SECTION 2.1: REVIEW OF LINES

- Slope of a line between two points, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\dfrac{\text{rise}}{\text{run}}$
- Equations of a Line,
  - Point-Slope Form: $y - y_1 = m(x - x_1)$ → given m and (x_1, y_1)
  - Slope-Intercept Form: $y = mx + b$ ← $y = mx+b$
  - Standard Form: $Ax + BY = C$ → A, B, C are whole numbers
  - Vertical Line: $x = a$ → slope is undefined
  - Horizontal Line: $y = b$ → slope is 0
- Intercepts of a Line
  - x-intercept: $(x, 0)$
  - y-intercept: $(0, y)$
- Interpreting Change, $m = \dfrac{\Delta y}{\Delta x}$

Pr 1. Write the equation of the line given the slope which passes through the given point in the stated form.

(a) $m = \frac{2}{7}$ and $(-9, 11)$, in point-slope form

don't forget the parentheses

$2(1+1) \neq 2\cdot 1 + 1$

$4 \neq 3$

$$y - y_1 = m(x - x_1)$$

$$y - 11 = \frac{2}{7}(x - (-9))$$

$$\boxed{y - 11 = \frac{2}{7}(x + 9)}$$

(b) $m = -\frac{5}{2}$ and $(4, -7)$, in slope-intercept form

$y = -\frac{5}{2}x + b$, then solve for b.

$$y - (-7) = -\frac{5}{2}(x - 4) = -\frac{5}{2}x - \frac{5}{2}(-4)$$

$$= -\frac{5}{2}x + 5\cdot 2$$

$+7$, -7, -7

$$y = -\frac{5}{2}x + 10 - 7$$

$$\boxed{y = -\frac{5}{2}x + 3}$$

(c) $m = \frac{6}{7}$ and $\left(\frac{7}{2}, 0\right)$, in standard form

$Ax + By = C$

$$y - 0 = \frac{6}{7}\left(x - \frac{7}{2}\right)$$

$$y = \frac{6}{7}x - \frac{\cancel{6}^{3}}{\cancel{7}}\cdot\frac{\cancel{7}}{\cancel{2}}$$

$$y = \frac{6}{7}x - 3 \rightarrow -\frac{6}{7}x + y = -3$$

$-\frac{6}{7}x$ $\quad -\frac{6}{7}x$

$$-6x + 7y = -3\cdot 7$$

$$\boxed{-6x + 7y = -21}$$

(d) $m = 0$ and $(17, 20)$, in standard form

$\frac{\text{rise}}{\text{run}}$ ← no rise

horizontal or vertical → $y = a$ constant

$$\boxed{y = 20}$$

e) m is undefined → vertical line $x = 17$

Pr 2. Write the equation of the line that passes through the given pair of points, in the stated form.

we need $m = \frac{y_2 - y_1}{x_2 - x_1}$

(a) $(2, -5)$ and $(-9, 11)$ in point-slope form

$y - y_1 = m(x - x_1)$

$m = \frac{\Delta y}{\Delta x} = \frac{11 - (-5)}{-9 - 2} = \frac{11 + 5}{-9 - 2} = \frac{16}{-11} = -\frac{16}{11}$

$\boxed{y - (-5) = -\frac{16}{11}(x - 2)}$ or $\boxed{y - 11 = -\frac{16}{11}(x - (-9))}$

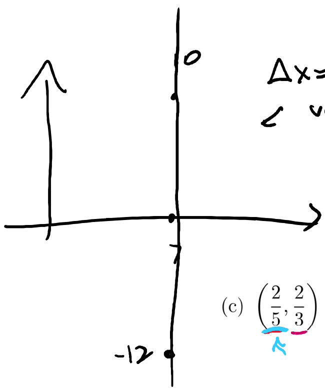
(b) $(7, 10)$ and $(7, -12)$ in slope-intercept form

$y = mx + b$

$m = \frac{-12 - 10}{7 - 7} = \frac{-22}{0}$? There is no slope

$\Delta x = 0$

vertical



The slope-intercept form does not exist

standard form $\rightarrow$ $x = 7$

(c) $\left(\frac{2}{5}, \frac{2}{3}\right)$ and $\left(\frac{2}{5}, -\frac{7}{11}\right)$ in standard form

$m = \frac{-\frac{7}{11} - \frac{2}{3}}{\frac{2}{5} - \frac{2}{5}} \rightarrow 0$

