Week in Review
Math 152

Week 05

Trigonometric Integrals

Trigonometric Substitution

Integration by Partial Fractions

Trigonometric Integrals

Compute $\int_0^{\pi/2} \sin^2(\theta)\cos^3(\theta)\, d\theta$.

(a) $\frac{2}{5}$

(b) $\frac{4}{5}$

(c) $\frac{2}{15}$

(d) $\frac{8}{5}$

(e) None of the above

Trigonometric Integrals

Compute $\int \cos^2(x)\sin^2(x)\ dx$

Trigonometric Integrals

The region bounded by $y = \cos x$ and the x-axis on the interval $\left[0, \frac{\pi}{2}\right]$ is rotated about the x-axis. Find the volume of the resulting solid.

(a) $\frac{\pi}{4}$

(b) $\frac{\pi^2}{4}$

(c) 1

(d) $\frac{\pi^2}{2}$

(e) $\frac{\pi}{2}$

Trigonometric Integrals

Which of the following is equal to $\int_0^{\pi/4} \tan^2(\theta)\sec^4(\theta)\,d\theta$?

(a) $\int_0^{\pi/4} u^2(u^2-1)\,du$

(b) $\int_0^{\pi/4} u^2(1+u^2)\,du$

(c) $\int_0^{\sqrt{2}/2} u^2(1+u^2)\,du$

(d) $\int_0^{1} u^2(u^2-1)\,du$

(e) $\int_0^{1} u^2(1+u^2)\,du$

Trigonometric Integrals

Compute $\int \tan^3(x)\sec^3(x)\; dx$

(a) $\frac{1}{5}\sec^5(x) - \frac{1}{3}\sec^3(x) + C$

(b) $-\frac{1}{5}\sec^5(x) + \frac{1}{3}\sec^3(x) + C$

(c) $\frac{1}{5}\tan^5(x) - \frac{1}{3}\tan^3(x) + C$

(d) $-\frac{1}{5}\tan^5(x) + \frac{1}{3}\tan^3(x) + C$

(e) $-\sec^4(x) + \sec^2(x) + C$

Trigonometric Integrals (Exercise)

Compute $\int \cos^4(x)\sin^5(x)\;dx$

(a) $-\frac{1}{5}\cos^5(x)+\frac{1}{9}\cos^9(x)+C$

(b) $\frac{1}{6}\sin^6(x)-\frac{1}{4}\sin^8(x)+\frac{1}{10}\sin^{10}(x)+C$

(c) $\frac{1}{6}\sin^6(x)-\frac{1}{10}\sin^{10}(x)+C$

(d) $-\frac{1}{5}\cos^5(x)+\frac{2}{7}\cos^7(x)-\frac{1}{9}\cos^9(x)+C$

(e) None of these.

Compute $\int \cos^3(2x)\;dx$

(a) $-\sin(2x)+\frac{1}{3}\sin^3(2x)+C$

(b) $\frac{-1}{2}\sin(2x)+\frac{1}{6}\cos^3(2x)+C$

(c) None of these.

(d) $\sin(2x)-\frac{1}{3}\sin^3(2x)+C$

(e) $\frac{1}{2}\sin(2x)-\frac{1}{6}\sin^3(2x)+C$

Evaluate $\int \tan^3(x)\sec^5(x)\;dx.$

(a) $\frac{1}{7}\tan^7 x-\frac{1}{5}\sec^5 x+C$

(b) $\frac{1}{7}\sec^7 x-\frac{1}{5}\tan^5 x+C$

(c) $\frac{1}{4}\sec^6 x-\frac{1}{6}\tan^{10} x+C$

(d) $\frac{1}{4}\sec^4 x-\frac{1}{6}\tan^6 x+C$

(e) $\frac{1}{7}\sec^7 x-\frac{1}{5}\sec^5 x+C$

Compute $\int_0^{\pi/4} \sec^4(x)\;dx$

(a) $\frac{2}{3}$

(b) $\frac{32}{5}$

(c) $\frac{4}{3}$

(d) $\frac{4\sqrt{2}}{5}$

(e) $\frac{2\sqrt{2}-1}{3}$

Trigonometric Integrals (Exercise)

Evaluate $\int \sin^2(x)\,dx$.

(a) $\frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
(b) $\frac{1}{3}\sin x \cos x + C$
(c) $\frac{1}{2}x + \frac{1}{2}\sin x + C$
(d) $\frac{1}{2}x - \frac{1}{4}\sin x + C$
(e) $\frac{1}{3}\cos^3 x + C$

Compute $\int \sin^7\theta \cos^5\theta\, d\theta$.

