Week in Review
Math 152

Week 11

Alternating series

Absolute Convergence and the Ratio Test

Power series

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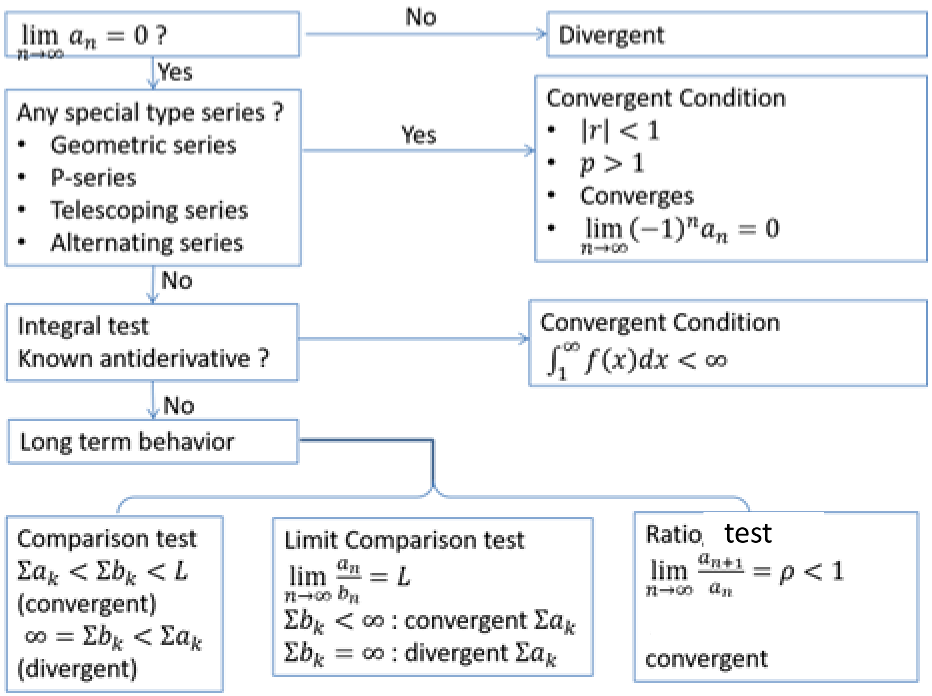
Alternating series



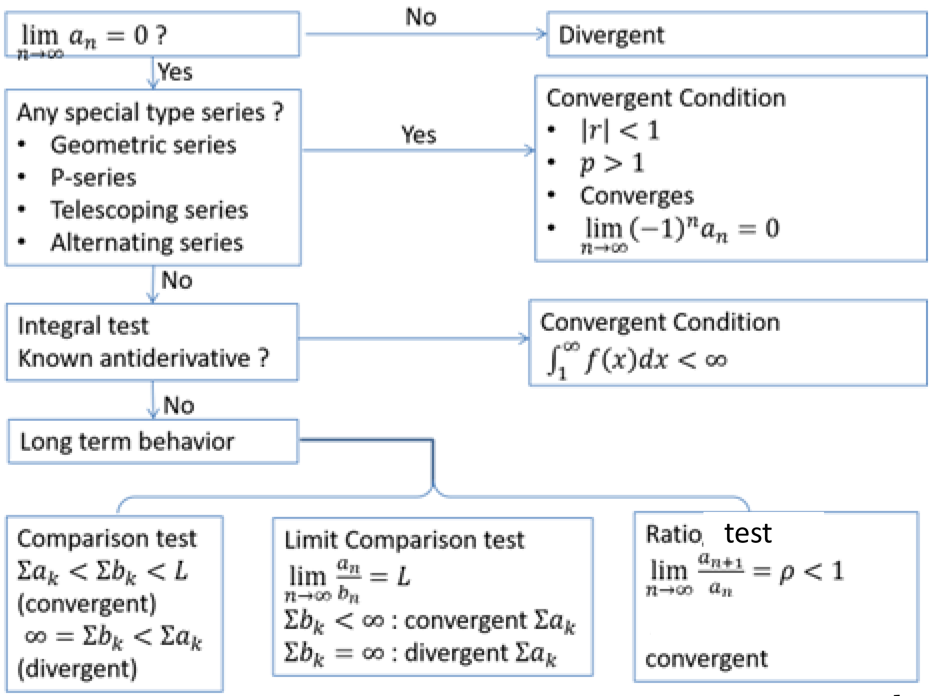
+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine if the alternating series converges or diverges