$-\frac{7}{11} - \frac{2}{3} = \frac{3 \cdot (-7)}{3 \cdot 11} - \frac{2 \cdot 11}{3 \cdot 11}$

$= \frac{-21 - 22}{33}$

$=$ messy

$= \frac{-43}{33}$

m is undefined

vertical line

$\boxed{x = \frac{2}{5}}$

$Ax + By = C$

$A = 1, \; B = 0, \; C = \frac{2}{5}$

$1x = x$ $\qquad x + 0y = \frac{2}{5}$

$x + 0 = 2/5$

$x = 2/5$

(d) intersects the y-axis at $y = 7$ and the x-axis at $x = -6$ in standard form

y-intercept $(0, 7)$ $\qquad$ x-intercept $(-6, 0)$

$Ax + By = C$

$m = \frac{0 - 7}{-6 - 0} = \frac{-7}{-6} = \frac{7}{6}$

$y = \frac{7}{6}x + b$ $\rightarrow$ y-coordinate of y-intercept

$y = \frac{7}{6}x + 7$

$-\frac{7}{6}x \qquad -\frac{7}{6}x$

$\frac{-7}{6}x + y = 7$ $\qquad$ (multiply through by 6)

$\cancel{6}\frac{-7}{\cancel{6}}x + 6y = 6 \cdot 7$

$\boxed{-7x + 6y = 42}$

$\boxed{7x - 6y = -42}$

Pr 3. Determine the slope, and the x- and y-intercepts without graphing. Write the coordinates of each intercept. Then use the points to graph each line.

line1 → (a) $5x - 6y = 30$

Standard form

$m = \frac{5}{6}$

$5x - 6y = 30$ → convert from standard form to slope-intercept

$-5x \quad -5x$

$\frac{-6y}{-6} = \frac{-5x + 30}{-6}$

$y = \frac{-5}{-6}x + \frac{\cancel{30}^{5}}{\cancel{-6}} = \frac{5}{6}x - 5$

Approach1: y-intercept: $(0,-5)$

Approach 2: set $x=0$

$5\cdot(0) - 6y = 30$

x-intercept: set $y=0$ $(a,0) = \boxed{(6,0)}$

$5x - 6(0) = 30$

$5x - 0 = 30$

$\frac{5x}{5} = \frac{30}{5} \rightarrow x = 6$

line2 → (b) $\frac{2}{3}y = -\frac{2}{3}x + 12$

$\frac{2}{3}y = -\frac{2}{3}x + 12$

$m \neq -\frac{2}{3}$

$\frac{3}{2}\frac{2}{3}y = \frac{3}{2}\left(-\frac{2}{3}x + 12\right)$

$y = -\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}x + \frac{3}{\cancel{2}}\cdot\cancel{12}^{6}$