Compute $\int \cos^2(x)\sin^2(x)\,dx$

Find $\int \cos^4 x\,dx$

Trigonometric Integrals (Exercise)

Compute $\int 2\sin^2(2\theta)\,d\theta$

(a) $\theta - \frac{1}{2}\sin(2\theta) + C$

(b) $\theta - \frac{1}{4}\sin(4\theta) + C$

(c) $\theta + \frac{1}{2}\sin(2\theta) + C$

(d) $\theta + \frac{1}{4}\sin(4\theta) + C$

(e) None of the above

Compute $\int_0^{\pi/3} \tan^3(\theta)\sec(\theta)\,d\theta$

(a) $\frac{4}{3}$

(b) $\frac{16-9\sqrt{3}}{24}$

(c) $\frac{2}{3}$

(d) $\frac{-3\sqrt{3}}{8}$

(e) None of the above

$\int_0^{\pi/2} \cos^3 x \sin^3 x\,dx =$

(a) $\frac{1}{12}$

(b) $\frac{2}{15}$

(c) $\frac{5}{12}$

(d) $\frac{-1}{12}$

(e) $\frac{-2}{15}$

$\int \tan^4 x \sec^4 x\,dx =$

(a) $\frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$

(b) $\frac{\tan^7 x}{7} - \frac{\tan^5 x}{5} + C$

(c) $\frac{\tan^9 x}{9} + \frac{\tan^5 x}{5} + C$

(d) $\frac{\tan^9 x}{9} - \frac{\tan^5 x}{5} + C$

(e) None of these

Trigonometric Integrals (Exercise)

Compute $\int_0^{\pi/2} \sin(2x)\cos x \; dx$.

(a) $\frac{3}{2}$

(b) $\frac{2}{3}$

(c) 0

(d) 1

(e) $\frac{1}{2}$

$\int_0^{\pi/4} \sin^2(x)\, dx =$

(a) $\dfrac{\pi}{8} - \dfrac{1}{4}$

(b) $\dfrac{\pi}{8}$

(c) $\dfrac{\pi}{8} - \dfrac{1}{2}$

(d) $\dfrac{2}{\sqrt{2}} - \dfrac{\pi}{2\sqrt{2}}$

(e) $\dfrac{\pi}{8} + \dfrac{1}{4}$

Trigonometric Substitution

Evaluate $\displaystyle\int \frac{1+x}{1+x^2}\,dx$.

(a) $\frac{1}{2}\ln(1+x^2)+C$

(b) $\frac{3}{2}\ln(1+x^2)+C$

(c) $\ln(1+x^2)+C$

(d) $\frac{1}{2}\ln(1+x^2)+\arctan x+C$

(e) $\arctan x+\arcsin(x^2)+C$

Trigonometric Substitution

After an appropriate substitution, the integral $\int \sqrt{x^2 + x}\, dx$ is equivalent to which of the following?

(a) $\int \tan^2\theta \sec\theta \, d\theta$

(b) $\frac{1}{4}\int \sec^3\theta \, d\theta$

(c) $-\frac{1}{4}\int \sin^2\theta \, d\theta$

(d) $\frac{1}{4}\int \tan^2\theta \sec\theta \, d\theta$

(e) $\int \cos^2\theta \, d\theta$

Trigonometric Substitution

Compute the following integral showing all necessary work clearly.

$$\int \frac{1}{(x^2+9)^{5/2}}\,dx$$

Trigonometric Substitution

Which of these substitutions would be used to evaluate $\int x^2\sqrt{x^2+4x+13}\,dx$?

(a) $x+4=\sqrt{13}\sec\theta$

(b) $x+2=3\tan\theta$

(c) $x^2+4x=\sqrt{13}\tan\theta$

(d) none of these.

(e) $x+2=3\sec\theta$

Trigonometric Substitution

After an appropriate trigonometric substitution,

$$\int_{2\sqrt{2}}^{4} \frac{\sqrt{x^2-4}}{x}\,dx \text{ is equivalent to}$$

(a) $2\displaystyle\int_{\pi/4}^{\pi/3} \tan^2\theta\,d\theta$

(b) $\displaystyle\int_{\pi/4}^{\pi/3} \sin(\theta)\,d\theta$

(c) $2\displaystyle\int_{\pi/4}^{\pi/6} \tan^2\theta\,d\theta$

(d) $\displaystyle\int_{\pi/4}^{\pi/6} \sin(\theta)\,d\theta$

(e) None of the above

Trigonometric Substitution (Exercise)

After an appropriate substitution, the integral $\int x^2\sqrt{9-x^2}\,dx$ is equivalent to which of the following?

(a) $9\int \cos^2\theta\,d\theta$

(b) $81\int \sin^2\theta\cos^2\theta d\theta$

(c) $27\int \sin^2\theta\cos\theta\,d\theta$

(d) $81\int \sec^3\theta\tan^2\theta\,d\theta$

(e) $27\int \sec^2\theta\tan\theta\,d\theta$

Which of the following is an appropriate substitution to use when solving the integral $\int\sqrt{16x^2-9}\,dx$?