$$\sum_{i=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$$

$$\sum_{i=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^2+1}$$

Alternating series



+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine if the alternating series converges or diverges

$$\sum_{i=1}^{\infty}(-1)^n \frac{\ln n}{n}$$

$$\sum_{i=1}^{\infty}(-1)^{n+1} \ln\left(1+\frac{1}{n}\right)$$

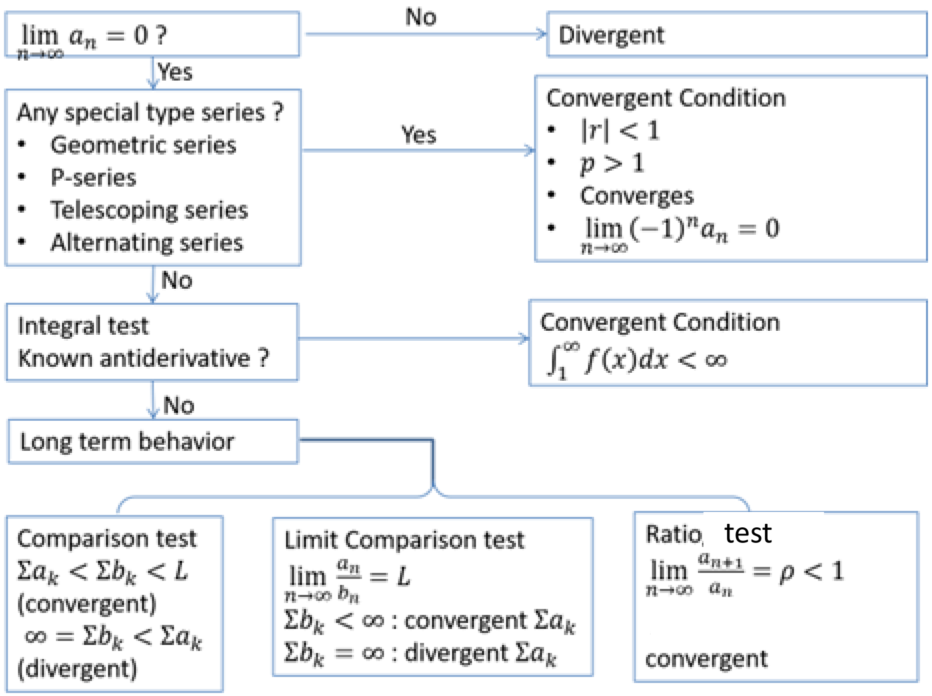
$$\lim_{n\to\infty} \frac{\ln n}{n} = 0$$

$\Rightarrow \sum_{i=1}^{\infty}(-1)^n \frac{\ln n}{n}$ converges

$$\lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right) = 0$$

$\Rightarrow \sum_{i=1}^{\infty}(-1)^{n+1} \ln\left(1+\frac{1}{n}\right)$ converges

Alternating series



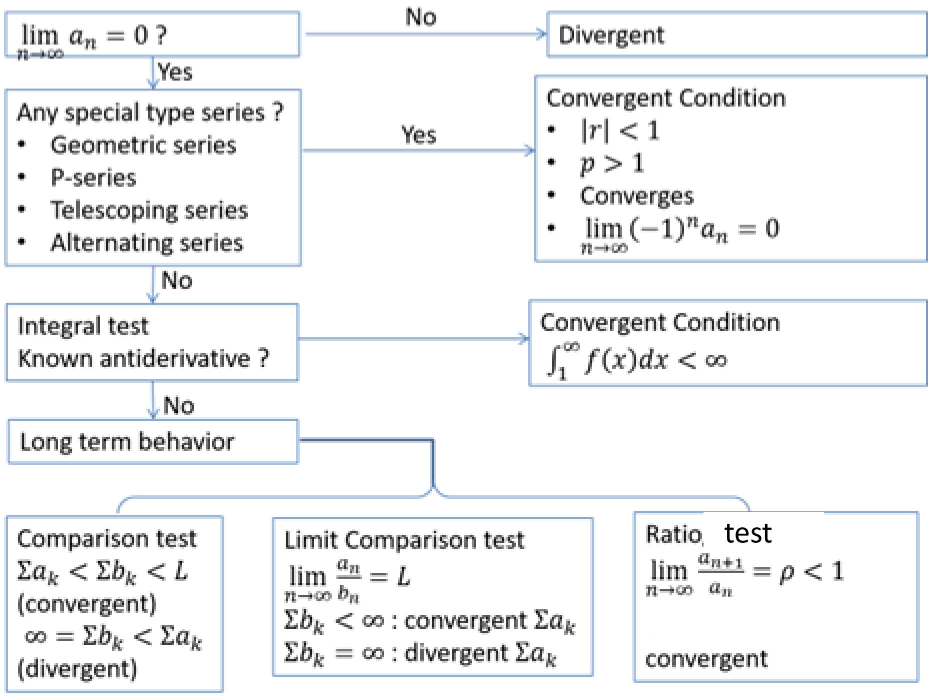
+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine if the alternating series converges or diverges