$y = -1\cdot x + 18$ → y-intercept $\boxed{(0, 18)}$

$m = -1$

$\frac{b}{a}\cdot\frac{a}{b}x = c\cdot\frac{b}{a}$

$x = \frac{b}{a}\cdot c$

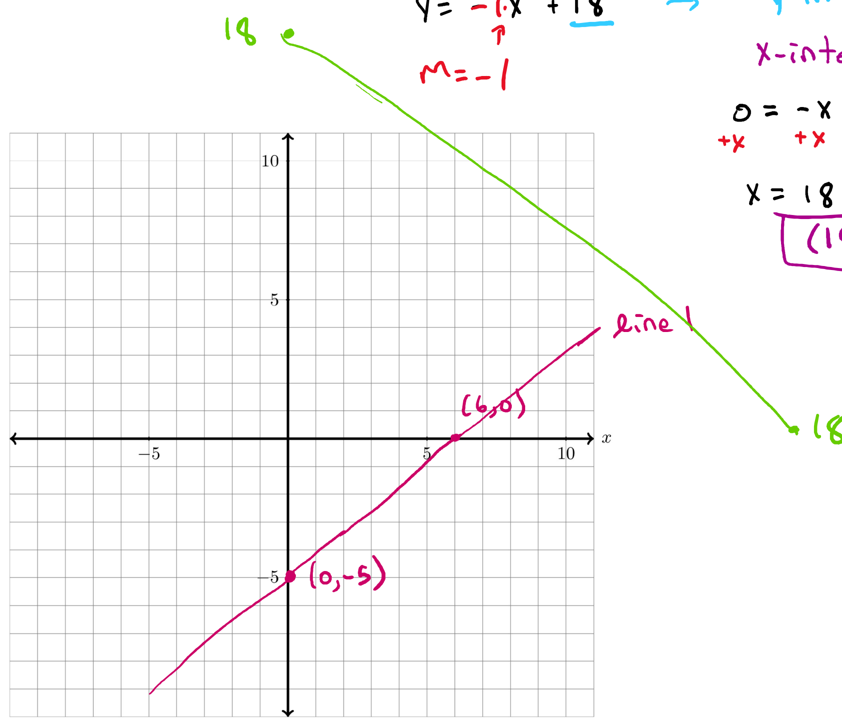
x-intercept: set $y=0$

$0 = -x + 18$

$+x \quad +x$

$x = 18$ not x-intercept

$\boxed{(18,0)}$



find slope, x - and y- intercepts

(c) $x = -4$

$x = c$ → vertical → $(c, 0)$

$\boxed{\text{slope is undefined}}$ (DNE for "does not exist" in WebAssign

x- intercept: set $y=0$

$x=-4$ (no y variable)

$\boxed{(-4, 0)}$

y- intercept: set $x=0$ → $0=-4$

$\boxed{\text{No y-intercept}}$

(d) $y = 7$

↓

horizontal line

$y = 0x + 7$

$\boxed{m=0}$

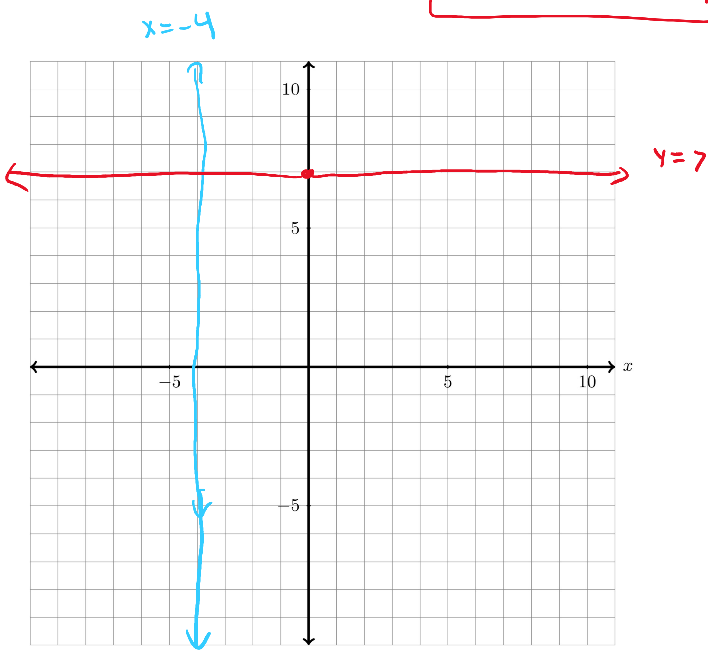
$y = c$

y-intercept: set $x=0$, $y=7$

$\boxed{(0,7)}$

$(0, c)$

$\boxed{\text{No x-intercept}}$ → set $y=0$, $0=7$?



standard form. Δx

Pr 4. (a) Given the line $3x - 2y = -8$, if x increases by 3 units, what is the corresponding change in y? Δy

$m = \dfrac{\Delta y}{\Delta x}$

what is m?

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{\Delta y}{+3}$

$3x - 2y = -8$
$-3x \qquad -3x$

$-2y = -3x - 8$
$\overline{-2} \qquad \overline{-2}$

$y = \dfrac{-3}{-2}x - \dfrac{\cancel{8}\,4}{\cancel{-2}} = \dfrac{3}{2}x + 4$

$m = \dfrac{3}{2} = \dfrac{\Delta y}{3}$

$\dfrac{3}{2} \times \dfrac{\Delta y}{3}$

$3 \cdot \dfrac{\Delta y}{\cancel{3}} = \dfrac{3}{2} \cdot 3$

$\dfrac{3 \cdot 3}{2} = \dfrac{2 \cdot \Delta y}{2}$

$\Delta y = \dfrac{3}{2} \cdot 3 = \dfrac{9}{2}$

$\left(\dfrac{-a}{-b} = \dfrac{a}{b}\right)$

$\boxed{y \text{ increases by } 9/2 \text{ units}}$

(b) Given the line $y = \frac{1}{2}x + 4$, if x decreases by 7 units, what is the corresponding change in y?

