2024_Fall_WeekInReview_Math140_Week6

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2024_Fall_WeekInRe...

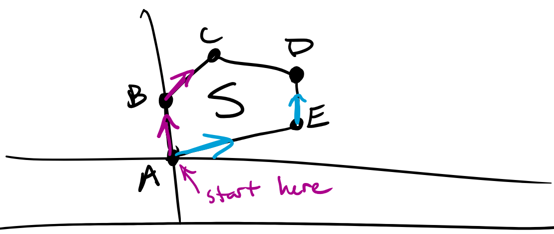
2024 Fall Math 140 Week-In-Review

Week 6: Sections 3.4 and 4.1

Sections 3.4: The Simplex Method

Some Key Words and Terms: Simplex Method, Standard Maximization Problem, Slack Variables, Initial Tableau, Pivot Column/Row/Element, Basic and Non-Basic Variables, Corner Point, Final Tableau, Solution, Leftovers

Simplex Method: Use a tableau to cycly thru the corner points of the "S" region until it finds the max



- Confirming "Standard Max"
- Rewrite inequalities as equations w/ slack variables (possibility of leftovers)
- Construct Initial Tableau
- Determine where to pivot
- Interpret any tableau

Standard Maximization Problem:

1. "Maximize" P/I/C... = $\underbrace{\quad\quad}_{\text{positive coefficients}}$
2. All constraints must be of the form: $\underbrace{\quad\quad}_{\text{variables}} \le \underbrace{\quad}_{\text{non-negative \#}}$
   ★ Being able to convert to the correct form
3. Non-negativity for "normal variables": x, y, z, ...

Slack Variables:

these represent the possibility of leftovers

$2x + 3y \le 20$ (electronics inequality)

$$\underbrace{2x + 3y}_{\text{amount used}} + \underbrace{s_1}_{\text{any leftover}} = \underbrace{20}_{\text{total}}$$

1

Initial Tableau:

- A row for each inequality that was a constraint (except for non-negativity)
- Bottom row for the rewritten "Maximize" equation
- A column for each "normal" variable (x, y, z), for each slack variable ($s_1, s_2, \ldots$), the maximized variable, and the constant

Pivot Column/Row/Element:
① ② ③

① Column: the column w/ biggest negative in bottom row
★ if no negatives in bottom row → final tableau

② Row: divide constant column by the entries in the pivot column (discard any ratios where we divide by 0 or negative) and choose smallest result

③ Entry: intersection of the pivot column & pivot row

Basic and Non-Basic Variables:

★ We can classify variables at any step of the tableau

We look at each column and:

a) if column has a single 1 & all other entries 0, then basic

b) if a variable is not basic → non-basic
★ automatically assigned a value of zero ★

Corner Point:

★ We can read a corner point from any step of the tableau

we only look at variables before slack variables

- if the variable is basic, we determine the value from the tableau
- if the variable is non-basic, assigned a value of 0

Final Tableau:

☆ We know we have a final tableau if no negatives in bottom row ☆

A final tableau gives the "optimal solution"

- determine the corner point
- assign values to slack variables
- assign a value to the maximized variable

Solution:

" Maximum of P/I/R/C... = (max value) when $(x, y) =$ (corner point)" (w/o context)

" Maximum revenue/profit/... of (#) when ___ of product 1, ___ product, ----" (w/ context)

Leftovers:

$s_1 =$ leftovers from inequality 1

$s_2 =$ leftovers from inequality 2

⋮

☆ the more difficult part of leftovers is the context"

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Examples:

1. Determine if the following Linear Programming Problems are Standard Maximization Problems.

① "Max" P/I/C = positive coefficients
② (variables) ≤ (non-negative #)
★ might have to convert ★
③ non-negativity

(a) Objective: **Maximize** $A = 5x + 4y$

Subject to: $-2x + 2y \le 4$

$3x + 2y \le 12$

$2 \le y \le 5$ → $2 \le y$ & $y \le 5$

$x \ge 0$

if $y \ge 2$ then $y \ge 0$ by default

this is a simplex setup

(b) Objective: **Minimize** $Z = 12x + 15y + 10z$ ← not max

Subject to: $x + y + z \le 10$

$-5x + 2y + 2z \le 14$

$3y + 6z \le 24$

$x \ge 0,\ y \ge 0,\ z \ge 0$

not a standard max problem

(c) Objective: **Maximize** $Z = 12x + 15y + 10z$

Subject to: $x + y + z \le 10$

$\boxed{5x - 2y - 2z \ge -14}$

$3y + 6z \le 24$

$x \ge 0,\ y \ge 0,\ z \ge 0$

Be careful! If we multiply the left & right by (-1), then:

$(-1)(5x - 2y - 2z \ge -14)$

$-5x + 2y + 2z \le 14$ ✓

standard max problem

(d) Objective: **Maximize** $P = 0.12x + 0.05y + 0.18z$ good ✓

Subject to: $-2x + 2y + z \le 0$ good ✓

$x - 5 \le y + z$ → $x - y - z \le 5$ good ✓

$3z - 5 \ge x + z$ → $-x + 2z \ge 5$ → $x - 2z \le -5$

no good ×

Not a standard max

2. Convert the following Standard Maximization Problems into an Initial Tableau.

subtract the variables from the right to the left

(a) Objective: **Maximize** $Z = 12x + 15y + 10z$ Last Row

Subject to: $x + y + z \le 10$ (s_1)

$-5x + 2y + 2z \le 14$ (s_2)

$3y + 6z \le 24$ (s_3)

First Rows

$x + y + z + s_1 = 10$

$-5x + 2y + 2z + s_2 = 14$

$3y + 6z + s_3 = 24$

$-5x + 2y + 2z \leq 14$ (s_2)

$3y + 6z \leq 24$ (s_3)

$x \geq 0,\ y \geq 0,\ z \geq 0$

Rows

$-5x + 2y + 2z + s_2 = 14$

$3y + 6z + s_3 = 24$

$-12x - 15y - 10z + Z = 0$

$x, y, z, s_1, s_2, s_3, Z$, constant

$$
\begin{array}{c}
\begin{array}{cccccccc} x & y & z & s_1 & s_2 & s_3 & Z & \text{constant} \end{array} \\
\left[\begin{array}{ccccccc|c}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 10 \\
-5 & 2 & 2 & 0 & 1 & 0 & 0 & 14 \\
0 & 3 & 6 & 0 & 0 & 1 & 0 & 24 \\
\hline
-12 & -15 & -10 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}
$$

(b) Objective: **Maximize** $P = 0.15x + 0.09y$

Subject to: $-x + 2y \leq 0$

$3x + 2y \leq 20$

$5x + 3y \leq 30$

$-x \geq -12 \longrightarrow x \leq 12$

$x \geq 0,\ y \geq 0$

$-x + 2y + s_1 = 0$

$3x + 2y + s_2 = 20$

$5x + 3y + s_3 = 30$

$x + s_4 = 12$

$-0.15x - 0.09y + P = 0$

$$
\begin{array}{c}
\begin{array}{cccccccc} x & y & s_1 & s_2 & s_3 & s_4 & P & \text{constant} \end{array} \\
\left[\begin{array}{ccccccc|c}
-1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
3 & 2 & 0 & 1 & 0 & 0 & 0 & 20 \\
5 & 3 & 0 & 0 & 1 & 0 & 0 & 30 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 12 \\
\hline
-0.15 & -0.09 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}
$$

3. For each of the following: classify each variable as basic or non-basic, determine the corner point, pivot column/row/element, and pivot on the selected element.

(a)
$$
\begin{array}{c}
\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & R & C \end{array}\\
\left[\begin{array}{cccccc|c}
3 & 5 & 1 & 0 & 0 & 0 & 390\\
\boxed{6} & 3 & 0 & 1 & 0 & 0 & 450\\
4 & 4 & 0 & 0 & 1 & 0 & 360\\ \hline
-115 & -75 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}
$$

corner point

pivot column

$\frac{390}{3} = 130$

$\frac{450}{6} = 75$

$\frac{360}{4} = 90$

basic: s_1, s_2, s_3, R

non-basic: x, y

automatically assigned zero

corner point: $(x,y) = (0,0)$

pivot: column 1 (x), row 2, entry 6

(b)
$$
\begin{array}{c}
\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & R & C \end{array}\\
\left[\begin{array}{cccccc|c}
0 & 7/2 & 1 & -1/2 & 0 & 0 & 165\\
1 & 1/2 & 0 & 1/6 & 0 & 0 & 75\\
0 & \boxed{2} & 0 & -2/3 & 1 & 0 & 60\\ \hline
0 & -35/2 & 0 & 115/6 & 0 & 1 & 8625
\end{array}\right]
\end{array}
$$

$165/(7/2) \approx 47$

$75/(1/2) = 150$

$60/2 = 30$

basic: x, s_1, s_3, R

non-basic: y, s_2

corner point: $(x,y) = (75, 0)$

pivot: column 2 (y), row 3, entry 2

(c)
$$
\begin{array}{c}
\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & P & C \end{array}\\
\left[\begin{array}{cccccc|c}
-2/5 & 0 & 1 & 4/5 & 0 & 0 & 66\\
-3/5 & 1 & 0 & 1/5 & 0 & 0 & 10\\
\boxed{8/5} & 0 & 0 & -1/5 & 1 & 0 & 26\\ \hline
-11/2 & 0 & 0 & 1 & 0 & 1 & 50
\end{array}\right]
\end{array}
$$

