Math 151 - Week-In-Review 2

V. Coffelt

Topics for the week:

J.2 The Dot Product
J.3 Vector Functions and Parametric Curves
2.2 The Limit of a Function

J.2 The Dot Product

1. Given vectors $\mathbf{u} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = -2\mathbf{j}$, compute each of the following.

   (a) $\mathbf{u} \cdot \mathbf{v}$

   (b) $\left(-\frac{2}{7}\mathbf{v}\right) \cdot \left(\frac{4}{5}\mathbf{u}\right)$

   (c) $(\mathbf{v} \cdot \mathbf{u})\,\mathbf{u}$

   (d) Compute the angle between vectors $\mathbf{u}$ and $\mathbf{v}$, leave your answer in exact form.

2. Given vectors $\mathbf{u} = \langle -5, 12 \rangle$, $\mathbf{v} = \langle 6, 0 \rangle$, and $\mathbf{w} = \left\langle -\frac{2}{5}, -\frac{3}{2} \right\rangle$, compute each of the following.

(a) $\mathbf{u} \cdot \mathbf{w}$

(b) $2\left(\mathbf{v} \cdot \mathbf{u}\right)$

(c) $(2\mathbf{v}) \cdot \mathbf{u}$

(d) The angle between vectors $\mathbf{v}$ and $\mathbf{w}$, leave your answer in exact form.

(e) The orthogonal complement of $\mathbf{w}$

3. Determine the scalar product of two vectors, $\vec{a}$ and $\vec{b}$, if $\vec{a}$ has a length of 6 and $|\vec{b}| = 15$ and the smaller angle between the vectors is $\dfrac{3\pi}{4}$.

4. $\mathbf{u} = \left\langle -5, \dfrac{1}{2} \right\rangle$ and $\mathbf{v} = \langle 3, c \rangle$, determine the value of c such that the vectors are orthogonal.

5. Given vectors $\vec{u} = \left\langle \dfrac{3}{2}, -\dfrac{1}{2} \right\rangle$ and $\vec{w} = \langle 4, 8 \rangle$, compute each of the following.

   (a) Scalar projection of $\vec{u}$ onto $\vec{w}$

   (b) Vector projection of $\vec{u}$ onto $\vec{w}$

6. Determine the distance between the line, $3x + y = 5$ and the point $(4, 1)$, using vectors.

7. A kid pushes a sled 30 ft up a hill with an incline of $5°$. The horizontal force exerted on the sled is 10 lbs. Determine the work done on the sled.

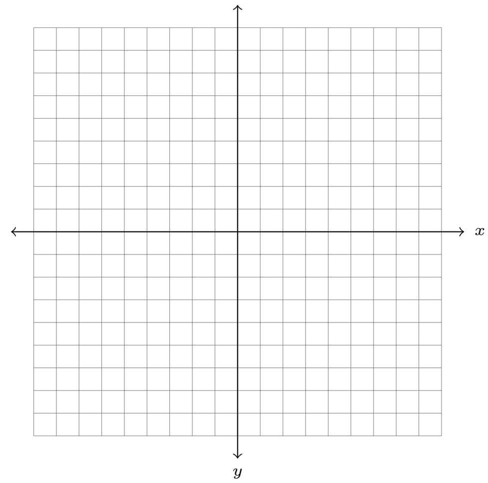
8. An object is moved along a straight line from the point $(1, 7)$ to the point $(5, 16)$ at a constant force $\mathbf{F} = 14\mathbf{i} + 20\mathbf{j}$. Assuming the distance is measured in meters and the magnitude of the force is measured in newtons, determine each of the following:

   (a) Displacement vector, $\mathbf{D}$.

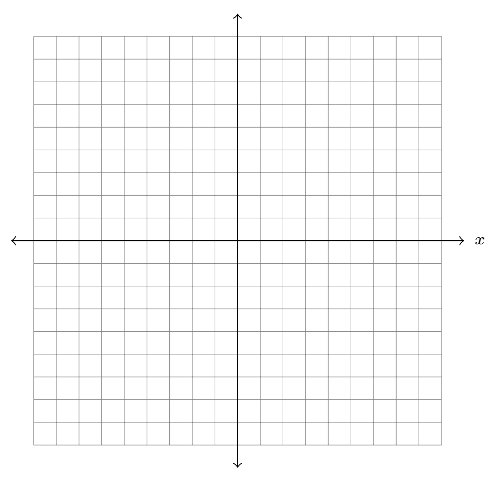
   (b) Work, W.

J.3 Vector Functions and Parametric Curves

9. Sketch the curve generated by the parametric equations, $x = t^2 - 13$, $y = \sqrt{4 - t}$, for $-5 \leq t \leq 3$. Then write the Cartesian equation for the the curve.



10. Sketch the curve generated by the parametric equations, $x = \cos(t)$, $y = 3 \sec t$, for $\dfrac{\pi}{2} < t < \pi$. Then write the Cartesian equation for the the curve.



