2024_Fall_WeekInReview_Math140_Week8

Sunday, October 20, 2024 3:38 PM

2024_Fall_ WeekInR...

2024 Fall Math 140 Week-In-Review

Week 8: Sections 5.1 and 5.2

Some Key Words and Terms: Interval Notation, Relation, Function, Domain, Range, Polynomial, Degree, Leading Term, Leading Coefficient, Constant Term, End-Behavior of a Polynomial, Root/Zero, Quadratic Function, Vertex, Symmetry, Min/Max of a Quadratic.

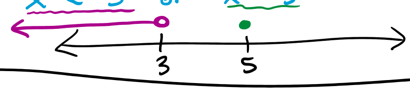
Interval Notation: How we write a range of values using ( ) & [ ] w/ a union "U"

( ) exclusive (we don't get the #) $<$ & $>$

[ ] inclusive (we do get the #) $\leq$ & $\geq$

★ if we have a single value, we use { }

ie.: $x < 3$ or $x = 5$ (or = union)



on $\pm\infty$, we only use parenthesis

$(-\infty, 3) \cup \{5\}$

Relation:

Any pairing of two things: usually x & y values

$R_1 = \{(0,0), (0,1), (0,2), \ldots\}$

$R_2 = \{x^2 + y^2 = 1\}$ (circle)

Function: A special kind of relation where no input value (x) is repeated with distinct output values (y)

★ a single y-value can have multiple x-values BUT a single x-value cannot have multiple y-values ★

Graphically: if a graph passes the vertical line test, it is a function

Domain: The set of all input values for a function (a relation that is not a function does not have a domain, just inputs)

★ the most important

3 domain restrictions:

1. denominator ($\neq 0$)
2. even root (inside ≥ 0)
3. logarithms (inside > 0)

Range: The set of all outputs for a function

Polynomial: Any function where there are no radicals, logs, fractions w/ variables in bottom, ... and all variables are raised to positive, whole number powers.

$$f(x) = \underbrace{a_n x^n}_{\text{leading term b/c it has the highest power}} + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x^1 + \underbrace{a_0x^0}_{\text{b/c } x^0=1 \text{ this is the constant term}}$$

Degree:

the largest power on the variable

→ the degree is "n"

Leading Term: the whole term $\underbrace{a_n}_{\text{coefficient}}\underbrace{x^n}_{\text{variable}}$

position doesn't, just where the biggest power is

$f(x) = 3x - 5x^4$ $a_nx^n = -5x^4$

Leading Coefficient:

Coefficient of the leading term

$a_nx^n \rightarrow$ then a_n is the leading coefficient

Constant Term:

the term w/ no variable attached

★ if no constant is listed, it is understood as zero

★ constant can be "gross": 2^{π} (constant b/c no variable)

e^2 (constant b/c no variable)

End-Behavior of a Polynomial: the end-behavior of ~~any~~ polynomial is determined by: 1) the degree & 2) leading coefficient

$\boxed{\text{I}}$ even degree / positive coefficient

as $x \to -\infty$, then $y \to +\infty$
as $x \to +\infty$, then $y \to +\infty$

$\boxed{\text{II}}$ even degree / negative coefficient

as $x \to -\infty$, $y \to -\infty$
as $x \to +\infty$, $y \to -\infty$

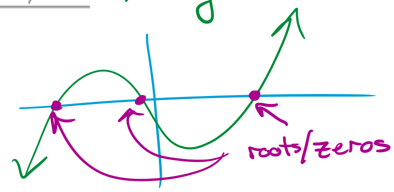
$\boxed{\text{III}}$ odd degree / positive coefficient

as $x \to -\infty$, $y \to -\infty$
as $x \to +\infty$, $y \to +\infty$

$\boxed{\text{IV}}$ odd degree / negative coefficient

as $x \to -\infty$, $y \to +\infty$
as $x \to +\infty$, $y \to -\infty$

Root/Zero: Fancy name for an x-intercept



① set function equal to zero

② factor if not already factored

★ $a \cdot b = 0 \rightarrow a = 0$ & $b = 0$ ★

③ set each factor equal to zero & solve

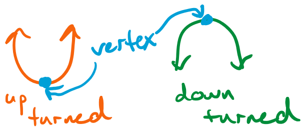
★ if a degree 2 polynomial (quadratic) does not factor, there is a formula ★

