-\dfrac{8}{x^2}}{\dfrac{x^2}{x^2}-\dfrac{x}{x^2}-\dfrac{6}{x^2}} = \lim_{x\to\infty} \frac{1-\overset{\nearrow 0}{\cancel{\dfrac{2}{x}}}-\overset{\nearrow 0}{\cancel{\dfrac{8}{x^2}}}}{1-\overset{\nearrow 0}{\cancel{\dfrac{1}{x}}}-\overset{\nearrow 0}{\cancel{\dfrac{6}{x^2}}}} = 1$$

$\boxed{y=1}$ horizontal asymptote.

vertical

$$f(x) = \frac{x^2-2x-8}{x^2-x-6} = \frac{(x-4)\cancel{(x+2)}}{(x-3)\cancel{(x+2)}}$$

$x=-2$ – removable discontinuity (a hole)

$x-3=0 \Rightarrow x=3$ is an infinite discontinuity.

$\boxed{x=3}$ is the vertical asymptote.

$a^2 - b^2 = (a-b)(a+b)$

3. Find $f'(x)$ by using the definition of derivative if

(a) $f(x) = (3-x)^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\overbrace{(3-(x+h))^2}^{f(x+h)} - (3-x)^2}{h} = \lim_{h\to 0} \frac{([3-x-h]+[3-x])([3-x-h]-[3-x])}{h}$$

$$= \lim_{h\to 0} \frac{(6-2x-h)(-\cancel{h})}{\cancel{h}} = -\lim_{h\to 0}(6-2x-h) = -(6-2x) = \boxed{2x-6}$$

(b) $f(x) = \sqrt{x-2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left(\sqrt{(x+h)-2} - \sqrt{x-2}\right)\left(\sqrt{(x+h)-2} + \sqrt{x-2}\right)}{h\left(\sqrt{(x+h)-2} + \sqrt{x-2}\right)}$$

$$= \lim_{h\to 0} \frac{(\cancel{x}+h-\cancel{2}) - (\cancel{x}-\cancel{2})}{h\left(\sqrt{(x+h)-2} + \sqrt{x-2}\right)} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{(x+h)-2} + \sqrt{x-2}\right)} = \boxed{\frac{1}{2\sqrt{x-2}}}$$

(c) $f(x) = \dfrac{1}{x+1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{x+1} - (\cancel{x}+h+\cancel{1})}{(x+h+1)(x+1)}}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(x+h+1)(x+1)} = \boxed{-\frac{1}{(x+1)^2}}$$

4. Let $f(x) = x|x|$

(a) For what values of x is f differentiable?

(b) Find a formula for f'.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x, & x \geq 0 \\ -2x, & x < 0 \end{cases}$$ differentiable for all x.

might not be differentiable @ $x=0$.

$$\lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h} \underset{f(x)=x^2}{=} \lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \lim_{h\to 0} \frac{(\cancel{x}+h-\cancel{x})(x+h+x)}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

$$\lim_{x\to 0^+} 2x = 0$$

$$\lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h} \quad f(x)=-x^2 \quad \lim_{h\to 0} \frac{-(x+h)^2+x^2}{h} = \lim_{h\to 0} \frac{[x-(x+h)][x+(x+h)]}{h} = \lim_{h\to 0} \frac{-h(2x+h)}{h} = -2x$$

$$\lim_{x\to 0^-}(-2x) = 0.$$

$$\boxed{f'(0)=0}$$

5. At what point on the curve $y = x^{3/2}$ is the tangent line parallel to the line $3x - y + 6 = 0$.

$y = 3x + 6$

slope $= 3$.

Find a point on the curve where the slope of a tangent line is 3.

$$f'(x) = \frac{3}{2} x^{\frac{3}{2}-1} = \boxed{\frac{3}{2} x^{1/2} = 3}$$

$$x^{1/2} = 2$$

$$\boxed{x = 4} \qquad y = (4)^{3/2} = 8.$$

$$\boxed{(4, 8)}$$

$$s(1) = 1 + 2 + \frac{1}{4} = \frac{13}{4}$$

6. The displacement of an object moving in a straight line is given by $s(t) = 1 + 2t + t^2/4$ (t is in seconds). Find the velocity of the object when $t = 1$.

$$\vec{v}(1) = \lim_{t \to 1} \frac{s(t) - s(1)}{t-1} = \lim_{t \to 1} \frac{1 + 2t + \frac{t^2}{4} - \frac{13}{4}}{t-1}$$

$$= \lim_{t \to 1} \frac{2t + \frac{t^2}{4} - \frac{9}{4}}{t-1} = \lim_{t \to 1} \frac{\frac{8t + t^2 - 9}{4}}{t-1} = \lim_{t \to 1} \frac{t^2 + 8t - 9}{4(t-1)}$$

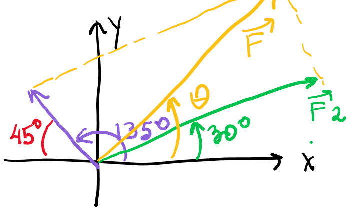
$$= \lim_{t \to 1} \frac{\cancel{(t-1)}(t+9)}{4\cancel{(t-1)}} = \frac{10}{4} = \boxed{2.5}$$

Review for Midterm 1.

1. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on an object. The force $\mathbf{F}_1$ has a magnitude of 16 lbs and a direction of 135° counterclockwise from the positive x-axis, and $\mathbf{F}_2$ has a magnitude of 60 lbs and a direction of 30 ° counterclockwise from the positive x-axis.

   (a) Find the resultant force $\mathbf{F}$.

   (b) Find the resultant angle θ as measured counterclockwise from the positive x-axis.



$|\vec{F}_1| = 16$

$|\vec{F}_2| = 60$

$\vec{F}_1 = 16 \langle -\cos\frac{\pi}{4}, \sin\frac{\pi}{4} \rangle$

$= 16 \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$\boxed{\vec{F}_1 = \langle -8\sqrt{2}, 8\sqrt{2} \rangle}$

$\vec{F}_2 = 60 \langle \cos\frac{\pi}{6}, \sin\frac{\pi}{6} \rangle$

$= 60 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

$\boxed{\vec{F}_2 = \langle 30\sqrt{3}, 30 \rangle}$

net force $\boxed{\vec{F} = \vec{F}_1 + \vec{F}_2 = \langle -8\sqrt{2} + 30\sqrt{3}, 30 + 8\sqrt{2} \rangle}$

$\tan\theta = \frac{F_y}{F_x} \Rightarrow \boxed{\theta = \arctan \frac{30 + 8\sqrt{2}}{30\sqrt{3} - 8\sqrt{2}}}$

1.1. Find a unit vector in the direction of $\vec{v} = \langle 4, -1 \rangle$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 4, -1 \rangle}{\sqrt{16+1}} = \langle \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}} \rangle$

a vector of length 4 in the direction opposite to $\vec{v}$

$4(-\vec{u}) = 4 \langle -\frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}} \rangle$

$= \boxed{\langle -\frac{16}{\sqrt{17}}, \frac{4}{\sqrt{17}} \rangle}$

2. A constant force $\mathbf{F} = 5\mathbf{i} + 6\mathbf{j}$ moves an object along a straight line from the point $A(-1,2)$ to the point $B(2,3)$. Find the work done by the force $\mathbf{F}$.

$$W = \vec{F} \cdot \vec{AB}$$

$$\vec{AB} = \langle 2-(-1), 3-2 \rangle = \langle 3, 1 \rangle, \quad \vec{F} = \langle 5, 6 \rangle.$$

$$W = \vec{F} \cdot \vec{AB} = \langle 3, 1 \rangle \cdot \langle 5, 6 \rangle = 3(5) + 1(6)$$
$$= 15 + 6 = 21$$

3. Find the scalar and vector projections of the vector $\underset{\vec{m}=}{2\mathbf{i} - 3\mathbf{j}}$ onto the vector $\mathbf{i} + 6\mathbf{j}$. $= \vec{n}$

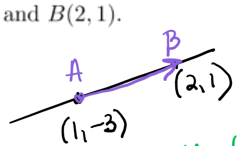
vector $\quad \text{proj}_{\vec{n}} \vec{m} = \dfrac{\vec{n} \cdot \vec{m}}{|\vec{n}|} \cdot \dfrac{\vec{n}}{|\vec{n}|} = \dfrac{\vec{n} \cdot \vec{m}}{|\vec{n}|^2} \cdot \vec{n}$

$$= \frac{\langle 2, -3 \rangle \cdot \langle 1, 6 \rangle}{1 + 6^2} \langle 1, 6 \rangle = \frac{2 - 18}{37} \langle 1, 6 \rangle$$

$$= \boxed{-\frac{17}{37} \langle 1, 6 \rangle}$$

scalar $\quad \text{comp}_{\vec{n}} \vec{m} = \dfrac{\vec{m} \cdot \vec{n}}{|\vec{n}|} = \dfrac{\langle 2, -3 \rangle \cdot \langle 1, 6 \rangle}{\sqrt{1+36}} = \boxed{-\dfrac{17}{\sqrt{37}}}$

4. Find the vector, parametric, and the Cartesian equations for the line passing through the points $A(1, -3)$ and $B(2, 1)$.