(a) $x=\frac{3}{4}\sin\theta$

(b) $x=\frac{4}{3}\sec\theta$

(c) $x=\frac{4}{3}\sin\theta$

(d) $x=\frac{3}{4}\sec\theta$

(e) $x=\frac{3}{4}\tan\theta$

Compute $\displaystyle\int_0^4 \frac{x+2}{x^2+4}\,dx$.

(a) $\frac{1}{2}(\ln 20-\ln 4)+\arctan(2)$

(b) $\ln 6-\ln 2$

(c) $\ln 20-\ln 4$

(d) $\frac{1}{2}(\ln 20-\ln 4)+2\arctan(4)$

(e) $\ln 20-\ln 4+2\arctan(4)$

Which of the following integrals is equivalent to $\int\sqrt{4x^2-9}\,dx$?

(a) $2\int \sec\theta\tan^2\theta\,d\theta$

(b) $\frac{9}{2}\int \tan\theta\,d\theta$

(c) $\frac{9}{2}\int \sec\theta\tan^2\theta\,d\theta$

(d) $\frac{9}{2}\int \sec^2\theta\tan\theta\,d\theta$

(e) $2\int \sec^2\theta\tan\theta\,d\theta$

Trigonometric Substitution (Exercise)

Compute $\int \frac{1}{x^4\sqrt{x^2-4}}\,dx$. In your final answer, any trig or inverse trig expressions that can be rewritten algebraically must be.

After an appropriate trigonometric substitution,

$\int \frac{dx}{\sqrt{x^2+8x+41}}$ is equivalent to which of the following?

(a) $\frac{1}{5}\int \cos(\theta)\,d\theta$

(b) $\int \sec(\theta)\,d\theta$

(c) $\int \sec^2(\theta)\,d\theta$

(d) $\int \tan(\theta)\,d\theta$

(e) $\frac{1}{5}\int \sin(\theta)\,d\theta$

After an appropriate trigonometric substitution,

$\int_{2\sqrt{2}}^{4} \frac{\sqrt{x^2-4}}{x}\,dx$ is equivalent to

(a) $2\int_{\pi/4}^{\pi/3} \tan^2\theta\,d\theta$

(b) $\int_{\pi/4}^{\pi/3} \sin(\theta)\,d\theta$

(c) $2\int_{\pi/4}^{\pi/6} \tan^2\theta\,d\theta$

(d) $\int_{\pi/4}^{\pi/6} \sin(\theta)\,d\theta$

(e) None of the above

Trigonometric Substitution (Exercise)

After an appropriate substitution, the integral $\int \sqrt{9-x^2}\,dx$ is equivalent to which of the following?

(a) $9\int \sec\theta\tan^2\theta\,d\theta$

(b) $3\int \cos\theta\,d\theta$

(c) $9\int \sec^3\theta\,d\theta$

(d) $9\int \cos^2\theta\,d\theta$

(e) $3\int \tan\theta\,d\theta$

Which of the following integrals is equivalent to $\int \frac{1}{(x^2-4x+5)^{3/2}}\,dx$?

(a) $\frac{1}{9}\int \cos\theta\,d\theta$

(b) $\int \cos^3\theta\,d\theta$

(c) $\frac{1}{27}\int \cos^3\theta\,d\theta$

(d) $\int \sec\theta\,d\theta$

(e) $\int \cos\theta\,d\theta$

If we use the appropriate trigonometric substitution to evaluate $\int_1^{2/\sqrt{3}} \left(\frac{\sqrt{x^2-1}}{x}\right) dx$, which of the following is the correct result?

(a) $\int_0^{\pi/6} \tan^2\theta\,d\theta$

(b) $\int_0^{\pi/6} \frac{\tan\theta}{\sec\theta}\,d\theta$

(c) $\int_0^{\pi/3} \tan^2\theta\,d\theta$

(d) $\int_{\pi/2}^{\pi/6} \sin^2\theta d\theta$

(e) $\int_{\pi/2}^{\pi/3} \sin^2\theta d\theta$

Trigonometric Substitution (Exercise)

Find $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

Evaluate $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$.