$$\sum_{i=1}^{\infty} (-1)^n \frac{\sqrt{n}+1}{n+1}$$

$$\sum_{i=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n} - \frac{1}{n^2}\right)$$

Alternating series



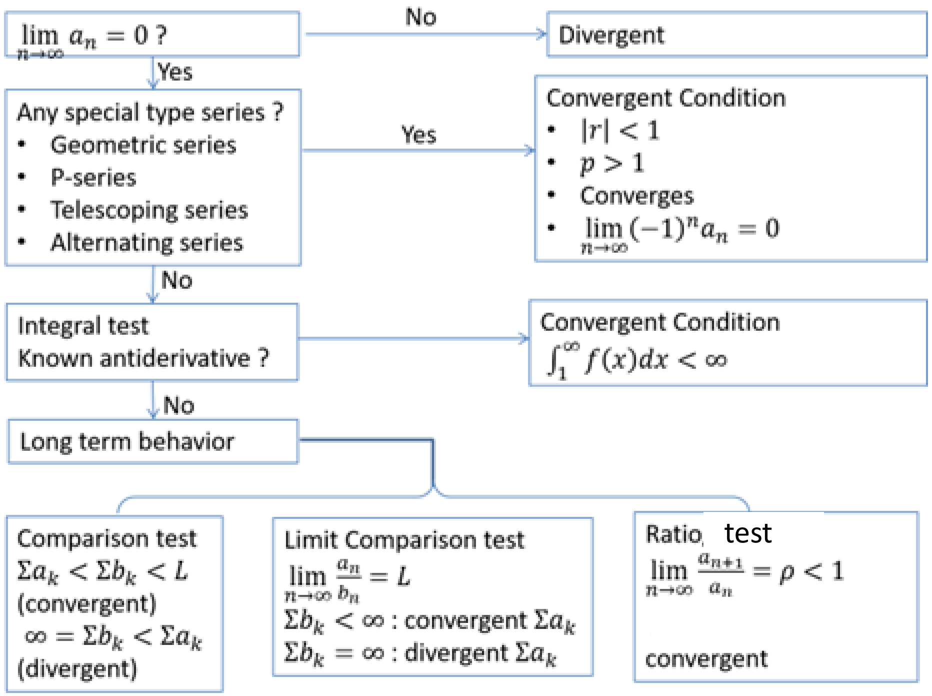
+/- terms but Not alternating
⇒ Absolute convergence