Quadratic Function:

- A degree 2 polynomial
- $f(x) = ax^2 + bx + c$ OR
- $f(x) = a(x-h)^2 + k$

★ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (if needed)

Vertex: For a quadratic/parabola, this is the most important (other than domain)



① if we have $f(x) = ax^2 + bx + c \rightarrow$ $\boxed{x = \dfrac{-b}{2a}}$ ★

if we have $f(x) = a(x-h)^2 + k \rightarrow x = h$

② if we have $f(x) = ax^2 + bx + c \rightarrow y = f(-b/2a)$

if we have $f(x) = a(x-h)^2 + k \rightarrow y = k$

Symmetry: is the vertical line that passes through the vertex: $\boxed{x = -\dfrac{b}{2a}}$ ← the line of symmetry must have "x ="

Min/Max of a Parabola: A parabola will have either a min OR a max, but not both, depending on if it opens up or down

With either $f(x) = ax^2 + bx + c$ OR $f(x) = a(x-h)^2 + k$, the value "a" determines which way it opens.

a is positive → opens up → a min & no max

a is negative → opens down → a max & no min

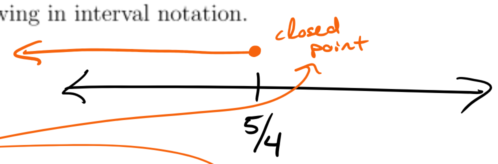
★ the min OR max is the y-value of the vertex ★

Examples:

1. Rewrite the following in interval notation.

   (a) $x \leq \frac{5}{4}$

   closed point

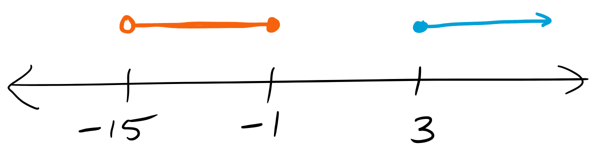


"equal to"

$\boxed{(-\infty, 5/4]}$ inclusive

   (b) $-15 < x \leq -1$ or $x \geq 3$

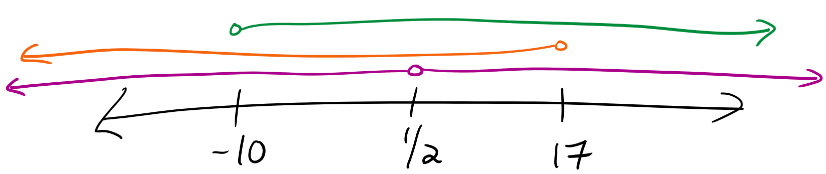
   union, if "and" then we do an intersection only where they overlap



$\boxed{(-15, -1] \cup [3, \infty)}$

   (c) $x > -10$ and $x < 17$, but $x \neq \frac{1}{2}$

   we only want what all 3 have in common



$(-10, \frac{1}{2}) \cup (\frac{1}{2}, 17)$

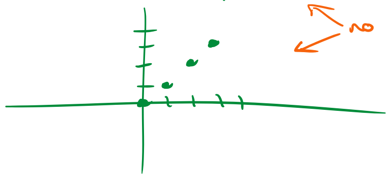
how we exclude a single value

2. Determine if the given relation is a function or not. Explain how you know.

$F = \{(0,0), (1,1), (2,2), (3,3)\}$

single point

"no repeated x-values"