a vector parallel to the line is

$\vec{AB} = \langle 2-1, 1-(-3) \rangle = \langle 1, 4 \rangle$

vector: $\boxed{\vec{r}(t) = \langle 1, -3 \rangle + t\langle 1, 4 \rangle}$ or $\boxed{\vec{r}(t) = \langle 2, 1 \rangle + t\langle 1, 4 \rangle}$

parametric: $\langle x, y \rangle = \langle 1+t, -3+4t \rangle$ or $\langle x, y \rangle = \langle 2+t, 1+4t \rangle$

$\boxed{\begin{cases} x = 1+t \\ y = -3+4t \end{cases}}$ $\boxed{\begin{cases} x = 2+t \\ y = 1+4t \end{cases}}$

Cartesian (eliminate t):

$\begin{cases} x = 1+t \\ y = -3+4t \end{cases} \rightarrow t = x-1$

$y = -3+4(x-1)$

$y = -3+4x-4$

$\boxed{y = 4x-7}$

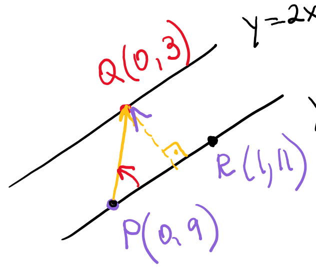
$\begin{cases} x = 2+t \\ y = 1+4t \end{cases} \rightarrow t = x-2$

$y = 1+4(x-2)$

$y = 1+4x-8$

$\boxed{y = 4x-7}$

5. Find the distance between the parallel lines $y = 2x + 3$ and $y - 2x = 9$.



$\Rightarrow y = 9+2x$

$\vec{PR} = \langle 1, 11-9 \rangle = \langle 1, 2 \rangle$

$\vec{PR}^{\perp} = \langle 2, -1 \rangle$

$\vec{PQ} = \langle 0, 3-9 \rangle = \langle 0, -6 \rangle$

$$d = \left| \operatorname{comp}_{\vec{PR}^{\perp}} \vec{PQ} \right| = \left| \frac{\vec{PQ} \cdot \vec{PR}^{\perp}}{|\vec{PR}^{\perp}|} \right| = \frac{|\langle 2, -1 \rangle \cdot \langle 0, -6 \rangle|}{\sqrt{2^2 + (-1)^2}} = \boxed{\frac{6}{\sqrt{5}}}$$

6. Given the parametric curve $x(t) = 1 + \cos t$, $y(t) = 1 - \sin^2 t$.

(a) Find a Cartesian equation for this curve.

(b) Does the parametric curve go through the point (1,0)? If yes, give the value(s) of t.

(c) Sketch the graph of the parametric curve on the interval $0 \le t \le \pi$, include the direction of the path.

(a)

$x(t) = 1 + \cos t$, $y(t) = 1 - \sin^2 t$

$\cos t = x - 1$ $\qquad$ $\sin^2 t = 1 - y$

$\cos^2 t + \sin^2 t = 1$

$\underbrace{(x-1)^2}_{\cos^2 t} + \underbrace{\cancel{1} - y}_{\sin^2 t} = \cancel{1}$

$\boxed{y = (x-1)^2}$ parabola

Domain

$|\cos t| \le 1 \Rightarrow |x-1| \le 1$

$-1 \le x-1 \le 1$

$\boxed{0 \le x \le 2}$

$0 \le \sin^2 t \le 1$

$0 \le 1 - y \le 1$ or $\boxed{0 \le y \le 1}$

(b) $\langle 1+\cos t, 1-\sin^2 t \rangle = \langle 1, 0 \rangle$ find t

$\begin{cases} 1 + \cos t = 1 \\ 1 - \sin^2 t = 0 \end{cases}$ $\qquad$ $\cos t = 0$ $\Rightarrow$ $\boxed{t = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots}$

$\sin^2 t = 1$ or $\sin t = 1$

$\sin t = -1$

(c) $\langle 1+\cos t, 1-\sin^2 t \rangle = \langle x, y \rangle$

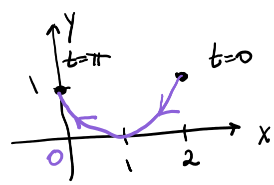
$0 \le t \le \pi$

$t=0$: $\langle x(0), y(0) \rangle = \langle 1 + \overset{1}{\cancel{\cos 0}}, 1 - \overset{0}{\cancel{\sin^2 0}} \rangle$

$= (2, 1)$

$t=\pi$: $\langle x(\pi), y(\pi) \rangle = \langle 1 + \overset{-1}{\cancel{\cos \pi}}, 1 - \overset{0}{\cancel{\sin^2 \pi}} \rangle$