$\Delta x = -7$ $\qquad \Delta y$

$\dfrac{1}{2} = m = \dfrac{\Delta y}{\Delta x} = \dfrac{\Delta y}{-7}$

$\dfrac{1}{2} \times \dfrac{\Delta y}{-7}$

$1 \cdot (-7) = 2 \cdot \Delta y$

$\dfrac{-7}{2} = \dfrac{2\Delta y}{2}$

$\Delta y = -7/2 \qquad y$ decreases by $+7/2$ units

point-slope

(c) Given the line $y = 3(x + 2) - 5$,If y decreases by 9 units, what is the corresponding change in x?

$\Delta y = -9$ $\qquad$ solve for Δx

$3 = m = \dfrac{\Delta y}{\Delta x} = \dfrac{-9}{\Delta x}$

$\dfrac{-9}{\Delta x} = \dfrac{3}{1} \rightarrow -9 \cdot 1 = 3 \cdot \Delta x$

$\dfrac{3\Delta x}{3} = \dfrac{-9}{3}$

or

$\cancel{\Delta x}\,\dfrac{-9}{\cancel{\Delta x}} = 3\,\Delta x$

$\Delta x = -9/3 = -3$

$\boxed{x \text{ decreases by } 3 \text{ units}}$

$m > 0 \longrightarrow \Delta x > 0$ if and only if $\Delta y > 0$
$m < 0 \longrightarrow \Delta x > 0$ if and only if $\Delta y < 0$

SECTION 2.2: MODELING WITH LINEAR FUNCTIONS

- Linear Depreciation, $V(t) = mt + b$ variable is t, in years after purchase
- Cost, variable cost + fixed costs $C(x) = mx + F$ m = production cost
- Revenue, price per item times ~~quantity sold~~ $R(x) = px$ lower case p
- Profit, revenue minus cost $P(x) = R(x) - C(x)$
- Demand, $D(x) = p(x) = mx + b$
- Supply, $S(x) = p(x) = mx + b$

time t (the "x" variable) the independ. Variable

Pr 1. A piece of machinery is purchased. After 15 months, it has a value of of \$225,000 and that same machinery has a value of \$165,000 after 5 years.

(a) Assuming the value of the machinery depreciates at a constant rate each year, determine the rate of depreciation.

rate of depreciation = $|m|$ $m<0$

$\frac{15}{12} = \frac{5\cdot 3}{4\cdot 3} = \frac{5}{4} = 1.25$

$\cancel{(15, 225000)}$ this gives $m>0$

$(5, 165000)$

$\left(\frac{15}{12}, 225000\right) = (1.25, 225000)$

$m = \frac{165000 - 225000}{5 - 1.25} = \frac{-60000}{3.75} = \boxed{-16000}$ → slope

$\boxed{\$16,000}$

(b) Write the linear depreciation model for the value of the machinery, V, after t years.

$V(t) = mt + b$

$m = -16000$

$V(t) - 165000 = -16000(t-5)$

$V(t) = -16000(t-5) + 165000$

$= -16000t + 16000 \times 5 + 165000 = \boxed{V(t) = -16000t + 245000}$

(c) What is the initial value of the machinery?

$V(0) = m\cdot 0 + b = b$

$V(0) = \$245,000$

$165000 = V(5) = -16000(5) + b$ and solve for b

(d) If the machinery reaches scrap value in 15 years, what is the scrap value of the machinery?

$V(15)$ is the scrap value

$V(15) = -16000(15) + 245000$

$= -240000 + 245000$

$= \boxed{\$5,000}$

$\boxed{\text{The scrap value is \$5,000}}$

$V(16) = 5,000$

cost / revenue / profit   Profit = revenue − cost

Pr 2. Ted runs a food truck that sells gyros. The cost of maintaining the food truck is \$255 per week. The stand makes a profit of \$145 when 50 gyros are sold in a week. If only 20 gyros are sold, Ted knows the total cost for that week is \$295.

profit

cost

(a) Write the cost function for producing x gyros at Ted's food truck (per week).