no repeated x-value & graph passes VLT

Function

3. Determine if the given relation is a function or not. Explain how you know.

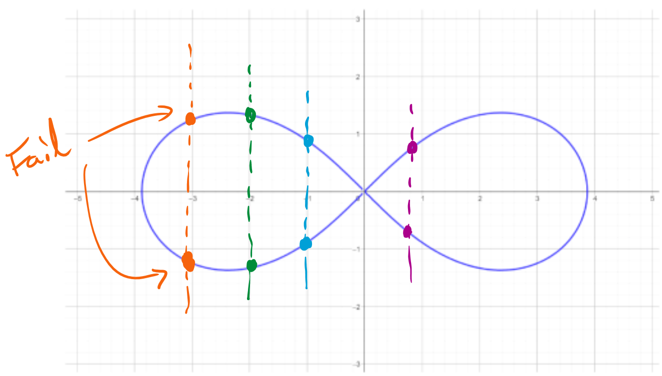
x	y
-2	4
-1	2
0	1
1	2
2	4

no repeated x-value

might as well not even look here unless we see a repeated x-value

Function

4. Determine if the given relation is a function or not. Explain how you know.



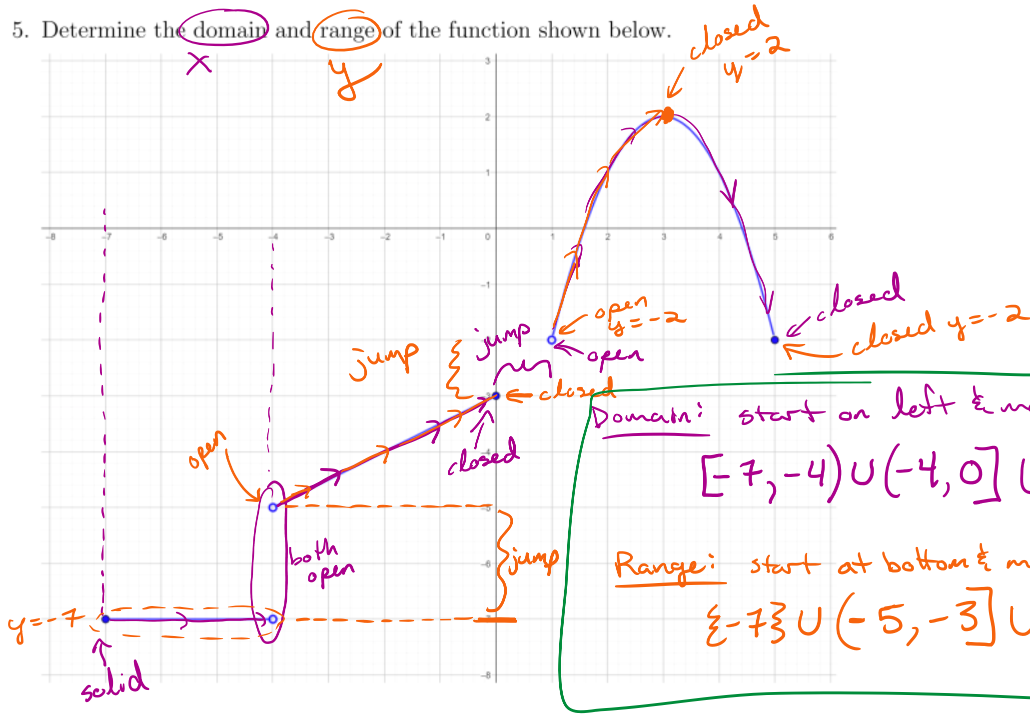
w/ a graph, go straight to VLT

☆ fails VLT in lots of places ....

Not a function

5. Determine the domain and range of the function shown below.

x y



Domain: start on left & move to right

$[-7,-4) \cup (-4,0] \cup (1,5]$

Range: start at bottom & move to top

$\{-7\} \cup (-5,-3] \cup [-2,2]$

6. For the given functions, if it is a polynomial, state the degree (n), the leading coefficient (a_n), and determine the end-behavior. If it is not a polynomial, state why not. Also, state the domain of each function in interval notation.

(a) $(f(x) = -3x^4 + 7x - 8x^7 + e^{11}$ D: $(-\infty, \infty)$

Polynomial: $n = 7$ $a_n = -8$ odd/negative

as $x \to -\infty$, $y \to +\infty$

as $x \to +\infty$, $y \to -\infty$

(b) $g(x) = 2x(x-1)^2(2x+3)$ x^2 D: $(-\infty, \infty)$ $2x \cdot x^2 \cdot 2x$ $4x^4$

