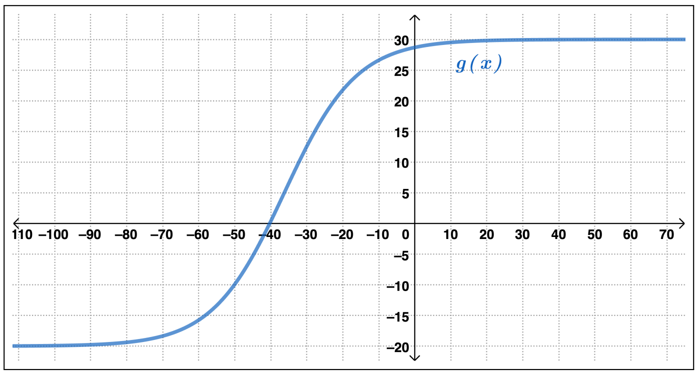
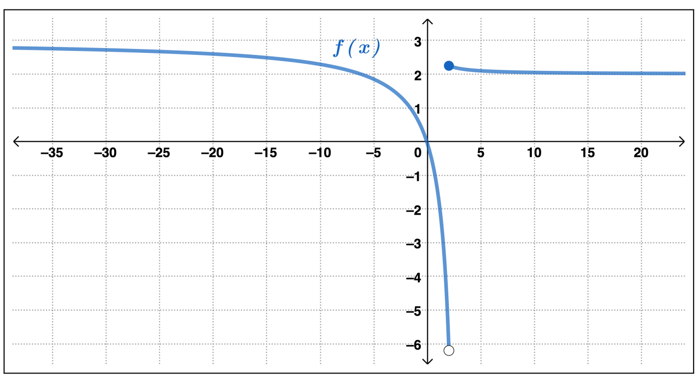
Session 2: Sections 1-3 and 1-4

1. Use the graphs below to estimate the given limits.

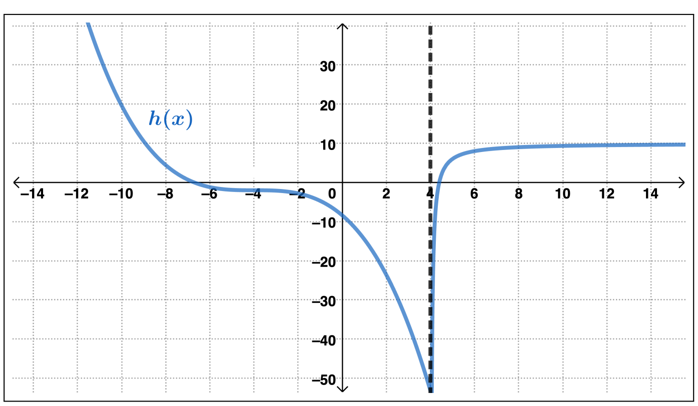
   (a) Estimate $\lim_{x\to\infty} g(x)$ and $\lim_{x\to-\infty} g(x)$. If a limit does not exist because the function has infinite behavior, use limit notation to describe the infinite behavior.



   (b) Estimate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. If a limit does not exist because the function has infinite behavior, use limit notation to describe the infinite behavior.



   (c) Estimate $\lim_{x\to\infty} h(x)$ and $\lim_{x\to-\infty} h(x)$. If a limit does not exist because the function has infinite behavior, use limit notation to describe the infinite behavior.



Finding Limits at Infinity Algebraically

- **Rational Functions:** To find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, for rational functions, we divide both the numerator and denominator by the term in the denominator with the highest power of x. Then simplify the function and use the fact that if n is a positive integer, then $\lim_{x\to\infty} \frac{1}{x^n} = 0$ and $\lim_{x\to-\infty} \frac{1}{x^n} = 0$.

- **Fractions containing Exponential Functions:** Use the fact that if n is a positive integer, then
  (a) $\lim_{x\to\infty} e^{nx} \to \infty$ and $\lim_{x\to-\infty} e^{nx} = 0$.
  (b) $\lim_{x\to\infty} e^{-nx} = 0$ and $\lim_{x\to-\infty} e^{-nx} \to \infty$.

2. Use algebric methods to find $\lim_{x\to\infty} \frac{4x^3 - 8x^{10} + 4x^6}{8x^2 - 7x + 5}$

3. Use algebraic methods to find $\lim_{x\to-\infty} \frac{x^2 - x - 72}{18 + 4x^2 - 38x}$

4. Use algebric methods to find $\lim_{x\to\infty} \dfrac{5e^{7x} - 4e^{-2x}}{e^{-2x} + 9e^{8x}}$

Method for Determining Holes and Vertical Asymptotes of Rational Functions

(a) Factor the numerator and denominator. Divide any common factors.