Integration by Partial Fractions

Which of the following is the form of the partial-fraction decomposition for the rational function?

$$\frac{1}{(x+1)(x^2-2x-3)(x^2-2x+2)}$$

(a) $\frac{A}{x+1} + \frac{Bx+C}{x^2-2x-3} + \frac{Dx+E}{x^2-2x+2}$

(b) $\frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{D}{x-3} + \frac{Ex+F}{x^2-2x+2}$

(c) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{D}{x^2-2x+2}$

(d) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2-2x+2}$

(e) $\frac{A}{x+1} + \frac{B}{x^2-2x-3} + \frac{C}{x^2-2x+2}$

$$\frac{1}{(x+1)(x-3)(x+1)(x^2-2x+2)}$$

$$= \frac{1}{(x+1)^2(x-3)(x^2-2x+2)}$$

$$= \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2-2x+2}$$

Integration by Partial Fractions

Compute the following integral showing all necessary work clearly.

$$\int \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)}\,dx$$

Integration by Partial Fractions

$$\int \frac{x^3+x}{x-1}\,dx =$$

(a) $\dfrac{x^3}{3}+\dfrac{x^2}{2}+2x-2\ln|x-1|+C$

(b) $\dfrac{x^3}{3}-\dfrac{x^2}{2}+2x+2\ln|x-1|+C$

(c) $\dfrac{x^3}{3}+\dfrac{x^2}{2}+2x+2\ln|x-1|+C$

(d) $\dfrac{x^3}{3}+\dfrac{x^2}{2}-2x+2\ln|x-1|+C$

(e) $\dfrac{x^3}{3}-\dfrac{x^2}{2}+2x-2\ln|x-1|+C$

Integration by Partial Fractions

Find $\displaystyle\int \frac{x+2}{x^2(x^2+1)}\,dx$

Integration by Partial Fractions (Exercise)

Write out the form of the partial fraction decomposition of the function

$$f(x) = \frac{x^3 - 2x^2 - 5x + 4}{(x+2)^2(x^2-1)(x^2+5x+7)}$$

(a) $\dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{x-1} + \dfrac{D}{x+1} + \dfrac{Ex+F}{x^2+5x+7}$

(b) $\dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{Cx+D}{x^2-1} + \dfrac{Ex+F}{x^2+5x+7}$

(c) $\dfrac{A}{(x+2)^2} + \dfrac{B}{x-1} + \dfrac{C}{x+1} + \dfrac{Dx+E}{x^2+5x+7}$

(d) $\dfrac{A}{(x+2)^2} + \dfrac{B}{x^2-1} + \dfrac{Cx+D}{x^2+5x+7}$

(e) $\dfrac{Ax+B}{(x+2)^2} + \dfrac{Cx+D}{x^2-1} + \dfrac{Ex+F}{x^2+5x+7}$

Integration by Partial Fractions (Exercise)

Which of the following is a proper Partial Fraction Decomposition for $\dfrac{x+1}{(x^2-16)(x-3)^2(x^2+1)}$?

(a) $\dfrac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \dfrac{Ax+B}{x^2-16} + \dfrac{C}{x-3} + \dfrac{D}{(x-3)^2} + \dfrac{Ex+F}{x^2+1}$

(b) $\dfrac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \dfrac{A}{x-4} + \dfrac{B}{x+4} + \dfrac{C}{x-3} + \dfrac{D}{(x-3)^2} + \dfrac{Ex+F}{x^2+1}$

(c) $\dfrac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \dfrac{Ax+B}{x^2-16} + \dfrac{Cx+D}{(x-3)^2} + \dfrac{Ex+F}{x^2+1}$

(d) $\dfrac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \dfrac{A}{x-4} + \dfrac{B}{x+4} + \dfrac{C}{x-3} + \dfrac{D}{(x-3)^2} + \dfrac{E}{x^2+1}$

(e) $\dfrac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \dfrac{A}{x-4} + \dfrac{B}{(x-4)^2} + \dfrac{C}{x-3} + \dfrac{D}{(x-3)^2} + \dfrac{Ex+F}{x^2+1}$

Which of the following is a proper Partial Fraction Decomposition for the rational function

$$\frac{5x+1}{(x+3)(x^2+4x+3)(x^2+4)}$$

(a) $\dfrac{A}{x+3} + \dfrac{B}{(x+3)^2} + \dfrac{C}{x+1} + \dfrac{Dx+E}{x^2+4}$

(b) $\dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+4x+3} + \dfrac{Dx+E}{x^2+4}$

(c) $\dfrac{A}{x+3} + \dfrac{Bx+C}{(x+3)^2} + \dfrac{D}{x+1} + \dfrac{Ex+F}{x^2+4}$

(d) $\dfrac{A}{x+3} + \dfrac{B}{(x+3)^2} + \dfrac{C}{x+1} + \dfrac{D}{x+2} + \dfrac{D}{x-2}$

(e) None of these.

Integration by Partial Fractions (Exercise)

Compute $\displaystyle\int \frac{2x^2+5x-5}{(x+1)(x+3)^2}\,dx$

Evaluate $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)}\,dx$

Evaluate $\displaystyle\int_0^1 \frac{4x^2+5}{2x+1}\,dx$

(a) $2\ln 3$

(b) $3\ln 3$

(c) $4\ln 3$

(d) $6\ln 3$

(e) None of these

Find $\displaystyle\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}\,dx$ showing all necessary work.