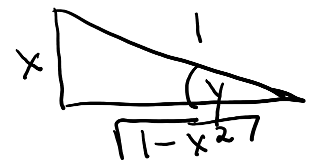
$= \langle 0, 1 \rangle$



7. Express $\tan(\underbrace{\arcsin(x)}_{y})$ without using trig or inverse trig functions.

$y = \arcsin x \implies \sin y = x$

Find $\tan y = ?$



$$\boxed{\tan y = \frac{x}{\sqrt{1-x^2}}}$$

8. Evaluate the limit (do no use the L'Hospital's Rule):

(a) $\lim_{x\to 5} \frac{x^2-5x+10}{x^2-25} = \frac{25-25+10}{25-25} = \frac{10}{0} = \boxed{\infty}$

(b) $\lim_{x\to 7} \frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x\to 7} \frac{4-(x-3)}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x\to 7} \frac{\cancel{(7-x)}}{\cancel{(x-7)}(x+7)(2+\sqrt{x-3})}$

$= -\lim_{x\to 7} \frac{1}{(x+7)(2+\sqrt{x-3})} = -\frac{1}{(7+7)(2+\sqrt{7-3})} = -\frac{1}{14(4)} = \boxed{-\frac{1}{56}}$

(c) $\lim_{x\to -2} \frac{x^2-4}{|x+2|}$ $\boxed{DNE}$

$|x+2| = \begin{cases} x+2, & x+2 \geq 0 \\ -(x+2), & x+2 < 0 \end{cases}$

$\lim_{x\to -2^+} f(x) \xrightarrow[|x+2|=x+2]{x>-2} \lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}} = \lim_{x\to -2} (-2-2) = -4$

$\lim_{x\to -2^-} f(x) \xrightarrow[|x+2|=-(x+2)]{x<-2} \lim_{x\to -2} \frac{x^2-4}{-(x+2)} = \lim_{x\to -2} \frac{(x-2)\cancel{(x+2)}}{-\cancel{(x+2)}} = -(-2-2) = 4$

$\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$

(d) $\lim_{x\to 0} \left(\frac{1}{x\sqrt{x+1}} - \frac{1}{x}\right) = \lim_{x\to 0} \frac{(1-\sqrt{x+1})(1+\sqrt{x+1})}{x\sqrt{x+1}\,(1+\sqrt{x+1})} = \lim_{x\to 0} \frac{1-(x+1)}{x\sqrt{x+1}\,(1+\sqrt{x+1})}$

$= \lim_{x\to 0} \frac{-x}{x\sqrt{x+1}\,(1+\sqrt{x+1})} = \lim_{x\to 0} \frac{-1}{\sqrt{x+1}\,(1+\sqrt{x+1})} = \boxed{\frac{1}{2}}$

9. Find and classify all points of discontinuity for the function

$$f(x) = \begin{cases} x^2+1 & , \text{ if } x < 2, \\ x+2 & , \text{ if } x \geq 2. \end{cases}$$

$x=2$

$$\lim_{x\to 2^+} f(x) \quad \frac{x>2}{f(x)=x+2} \quad \lim_{x\to 2}(x+2) = 4$$

$$\lim_{x\to 2^-} f(x) \quad \frac{x<2}{f(x)=x^2+1} \quad \lim_{x\to 2^-}(x^2+1) = 5$$

$4 \neq 5 \Rightarrow$ $f(x)$ has a jump discontinuity @ $x=2$.

10. Find the vertical and horizontal asymptotes of the curve $y = \dfrac{x^2+4}{3x^2-3}$.

horizontal.

$$\lim_{x\to\infty} \frac{x^2+4}{3x^2-3} = \lim_{x\to\infty} \frac{\cancel{x^2}\left(\frac{x^2+4}{x^2}\right)}{\cancel{x^2}\left(\frac{3x^2-3}{x^2}\right)}$$

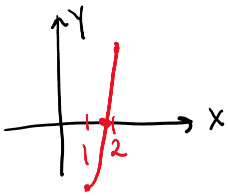
$$= \lim_{x\to\infty} \frac{\left(1+\cancel{\frac{4}{x^2}}^{\,0}\right)}{3-\cancel{\frac{3}{x^2}}^{\,0}} = \frac{1}{3} \Rightarrow \text{horizontal} \quad \boxed{y=\frac{1}{3}}$$

vertical

$$y = \frac{x^2+4}{3(x^2-1)} = \frac{x^2+4}{3(x-1)(x+1)}$$

$x-1=0$
$x+1=0$ $\Rightarrow$ $\boxed{x=\pm 1}$ vertical asymptotes.

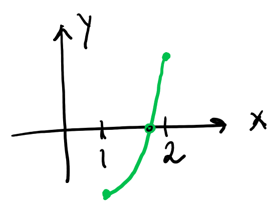
11. Use the Intermediate Value Theorem to show that there is a root of the equation $x^3-3x+1=0$ in the interval (1,2).



$f(x) = x^3-3x+1$

$f(1) = 1-3+1 = -2$

$f(2) = 8-6+1 = 3$



since $f(1)<0$ and $f(2)>0$, then there is a point $1<c<2$ such that $f(c)=0$.