Polynomial: $n = 4$ $a_n = 4$ even/positive

as $x \to -\infty$, $y \to +\infty$

as $x \to +\infty$, $y \to +\infty$

(c) $h(x) = 5x^2 - (x-1)^3 + x^\pi$ D: $(-\infty, \infty)$

Not polynomial

not a whole #

$\pi \approx 3.14159 \ldots\ldots$

domain

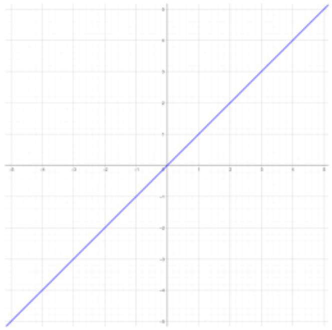
1. denominators w/ variables
2. even roots
3. log

if we don't see these, then the domain is $(-\infty, \infty)$

Polynomial:

- non-negative powers
- whole # powers

7. For each parent function graph shown below, write the function, domain, range, and end-behavior.



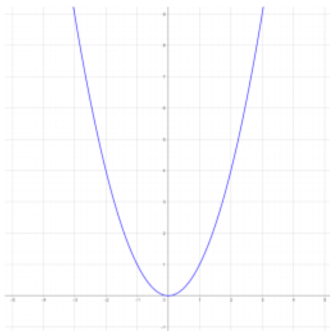
"Linear Parent Function"

$y = f(x) = x$ "first degree polynomial"

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

as $x \to -\infty$, $y \to -\infty$

as $x \to +\infty$, $y \to +\infty$



"Quadratic Parent Function"

$y = f(x) = x^2$ "second degree polynomial"

D: $(-\infty, \infty)$ R: $[0, \infty)$

as $x \to -\infty$, $y \to +\infty$

as $x \to +\infty$, $y \to +\infty$

"Cubic Parent Function"

$y = f(x) = x^3$ "third degree polynomial"

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

as $x \to -\infty$, $y \to -\infty$

as $x \to +\infty$, $y \to +\infty$

x-intercept $(x,0)$

8. For the given polynomial functions, determine any roots or zeros of the function.

set the function equal to zero & (usually) factor

9. $f(x) = x^2 - 5x + 6 = 0$

★ if a=1, factoring is not so bad ★

$c=6$: $1\cdot 6$, $2\cdot 3$, $-1\cdot -6$, $-2\cdot -3$ } which pair adds to $b=-5$?

$F_1=-2$ & $F_2=-3$

$x^2-5x+6 = (x+F_1)(x+F_2)$
$= (x+(-2))(x+(-3))$
$= (x-2)(x-3) = 0$

$x-2=0$, $x-3=0$
$x=2$, $x=3$

$\boxed{(2,0) \text{ \& } (3,0)}$

★ if no pairs add to the b-value then we must go w/ quadratic formula ★

10. $g(x) = 3x^2 - 5x - 2$
$a=3$ $c=-2$

$a\cdot c = -6$: $-1\cdot 6$, $-2\cdot 3$, $1\cdot -6$, $2\cdot -3$ } any pairs add to $b=-5$?

$F_1=1$, $F_2=-6$

★ $\frac{1}{a}(ax+F_1)(ax+F_2) = \frac{1}{3}(3x+1)(3x-6)$

distribute the 1/3 to the parenthesis that will result in whole #s

$\frac{1}{3}(3x-6) = (x-2)$

$(3x+1)(x-2)=0$

$(3x+1)(x-2)=0$
$3x+1=0$, $x-2=0$
$x=-1/3$ $x=2$

$\boxed{(-1/3,0) \text{ \& } (2,0)}$

★ more factoring examples @ vmlc.tamu.edu "Workshops" → "Algebra Series" → "Solving Quadratic Equations"

11. $h(x) = 2x(x+2)(4x-5)^2(3x+7) = 0$
pre-factored (yay!)