Estimate the magnitude of the error $|S - S_n|$

$\sum_{i=1}^{\infty} \frac{(-1)^n}{n}$

$\sum_{i=1}^{\infty} (-r)^n$ with $|r| < 1$

Absolute Convergence



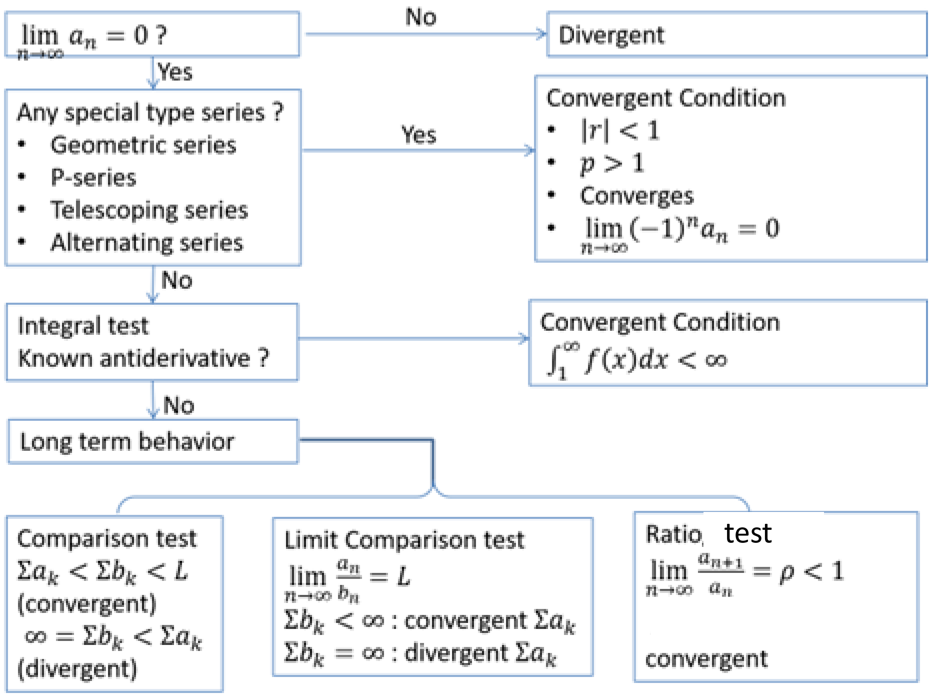
+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine how many terms should be used to estimate the sum of the entire series with an error of less than 0.001.

$$\sum_{i=1}^{\infty} \frac{(-1)^n n}{n^2+1}$$

$$\sum_{i=1}^{\infty} \frac{(-1)^n}{\ln(n+2)}$$

Absolute Convergence



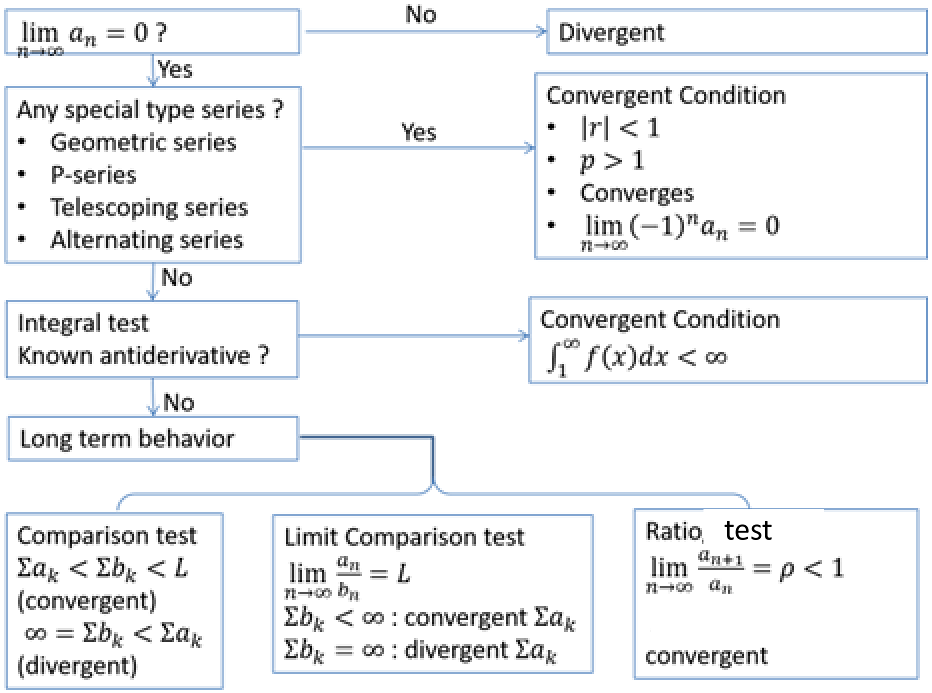
+/- terms but Not alternating
⇒ Absolute convergence

Determine which of the converge absolutely, converge, and diverge?

$$\sum_{i=1}^{\infty} (-1)^n \, (0.1)^n$$

$$\sum_{i=1}^{\infty} (-1)^{n+1} \frac{(0.1)^n}{n}$$

Absolute Convergence



+/- terms but Not alternating
⇒ Absolute convergence

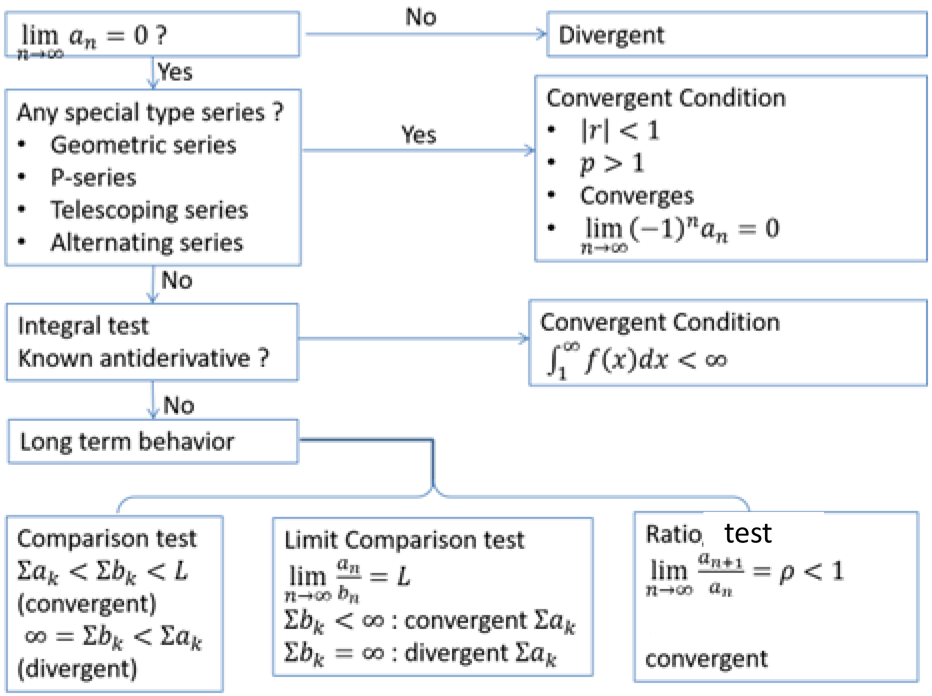
Determine which of the converge absolutely, converge, and diverge?

$$\sum_{i=1}^{\infty} \frac{(-1)^n\, n^3}{3^n}$$

$$\sum_{i=1}^{\infty} \frac{(-1)^n \sqrt{n}}{3^n}$$

$$\sum_{i=1}^{\infty} \frac{(-1)^n\, 3^n}{2^{2n}}$$

Absolute Convergence



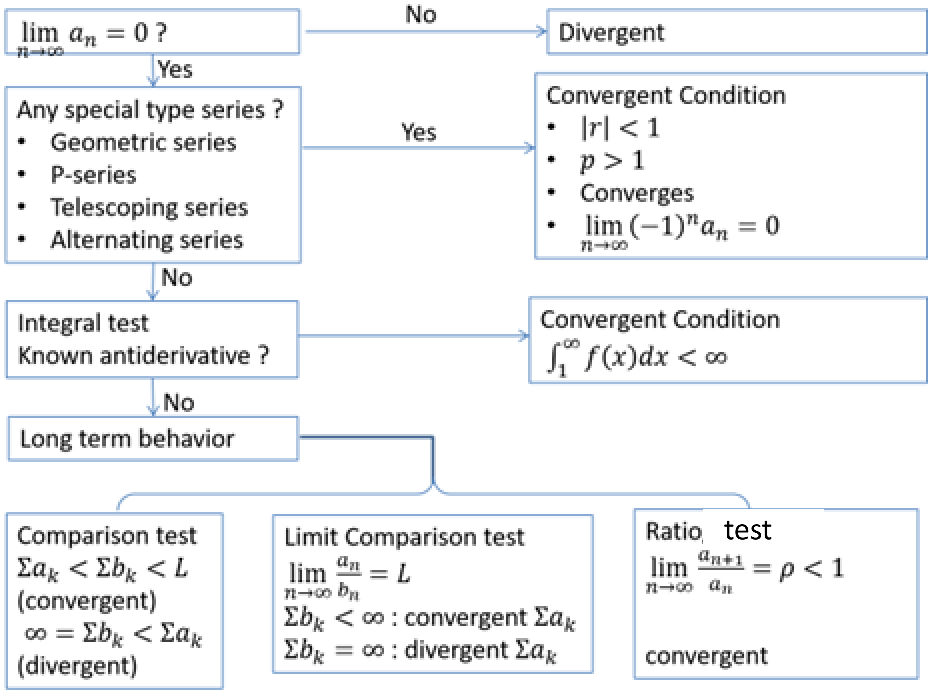
+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine which of the converge absolutely, converge, and diverge?

$$\sum_{i=2}^{\infty} (-1)^n \frac{1}{n \ln n}$$

$$\sum_{i=2}^{\infty} (-1)^n \frac{1}{n - \ln n}$$

Absolute Convergence



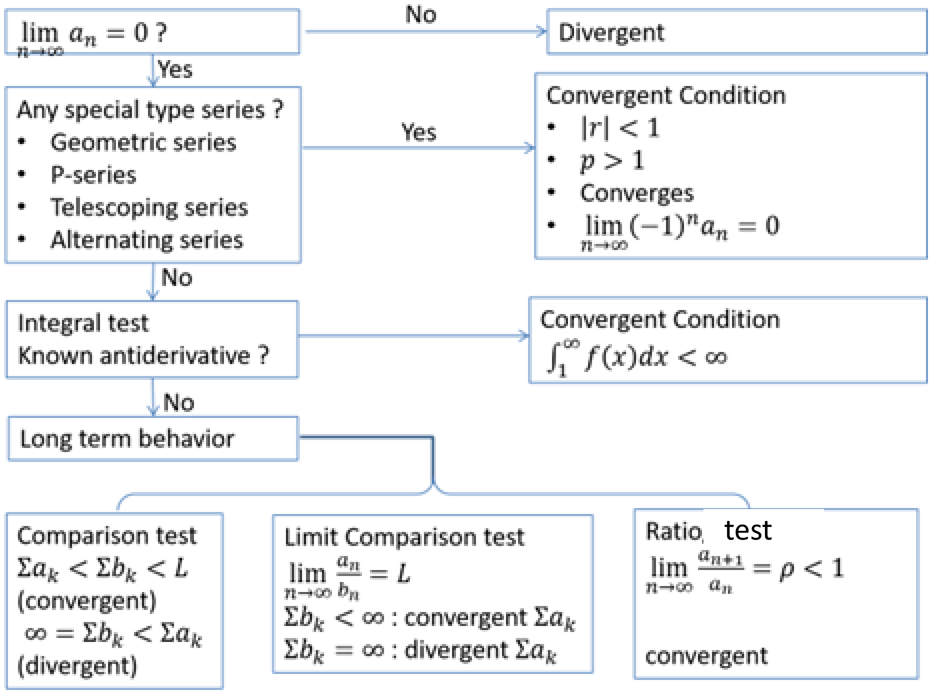
+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine which of the converge absolutely, converge, and diverge?

$$\sum_{i=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2}$$

$$\sum_{i=2}^{\infty} (-1)^n \frac{1}{(\ln n)^2}$$

Absolute Convergence



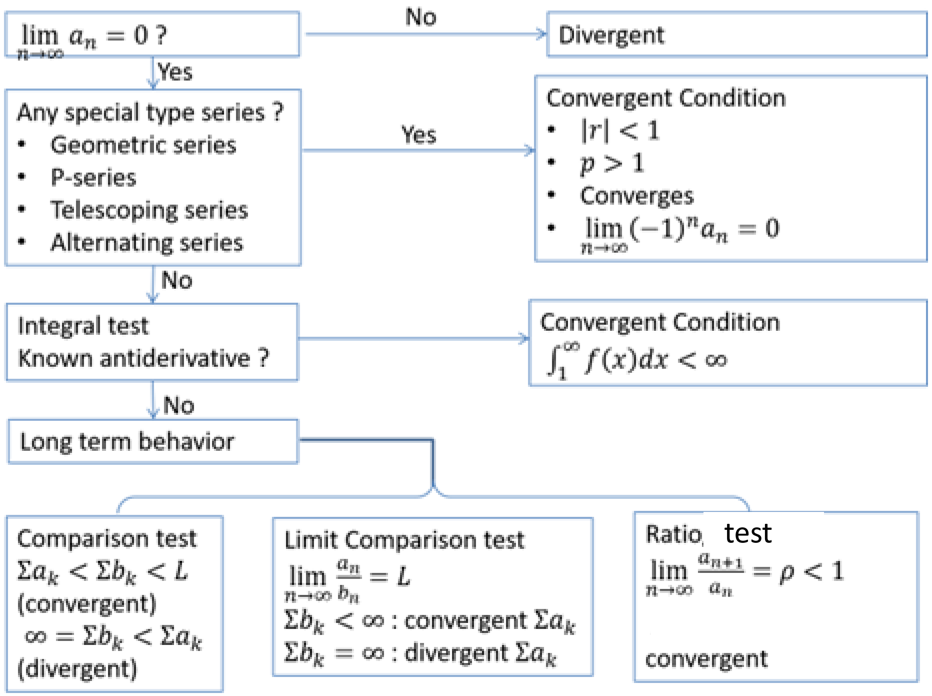
+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine which of the converge absolutely, converge, and diverge?

$$\sum_{i=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$$

$$\sum_{i=1}^{\infty} \frac{\cos n\pi}{n}$$

Absolute Convergence



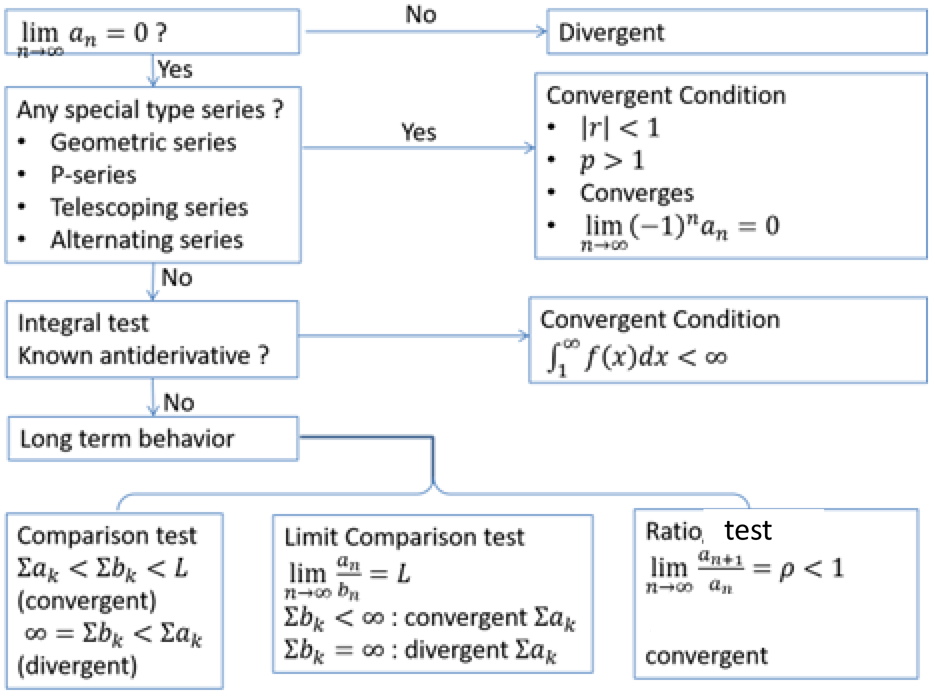
+/- terms but Not alternating
⇒ Absolute convergence

Determine which of the converge absolutely, converge, and diverge?

$\sum_{i=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$

$\sum_{i=1}^{\infty} (-1)^n \left(\sqrt{n^2+n} - n\right)$

Absolute Convergence



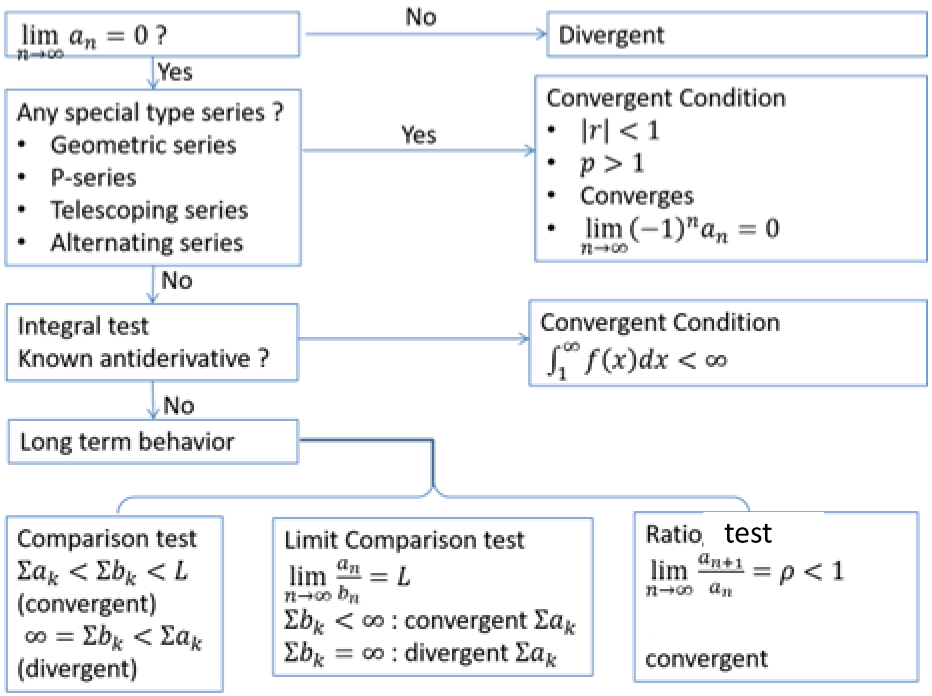
+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine which of the converge absolutely, converge, and diverge?

$$\sum_{i=1}^{\infty} \frac{(-1)^n}{(\sqrt{n+1}+\sqrt{n})}$$

$$\frac{1}{4} - \frac{1}{6} + \frac{1}{8} - \frac{1}{10} + \frac{1}{12} - \frac{1}{14} + \cdots$$

Absolute Convergence



+/- terms but Not alternating
$\Rightarrow$ Absolute convergence

Determine which of the converge absolutely, converge, and diverge?

$$\sum_{i=1}^{\infty} \frac{(-1)^n (n+1)^n}{(n)^n}$$

$$\sum_{i=1}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}$$

Absolute Convergence

Which of the following series is absolutely convergent by **the Ratio Test**?

(I) $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{3^n}$ (II) $\sum_{n=1}^{\infty} \frac{n^4(-2)^n}{n!}$ (III) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3+4}$

(a) I and II only

(b) I only

(c) II only

(d) II and III only

(e) I, II, and III

Convergence of power series

A power series about a, or just power series, is any series that can be written in the form,

$$S(x) = \sum_{n=1}^{\infty} c_n (x-a)^n \text{ where } a, c_n \in \mathbb{R}$$

- The c_n's are often called the coefficients of the series.
- A power series is that it is a function of x.
  - For different x, the power series may or may not converges
- There is a number R so that the power series will converge for, $|x-a| < R|$ and will diverge for $|x-a| > R$. This number, R is called the radius of convergence for the series
  - The series may or may not converge if $|x-a| = R$
- The interval of all x's, including the endpoints ($|x-a| = R$), for which the power series converges is called the interval of convergence of the series

- To find the radius of convergence, we apply ratio test for absolute convergence of the power series
- To find the interval of convergence, investigate the convergence at the endpoints $|x-a| = R$

Example: Plot the partial sums of $\sum_{n=1}^{\infty} x^n$

Convergence of power series

Determine the radius of convergence and interval of convergence for the following power series.

$\sum_{i=1}^{\infty} \frac{(-1)^n (x+1)^n}{4^n}$

$\sum_{i=1}^{\infty} \frac{(x-1)^n}{n!}$

$\sum_{i=1}^{\infty} n!\,(2x+1)^n$

Convergence of power series

Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(-2)^n x^2}{(n+1)!}$.

(a) 0

(b) ∞

(c) $\frac{1}{2}$

(d) 1

(e) 2

Convergence of power series

The series $\sum_{n=1}^{\infty} c_n(x+1)^n$ converges when $x = -4$. Which of the following series is guaranteed to converge?

(I) $\sum_{n=1}^{\infty} c_n \cdot 0^n$ (II) $\sum_{n=1}^{\infty} c_n$ (III) $\sum_{n=1}^{\infty} c_n 2^n$ (IV) $\sum_{n=1}^{\infty} c_n 3^n$

(a) I and II only

(b) I, II, and III only

(c) II and III only

(d) II, III, and IV only

(e) I, II, III and IV