$\cancel{66/(-2/5)}$

$\cancel{10/(-3/5)}$

$26/(8/5)$

basic: y, s_1, s_3, P

non-basic: x, s_2

corner point: $(x,y) = (0, 10)$

pivot: column 1 (x), row 3, entry $8/5$

4. The following Standard Maximization Problem represents a shoe company making two types of shoes: a running shoe and a walking shoe. Let x represent the number of running shoes produced, y represent the number of walking shoes produced, and R represent the weekly revenue made from selling the shoes. The first constraint represents the number of units of leather used in the production of the shoes for a given week, the second constraint represents the number of units of cloth used in the production of the shoes for a given week, and the third constraint represents the number of units of rubber used in the production of the shoes week. Express the solution of the Standard Maximization Problem in the context of this scenario, including discussing any weekly leftovers the company has.

Maximize: $R = 115x + 75y$

Subject to: $3x + 5y \le 390$ (leather)

$6x + 3y \le 450$ (cloth)

$4x + 4y \le 360$ (rubber)

$x \ge 0,\ y \ge 0$

The company will maximize weekly revenue at \$9,150 when 60 running shoes & 30 walking shoes are produced. There are 60 leftover units of leather (b/c $s_1 = 60$) and no leftovers for cloth & rubber (b/c s_2 & $s_3 = 0$)

$$\begin{array}{cccccccc} x & y & s_1 & s_2 & s_3 & R & C \\ \end{array}$$
$$\left[\begin{array}{cccccc|c} 0 & 0 & 1 & 2/3 & -7/4 & 0 & 60 \\ 1 & 0 & 0 & 1/3 & -1/4 & 0 & 60 \\ 0 & 1 & 0 & -1/3 & 1/2 & 0 & 30 \\ \hline 0 & 0 & 0 & 40/3 & 35/4 & 1 & 9150 \end{array}\right]$$

basic: x, y, s_1, R

non-basic: s_2, s_3 (zero)

$x = 60$, $y = 30$, $s_1 = 60$, $R = 9150$

Sections 4.1: Mathematical Experiments

Some Key Words and Terms: Sample Space, Outcomes, Event, Tree Diagram, Venn Diagram, Complement, Intersection, Union, Mutually Exclusive, Converting Between Symbolic and Verbal

Sample Space: Always write as "$S = \{\underbrace{\qquad\qquad}_{\text{sample space}}\}$"

the sample space is the set { } of possible outcomes in an experiment

"Flip a coin once" → S = {heads, tails}

"roll a 5 sided die once" ⇒ S = {1, 2, 3, 4, 5}

Outcomes: A single outcome is one possible result from performing a specific experiment

Events: Any collection of possible outcomes from an experiment

"roll a 5 sided die once"

let A be "the event we roll an even #" → A = {2, 4}

let B be "the event we roll a # greater than 1" → B = {2, 3, 4, 5}

let C be "the event we roll a 7" → C = { } "empty set"

Tree Diagram: A convenient way to graphically represent a multi-stage experiment

each set of "branches" represents one stage of the experiment

Venn Diagram: A convenient way to graphically represent different events in an experiment

We shade regions to represent the unions intersections, and/or complements of events

Complement: "the opposite of" but still within the sample space

$S = \{a, b, c, 1, 2, 3\}$

$A = \{a, c, 2\} \longrightarrow A^c$ the stuff in S that is <u>not</u> in A

$A^c = \{b, 1, 3\}$

Intersection: "what is in common to two set"

$\left.\begin{array}{l} A = \{1,2,3,4\} \\ B = \{2,4,6,8\} \end{array}\right\} \underset{\text{"intersect"}}{A \cap B} = \{2,4\}$ ★ $A \cap A^c = \{\ \}$ ★

Union: "take every element from both set (no repetition)"

$\left.\begin{array}{l} A = \{1,2,3,4\} \\ B = \{2,4,6,8\} \end{array}\right\} \underset{\text{"union"}}{A \cup B} = \{1,2,3,4,6,8\}$ ★ $A \cup A^c = S$ ★

Mutually Exclusive: two sets or events have nothing in common, or cannot occur simultaneously

★ A & B are mutually exclusive if and only if $A \cap B = \{\ \}$ ★

Converting Between Symbolic and Verbal: the key is associating specific words w/ specific symbols:

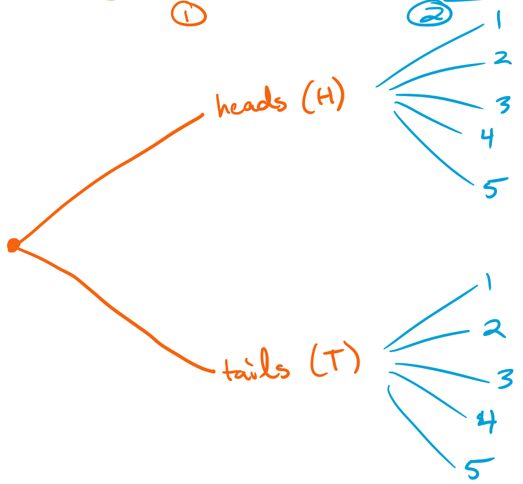
Symbolic		Verbal
"union" $\cup$	$\longleftrightarrow$	"or"
"intersection" $\cap$	$\longleftrightarrow$	"and" (sometimes "but")
"complement" ^{C}	$\longleftrightarrow$	"not"

Examples:

1. For the following experiments, draw a tree diagram to represent the experiment, determine the sample space, the number of simple events, and the total number of ~~outcomes.~~ events

   (a) Flipping a fair coin, then rolling a 5 sided die.

   1 2



★ Every outcome is a pair: H/T & 1/2/3/4/5

$S = \{(H,1),(H,2),(H,3),(H,4),(H,5),(T,1),(T,2),(T,3),(T,4),(T,5)\}$

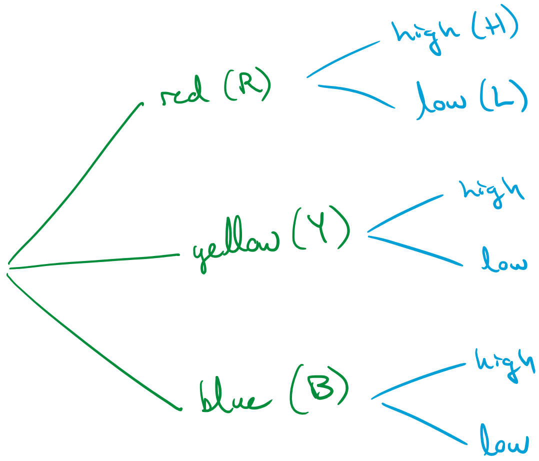
simple events = # outcomes in sample space

$n = 10$ simple events

total # events $= 2^n$ ★

$2^{10} = 1024$ total events

   (b) Spinning a fair spinner showing red, yellow, and blue, then rolling a 4-sided die and recording a 3 or 4 as a "high" result and a 1 or 2 as a "low" result.



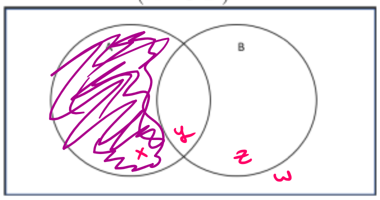
$S = \{(R,H),(R,L),(Y,H),(Y,L),(B,H),(B,L)\}$

$n = 6$ simple events

$2^6 = 64$ total events

2. Shade the following Venn Diagrams for the given event. ☆ label each unique region ☆

$(A^C \cup B)^C$



$A = \{x, y\} \rightarrow A^c = \{z, w\}$

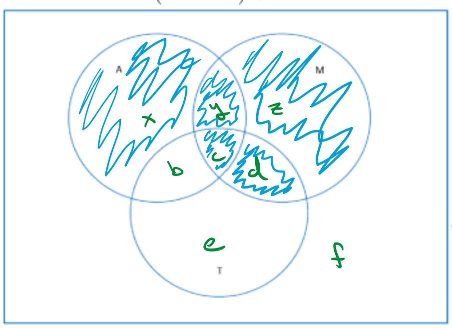
$B = \{y, z\}$

$A^c \cup B = \{z, w, y\}$

$(A^c \cup B)^c = \{x\}$

so shade x only

$(A \cap T^C) \cup M$



$A = \{x, y, b, c\}$

$T = \{b, c, d, e\} \rightarrow T^c = \{x, y, z, f\}$

$(A \cap T^c) = \{x, y\}$

$M = \{y, z, c, d\}$

$(A \cap T^c) \cup M = \{x, y, z, c, d\}$

so shade these

3. Let A be "the event that a randomly selected student likes chocolate ice cream", let B be "the event that a randomly selected student is involved in an org", and let C be "the event that a randomly selected students lives on campus". Use these definitions to answer the following.

(a) Write the event "a randomly selected student is involved in an org or lives on campus, but only likes strawberry ice cream"

B $\cup$ C   "and" $\cap$ comma natural split

A^c

$$\boxed{(B \cup C) \cap A^c}$$

(b) Write the event $A \cap C \cap B^C$ in words using the context given above.

A randomly selected student likes chocolate ice cream and lives on campus and is not involved in an org