$2x=0$, $x+2=0$, $4x-5=0$, $3x+7=0$
$x=0$, $x=-2$, $4x=5$, $3x=-7$
$x=5/4$, $x=-7/3$

$\boxed{(0,0) \text{ \& } (-2,0) \text{ \& } (5/4,0) \text{ \& } (-7/3,0)}$

12. $j(x) = x^2 - 5x + 2$
$a=1$ $b=-5$ $c=2$

$c=2$: $1\cdot 2$, $-1\cdot -2$ } no way to get -5 out of this → Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2-4(1)(2)}}{2a} = \frac{5 \pm \sqrt{25-8}}{2}$

$x = \frac{5 \pm \sqrt{17}}{2}$

two points

$\left(\frac{5+\sqrt{17}}{2}, 0\right)$
$\left(\frac{5-\sqrt{17}}{2}, 0\right)$

8

very common type of question

13. For the quadratic functions given, determine the domain, vertex, if it opens up or down, range, minimum value, and maximum value.

never be domain restrictions w/ quadratic (polynomial)

$f(x) = 2x^2 + 6x + 1$ (not vertex form)

D: $(-\infty, \infty)$

$x = \frac{-b}{2a} = \frac{-6}{2(2)} = \frac{-6}{4} = \frac{-3}{2}$ so $y = f(-\frac{3}{2}) = 2(-\frac{3}{2})^2 + 6(-\frac{3}{2}) + 1 = -\frac{7}{2}$

vertex: $(-3/2, -7/2)$

$a = 2 > 0 \rightarrow$ range [y-value of vertex, ∞)

* if opens down range ($-\infty$, y-value vertex]

opens: up

R: $[-7/2, \infty)$

Min: $y = -7/2$

Max: No max

14. For the quadratic functions given, determine the domain, vertex, if it opens up or down, range, minimum value, and maximum value. Then, sketch a graph of the function.

$f(x) = x^2 - 2x - 15$

$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$y = f(1) = 1^2 - 2(1) - 15$
$= 1 - 2 - 15 = -16$

D: $(-\infty, \infty)$

vertex: $(1, -16)$

opens: up $(a = 1 > 0)$

R: $[-16, \infty)$

Min: $y = -16$

Max: no max

* even though we weren't asked for them, we need the zeros to accurately graph this *

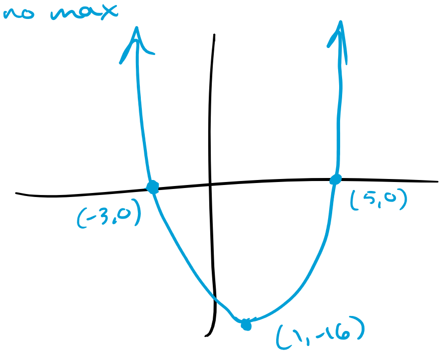
$f(x) = x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5, \ x = -3$

$\underbrace{(5,0) \ \& \ (-3,0)}$

the x-value of the vertex should be exactly halfway between these



Application of Quadratics

15. The weekly price-demand function for a company that supplies bottles of tattoo ink is given by $p(x) = -0.5x + 100$. The total weekly production cost for the company is given by $C(x) = 20x + 3000$. Determine:

(a) the weekly revenue function for the company — from Chp 2: Revenue = (price)(quantity) = $p(x) \cdot x$

(b) the weekly profit function for the company — from Chp 2: Profit = Revenue − Cost

(c) the number of bottles of ink the company should sell to maximize weekly profit

(b) the weekly profit function for the company from Chp 2: Profit = Revenue − Cost

(c) the number of bottles of ink the company should sell to maximize weekly profit

(d) the price each bottle should be sold for to maximize weekly profit

(a) $R(x) = p(x) \cdot x = (-0.5x + 100) \cdot x$

$\boxed{R(x) = -0.5x^2 + 100x}$ (quadratic)

(b) $P(x) = R(x) - C(x) = (-0.5x^2 + 100x) - (20x + 3000)$

$\boxed{P(x) = \underbrace{-0.5}_{a=-0.5<0}x^2 + 80x - 3000}$ (quadratic) (opens down)

max @ vertex

(c) max for profit & we want x-value (# bottles of ink)

$x = \frac{-b}{2a} = \frac{-80}{2(-0.5)} = \frac{-80}{-1} = \boxed{\text{80 bottles of ink to maximize profit}}$

(d) price per bottle, not the profit (make sure you plug-in to the correct formula)

$\text{price} = p(x) = -0.5x + 100$

$\text{price} = p(80) = -0.5(80) + 100$

$= -40 + 100 = \boxed{\text{\$60 per bottle of ink to maximize profit}}$