\$255 − fixed cost

$C(x) = mx + F$ $F = 255$

cost $(20, 295)$ — quantity, dollars

Find m

$295 = C(20) = m(20) + 255$

$295 = 20m + 255$

$-255 \quad -255$

$40 = 20m$

$\frac{40}{20} = \frac{20m}{20}$

$m = \$2$ ← production cost

$\boxed{C(x) = 2x + 255}$

(b) Write the revenue function for the sale of x gyros at Ted's food truck.

$(50, 145)$ — cost to make 50 years ≠ 145

$R(x) = mx \ (+0)$ ← given revenue of p dollars for x items

Profit = Revenue − cost

Profit + cost = Revenue

$P(50) = \$145$ $C(50) = ?$

$C(50) = 2 \cdot (50) + 255$

$= 100 + 255$

$= 355$

not revenue function

$R(50) = 145 + 355 = 500$ → ~~Rt~~ $(50, 500)$

$500 = R(50) = p \cdot 50$

$\frac{500}{50} = \frac{50p}{50}$

$p = 10$, p = sales price

$\boxed{R(x) = 10x}$

(c) Write the profit function for producing and selling x gyros.

Approach I: $P(x) = R(x) - C(x)$

$= 10x - (2x + 255)$ ← don't forget ( )

$= 10x - 2x - 255$

$\boxed{P(x) = 8x - 255}$

$C(x) = mx + F$ — both positive

$P(x) = mx + b$ — positive, negative

supplier Supply / demand y- coordinate is always money

Pr 3. Mimtando is a video game company that decides to make a new console, and GameStomp decides to carry it. Mimtando will supply 300 thousand consoles to GameStomp if the sales price of the console is \$200. If the sales price increases by \$50, then Mimtando will supply 25 thousand more consoles. Consumers will not buy the console at all if the price is \$400, but will buy 600 thousand consoles if the price is \$250.

supply - side

demand

$S(x) = mx + b$

positive

(a) Write the demand function for consumers demanding x thousand consoles at a price of p dollars.

$D(x) = mx + b$

$(600, \$250)$

$(0, \$400)$

y- intercept

$m = \frac{400 - 250}{0 - 600} = \frac{150}{-600} = -\frac{150}{600} = -\frac{1}{4}$

$D(x) = mx + b$

negative

$$\boxed{D(x) = -\frac{1}{4}x + 400}$$

x is thousands of consoles

$D(x) - y_1 = -\frac{1}{4}(x - x_1) \ldots$

y is dollars..

(b) Write the supply function for Mimtando to provide x thousand consoles when the sales price of the console is p dollars.

$(300, \$200)$

X $\cancel{(25, \$50)}$ → increase price by \$50, → Δp

supply increases by 25 → Δx

not a point

$m = \frac{\Delta p}{\Delta x}$ ← money always on top

$m = \frac{50}{25} = 2$ ✓

$S(x) = mx + b$

$S(x) - \$200 = 2(x - 300)$

$S(x) = 2(x - 300) + 200$

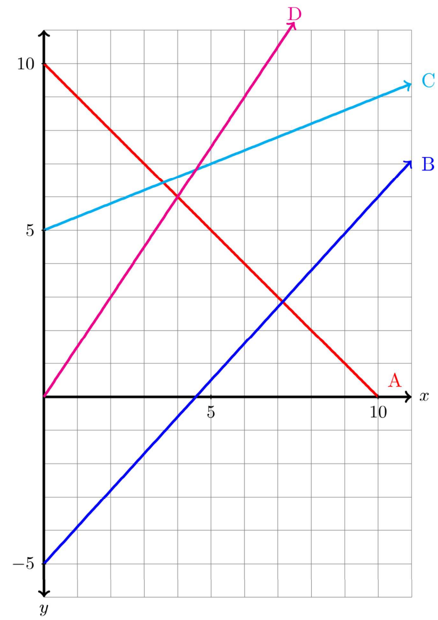
$= 2x - 2 \cdot 300 + 200$

$= 2x - 600 + 200$

$= 2x - 400$

$$\boxed{S(x) = 2x - 400}$$

Pr 4. Which of the following lines graphed below could be the graphs of a supply, demand, cost, revenue, or profit function? Explain your answer.



(a) Lines that could be graphs of Cost functions:

(b) Lines that could be graphs of Revenue functions:

(c) Lines that could be graphs of Profit functions:

(d) Lines that could be graphs of demand functions:

(e) Lines that could be graphs of supply functions: