1) $\sqrt{y} = e^{3x+5}$

$\ln(\sqrt{y}) = \ln\left(e^{3x+5}\right)$

$\ln(y^{1/2}) = (3x+5)\ln(e)$

$\boxed{\frac{1}{2}\ln(y) = 3x+5}$

$\ln(e) = 1$

$\ln(y) = 6x+10$

2) Expand $\log_2\left(\dfrac{2\cdot w^5 x^3}{\sqrt{y}\, z}\right)$

$$= \log_2(2) + \log_2(w^5) + \log_2(x^3) - \log_2(\sqrt{y}) - \log_2(z)$$

$\log_b b = 1$

$$= \boxed{\begin{aligned} &1 + 5\log_2(w) + 3\log_2(x) \\ &- \frac{1}{2}\log_2(y) - \log_2(z) \end{aligned}}$$

3) Solve $\underline{2}^{(3x+5)} = \underline{3}^{(2x-1)}$ for x

$$\ln\left(2^{(3x+5)}\right) = \ln\left(3^{(2x-1)}\right)$$

$$(3x+5)\ln(2) = (2x-1)\ln(3)$$

$$3\underline{x}\ln(2) + 5\ln(2) = 2\underline{x}\ln(3) - \ln(3)$$

$$3x\ln(2) - 2x\ln(3) = -5\ln(2) - \ln(3)$$

$$x\left(3\ln(2) - 2\ln(3)\right) = -5\ln(2) - \ln(3)$$

$$\boxed{x = \frac{-5\ln(2) - \ln(3)}{3\ln(2) - 2\ln(3)}}$$

4) Solve $\ln(x-3) + \ln(3x+1) = \ln(x+9)$

$$\ln\big((x-3)(3x+1)\big) = \ln(x+9)$$

$$(x-3)(3x+1) = x+9$$

$$3x^2 - 9x + x - 3 = x + 9$$

$$3x^2 - 8x - 3 - x - 9 = 0$$

$$3x^2 - 9x - 12 = 0$$

$$3(x^2 - 3x - 4) = 0$$

$$3(x-4)(x+1) = 0$$

$\rightarrow$ $\boxed{x=4}$, $\cancel{x=-1}$ $\rightarrow$ $\ln(-4)$ DNE

plug values into original equation

5) a) $P = 2000$, $t = 3$, years ago $r = .35\% = .0035$

$$A(t) = 2000e^{.0035t}$$

$$A(3) = 2000e^{.0035 \times 3} = \$2021.11$$

b) Approach 1: Find $A(t) = 5000$,

t = years since initial deposit 3 years ago

answer → $t-3$ years from now

$$5000 = 2000e^{.0035t}$$

$$2.5 = e^{.0035t}$$

$$\ln(2.5) = .0035t$$

$$t = \frac{\ln(2.5)}{.0035}$$

Answer: $\frac{\ln(2.5)}{.0035} - 3$

5b) Approach II: $P = 2021.11$

$$5000 = 2021.11\, e^{.0035t}$$

$$\frac{5000}{2021.11} = e^{.0035t}$$

$$\ln\left(\frac{5000}{2021.11}\right) = .0035t$$

$$t = \frac{1}{.0035} \ln\left(\frac{5000}{2021.11}\right)$$

6) Find the domain of

$$f(x) = \log_3 (2-5x).$$

Need $2-5x \geq 0$

$\quad -2 \qquad -2$

$$-5x \geq -2$$

$\quad \overline{-5} \qquad \overline{-5}$

$$x \leq 2/5$$

$$[-\infty, 2/5]$$

7) Domain: $(-\infty, -5] \cup [-3, \infty)$

Range: $[-4, 4] \cup [-8, \infty)$

fails horizontal line test

8) $f(x) = \dfrac{x-2}{x+3}$, $g(x) = \dfrac{x+5}{x-2}$, $h(x) = x^2$

a) $$f(x) + g(x) = \frac{x-2}{x+3} + \frac{x+5}{x-2} = \frac{(x-2)}{(x+3)}\frac{(x-2)}{(x-2)} + \frac{(x+5)}{(x-2)}\frac{(x+3)}{(x+3)}$$

$$= \frac{(x-2)^2 + (x+5)(x+3)}{(x-2)(x+3)} = \frac{x^2-4x+4 + x^2+8x+15}{x^2+x-6}$$

$$= \frac{2x^2+4x+19}{x^2+x-6}$$

b) $$f(x)h(x) = \frac{(x-2)}{x+3} \cdot x^2 = \frac{x^3-2x^2}{x+3}$$

8) $f(x) = \frac{x-2}{x+3}$, $g(x) = \frac{x+5}{x-2}$, $h(x) = x^2$

c) $f(x) \div g(x) = \frac{x-2}{x+3} \div \frac{x+5}{x-2} = \frac{(x-2)(x-2)}{(x+3)(x+5)}$

$$= \frac{x^2-4x+4}{x^2+8x+15}$$

d) $g \circ h(x) = \frac{x^2+5}{x^2-2}$

e) $h \circ f(x) = \left(\frac{x-2}{x+3}\right)^2 = \frac{(x-2)^2}{(x+3)^2} = \frac{x^2-4x+4}{x^2+6x+9}$

Pr 9

It is a polynomial.

degree 7

leading term $= -17$

constant term $= 2^5 + 12$

$= 32 + 12 = 44$

domain: $(-\infty, \infty)$

10) $p(x) = -.06x + 56$

production cost $= \$5$

fixed costs $= \$150$

Determine Maximum Profit

$P(x) = R(x) - C(x)$

$C(x) = 5x + 150$

$R(x) = p \cdot x = (-.06x + 56)x$

$= -.06x^2 + 56x$

$P(x) = -.006x^2 + 51x - 150$

need vertex

$\frac{-b}{2a} = \frac{-51}{2(-.006)}$

$= \frac{51}{.012}$

$= 4250$ items

$P(4250)$

Determine Maximum Profit

$P(x) = R(x) - C(x)$

$C(x) = 5x + 150$

$R(x) = p \cdot x = (-.06x + 56)x$

$= -.06x^2 + 56x$

$P(x) = -.06x^2 + 51x - 150$

$= \frac{51}{.12}$

$= 425$ items

$P(425)$

$P(425) = -.06(425)^2 + 51(425) - 150$

$= \$10,687.50$

11) Domain of $f(x) = \dfrac{(3x-2)(2x-5)(x-5)}{(x-5)(2x+5)(x+1)}$

denominator $\neq 0$ $\quad (x-5)(2x+5)(x+1) = 0$

$x-5=0 \qquad 2x+5=0 \qquad x+1=0$

$x=5 \qquad 2x=-5 \qquad x=-1$

$x = \dfrac{-5}{2}$

Domain: $\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, -1\right) \cup (-1, 5) \cup (5, \infty)$

$x = -5/2$, $x=-1$ are asymptotes

hole @ $x=5$

$\boxed{\left(5, \frac{65}{90}\right)}$

$$\frac{(3\cdot5-2)(2\cdot5-5)\cancel{(5-5)}}{\cancel{(5-5)}(2\cdot5+5)(5+1)} = \frac{13\cdot5}{15\cdot6} = \boxed{\frac{65}{90}}$$

12) $g(x) = \dfrac{2x}{3x-1}$ Find the difference quotient

$$\frac{g(x+h)-g(x)}{h} = \frac{1}{h}\left[g(x+h) - g(x)\right]$$

$$= \frac{1}{h}\left[\frac{2(x+h)}{3(x+h)-1} - \frac{2x}{3x-1}\right]$$

$$= \frac{1}{h}\left[\frac{2x+2h}{3x+3h-1} - \frac{2x}{3x-1}\right]$$ → common denominator

$\sqrt{ax+b}$ → conjugate

$$= \frac{1}{h}\left[\frac{2x+2h}{3x+3h-1} - \frac{2x}{3x-1}\right] \rightarrow \text{common denominator}$$

$$= \frac{1}{h}\left[\frac{(2x+2h)}{(3x+3h-1)}\frac{(3x-1)}{(3x-1)} - \frac{(2x)}{(3x-1)}\frac{(3x+3h-1)}{(3x+3h-1)}\right]$$

$$= \frac{1}{h}\left[\frac{6x^2-2x+6xh-2h-6x^2-6xh+2x}{(3x+3h-1)(3x-1)}\right]$$

$$= \frac{1}{\cancel{h}}\frac{-2\cancel{h}}{(3x+3h-1)(3x-1)} = \boxed{\frac{-2}{(3x+3h-1)(3x-1)}}$$

13) domain of $f(x) = \dfrac{(3x-2)\sqrt{1-2x}}{(x+5)^{4/7}}$

Denominator $\neq 0$ $\qquad (x+5)^{4/7} = \sqrt[7]{(x+5)^4}$

$(x+5)^{4/7} \neq 0 \rightarrow \quad x+5 \neq 0$

$\rightarrow x \neq -5$

$1-2x \geq 0 \rightarrow \quad 1 \geq 2x \qquad \frac{1}{2} \geq x$

$+2x \quad +2x \qquad \frac{1}{2} \quad \frac{2x}{2}$

or $\quad x \leq 1/2$

$\boxed{(-\infty, -5) \cup (-5, 1/2]}$

14) $f(x) = 2(x+4)^{3/11}$ into radical

Convert

notation.

$$2 \cdot (x+4)^{3/11} = 2 \cdot \sqrt[11]{(x+4)^3}$$

15)

$$f(x) = \begin{cases} \dfrac{1}{(x+5)(x-3)} & \text{if } x < -3 \\ \ln(12-2x) & \text{if } x \geq 3 \end{cases}$$

a) $f(-10)$ $\leftarrow$ apply rule 1

$$= \frac{1}{(-10+5)(-10-3)} = \frac{1}{-5\cdot(-13)} = \frac{1}{5\cdot 13}$$

$$= \frac{1}{65}$$

b) $f(-5) = \dfrac{1}{(-5+5)(-5-3)} = \dfrac{1}{0\cdot(-8)}$

= D.N.E.

c) $f(-3)$ DNE $\quad -3 < -3$ false

$-3 \geq 3$ false

$$f(x) = \begin{cases} \dfrac{1}{(x+5)(x-3)} & \text{if } x < -3 \\ \ln(12-2x) & \text{if } x \geq 3 \end{cases}$$

d) $f(3) = \ln(12-2\cdot 3)$ rule 2

$= \ln(12-6)$

$= \ln(6)$

e) $f(10) = \ln(12-2\cdot 10) = \ln(-8)$

DNE

16) $f(x) = 2|4-2x|$ as a piecewise function

$$2|4-2x| = \begin{cases} -2(4-2x) & \text{if } \underline{4-2x<0} \\ & (x>2) \\ \underline{2(4-2x)} & \text{if } \underline{4-2x \geq 0} \\ & \underline{(x \leq 2)} \end{cases}$$

should be first

$$4-2x<0 \rightarrow 4<2x$$
$$\rightarrow 2<x$$
$$\rightarrow x>2$$

$$f(x) = \begin{cases} 2(4-2x) & \text{if } x \leq 2 \\ -2(4-2x) & \text{if } x > 2 \end{cases}$$

17) domain of $h(x) = 2^{\sqrt{3-4x}}$

domain of $a^{f(x)}$

= domain of $f(x)$

$\sqrt{3-4x} \rightarrow 3-4x \geq 0$

$-3 \qquad -3$

$\frac{-4x}{-4} \geq \frac{-3}{-4}$

$x \leq 3/4$

$[-\infty, 3/4]$

18) Algebraically solve

$$27 \cdot 9^{2x-1} = 81$$

common base : 3

Approach 1: use lns

$$27 \cdot 9^{2x-1} = 3^3 \cdot (3^2)^{(2x-1)}$$

$(a^b)^c = a^{b \cdot c}$

$$= 3^3 \cdot 3^{2(2x-1)}$$

$9^{2x-1} = 9\wedge(2x-1)$

$$= 3^{3+4x-2} = 3^{4x+1} = \text{"LHS"}$$

$$81 = 3^4$$

$$3^{4x+1} = 3^4 \longrightarrow 4x+1 = 4 \rightarrow 4x = 3 \rightarrow \boxed{x = 3/4}$$

3)