(b) The factors in the denominator that *divide completely* will determine the holes. Set each factor in the denominator that divides completely equal to zero to find the x-value of the **hole**.

(c) The factors that *remain in the denominator* (and denominator only!) will determine the vertical asymptotes. Again, set each factor that remains in the denominator equal to zero to find the x-value of the **vertical asymptote**.

5. Find any (a) horizontal asymptotes, (b) holes, and (c) vertical asymptotes of the functions given below. If there are no horizontal asymptotes, describe he end behavior using limit notation. For each vertical asymptote, describe the infinite behavior using limit notation.

   (a) $f(x) = \dfrac{(x+8)(x-9)}{(x-9)(4x-2)} = \dfrac{x^2 - x - 72}{4x^2 - 38x + 18}$

(b) $f(x) = \dfrac{(2x+7)^2(x-4)}{(x+3)(2x+7)} = \dfrac{4x^3+12x^2-63x-196}{2x^2+13x+21}$

(c) $f(x) = \dfrac{(5x-17)(x+4)}{(3x-8)(x+4)^2} = \dfrac{5x^2+3x-68}{3x^3+16x^2-16x-128}$

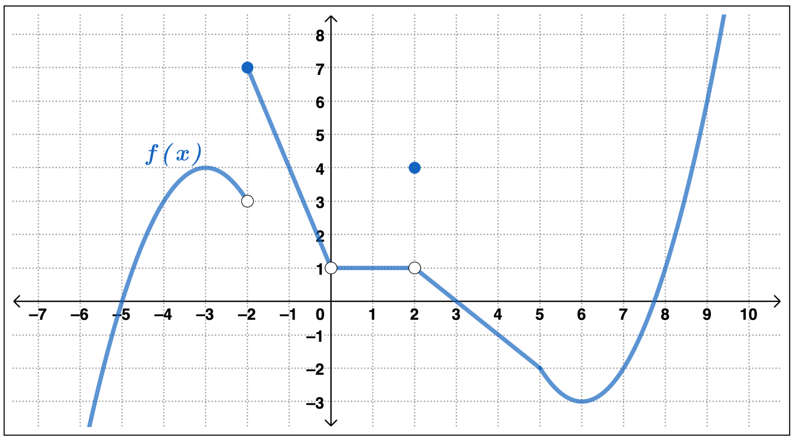
A function f is **continuous at a point** where $x = c$ if and only if the following three conditions are satisfied:

I. $f(c)$ is defined.

II. $\lim_{x \to c} f(x) =$ exists.

III. $\lim_{x \to c} f(x) = f(c)$.

6. Use the graph of f to determine the x-value(s) where f is discontinuous. State the condition in the definition of continuity at a point that fails first at each x-value.



7. Determine if the functions below are continuous at the given value of c. If the function is not continuous at $x = c$, also state the condition in the definition of continuity at a point that fails first mathematically.

   (a) $f(x) = \dfrac{2x^2 - x - 15}{x^2 - x - 6} = \dfrac{(2x+5)(x-3)}{(x-3)(x+2)}, \quad c = 10$

   (b) $f(x) = \begin{cases} 4x + 7 & x < 2 \\ 0 & x = 2 \\ 9x - 3 & x > 2 \end{cases}, \quad c = 2$

Polynomials, rational functions, power functions, exponential functions, logarithmic functions, and combinations of these are continuous on their domain.

Domain Restrictions

(a) The denominator must be nonzero.

(b) The argument of an even root must be nonnegative.

(c) The argument of a logarithm (of any base) must be positive.

8. Using algebraic methods to determine the intervals on which each functions is continuous. Write your answer using interval notation.

   (a) $f(x) = \sqrt{3x - 7}$

   (b) $f(x) = \ln(2x - 8)$

   (c) $f(x) = \dfrac{x^2 - 3x - 10}{\sqrt[6]{-2x - 7}}$

   (d) $f(x) = \dfrac{4\ln(x - 8)}{\sqrt{5x - 3}}$

(e) $f(x) = \begin{cases} \dfrac{4}{x-1} & x \leq 2 \\ x^2 & 2 < x < 4 \\ -8\log_2(x+12) & x \geq 4 \end{cases}$

9. Given $f(x) = \begin{cases} 2x^2 + k & x < 2 \\ 3x - 8 & x \geq 2 \end{cases}$, answer the questions that follow.

(a) Determine if $f(x)$ will be continuous on $(-\infty, \infty)$ if $k = 4$.

(b) Determine if $f(x)$ will be continuous on $(-\infty, \infty)$ if $k = -1$.

(c) Find the value(s) of k that make(s) the function continuous for all real numbers. If there is no such value of k, explain why.