11. Given the position of an object moving in the Cartesian plane is $\mathbf{r}(t) = \left\langle e^{2t}, e^{-t} \right\rangle$ after t-seconds, determine each of the following:

    (a) The position of the object after 3 seconds.

    (b) At what time is the object at the point $(1, 1)$?

    (c) Does the object ever pass through the point $\left(\frac{1}{e^2}, e\right)$?

12. Write a vector equation of the line $y = 4x - 8$. (Hint: Use $(0, -8)$ as P_0.)

13. Write a vector equation of the line perpendicular to the line $y = 6x - 7$ and passing through the point $(0, -7)$.

14. Write a parametric equation of the line passing through the points $(12, 5)$ and $(9, -2)$.

15. Determine the parametric equations for the line that passes through the point (3,-1) and is

    (a) is parallel to the vector $\langle -5, -4 \rangle$.

    (b) is perpendicular to the vector $\langle -5, -4 \rangle$.

16. State the slope of the line with corresponding vector equation $\mathbf{r}(t) = \langle 5 - 2t, -8 + 7t \rangle$.

17. Determine whether the lines, $L_1 = \mathbf{r}(t) = (-6 + 2t)\mathbf{i} + (7 - 6t)\mathbf{j}$ and $L_2 = \mathbf{r}(s) = \left(5 + \frac{1}{2}s\right)\mathbf{i} + \left(-8 + \frac{3}{2}s\right)\mathbf{j}$, are parallel, perpendicular, or neither.

2.2 The Limit of a Function

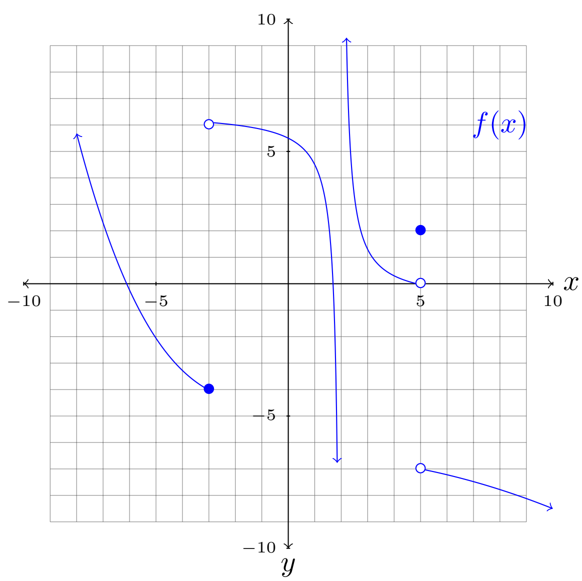
18. Use the graph provided to answer each of the following:

a. $\lim_{x \to -3^-} f(x)$

b. $\lim_{x \to -3^+} f(x)$

c. $\lim_{x \to 2^-} f(x)$

d. $\lim_{x \to 5} f(x)$



19. If $\lim_{x \to 4^-} f(x) = -25$ and $\lim_{x \to 4^+} f(x) = -26$, what can we say about $\lim_{x \to 4} f(x)$?

20. If $\lim_{x \to 4^-} f(x) = -\infty$ and $\lim_{x \to 4^+} f(x) = -\infty$, what can we say about the function $f(x)$ at $x = 4$ and the graph of $f(x)$ at $x = 4$.

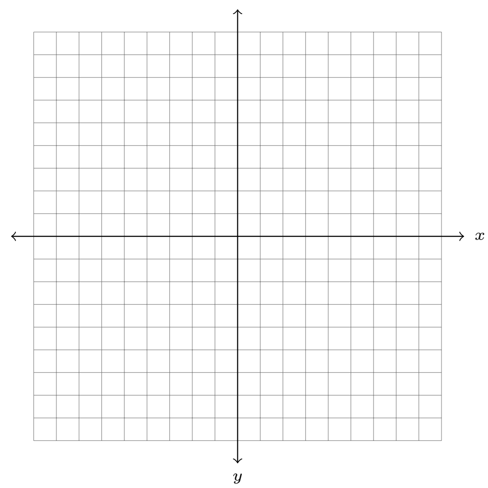
21. Evaluate each of the following limits.

    (a) $\lim_{x \to -8^+} \left(\dfrac{x^2 + 3x + 1}{8 + x} \right)$

    (b) $\lim_{x \to 3^-} \left(\dfrac{1 - x}{2x - 6} \right)$

22. Sketch the graph of a function, $g(x)$, that satisfy the conditions below:

- $g(-3) = -2$
- $g(5)$ is undefined
- $\lim_{x \to -6^-} g(x) = \infty$
- $\lim_{x \to -6^+} g(x) = -\infty$
- $\lim_{x \to -3^-} g(x) = 5$
- $\lim_{x \to -3^+} g(x) = 5$
- $\lim_{x \to 5} g(x) = 4$



23. Sketch the graph of $h(x) = \dfrac{x}{(3x-12)^2}$.