Exam 2 Review over Chapters 3 and 4

- Setting Up Linear Programming Problems
- Graphing Systems of Linear Inequalities in Two Variables
- Graphical Solution of Linear Programming Problems
- Mathematical Experiments
- Basics of Probability
- Rules of Probability
- Probability Distributions and Expected Value

Pr 1. In an experiment, a fair standard 10-sided die is rolled, noting whether or not the number facing is a multiple of 5, and then a card is drawn from a well-shuffled deck, and noting the suit. Write the sample space for the experiment. Let A be the event that a red card is drawn, and let B be the event that a multiple of 5 is rolled.

multiple of 5 — H, C, S, D
not multiple of 5 — H, C, S, D

# outcomes = 8
M = multiple of 5
N = not a multiple of 5

$S = \{(M,H), (M,C), (M,S), (M,D), (N,H), (N,C), (N,S), (N,D)\}$

(a) Write the sample space for the experiment that a red card is drawn.

$A = E = \{(M,H), (M,D), (N,H), (N,D)\}$

(b) Is A a simple event? No, because A has at least two outcomes.

(c) How many outcomes, simple events, and total events are there?

outcomes = 8
simple events = 8

total events = $2^{\#\text{outcomes}}$
$= 2^8 = 256$ total events

(d) Are A and B mutually exclusive? Why or why not?

Mutually exclusive = $A \cap B = \emptyset$

A = a red card is drawn
B = a multiple of 5 is rolled.

No, because $A \cap B = \{(M,H), (M,D)\}$

(e) Construct the symbolic notation for the event 'a multiple of 5 is rolled or a red card is not drawn'.

or → union
not → (red card)^c

and → $\cap$

$\boxed{B \cup A^c}$

Pr 2. Suppose that we surveyed 1000 Math 140 students. We found that 750 of them have attended at least one BMTA office hour, and 200 students have attended at least one WIR. Suppose that 150 students have not gone to any office hours or WIRs.

Let A be the event that a student attended an BMTA office hour, and let B be the event that a student attended at least one WIR.

B^c attended no WIR

(a) Verbally describe the event $A \cap B^C$.

" a student attended an office hour and the student never went to WIR "

$P(A) = \frac{750}{1000}$

(b) Compute $P(A \cap B^C)$. $= P(A) + P(B^c) - P(A \cup B^c)$?

- $P(A^c) = 1 - P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$+ P(A \cap B)$ $\quad + P(A \cap B)$

$- P(A \cup B)$ $\quad - P(A \cup B)$

- $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(B) = \frac{200}{1000}$

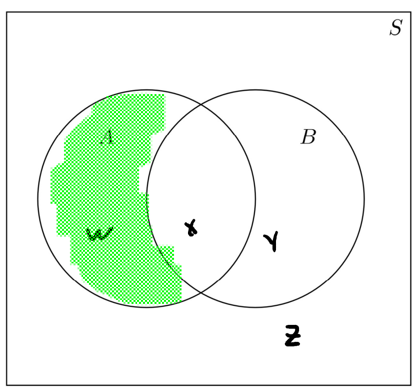
$P((A \cup B)^c) = \frac{150}{1000}$

$$\boxed{\text{Answer} = \frac{650}{1000}}$$

$P(A \cup B) = 1 - \frac{150}{1000} = \frac{850}{1000}$

$P(A \cap B) = \frac{750 + 200 - 850}{1000} = \frac{100}{1000}$

(c) Shade $A \cap B^C$ on the Venn Diagram. Your answer must be obvious to the instructor.



$A = \{w, x\}$ $\quad B = \{x, y\}$

$B^c = \{w, z\}$

$A \cap B^c = \{w\}$

$P(A \cap B^c) = P(A) - P(A \cap B)$

$= \frac{750}{1000} - \frac{100}{1000} = \frac{650}{1000}$

$$\left.\begin{aligned} w + x + y + z &= 1 \\ w + x &= P(A) \\ x + y &= P(B) \\ z &= \text{given} \end{aligned}\right\}$$

use RREF → gives $P(w)$, $P(x)$, $P(y)$, $P(z)$

Pr 3. A survey of 50 veterans from the Air Force and Navy was taken to gather information on their service career and what life is like outside of the miliatry. A breakdown of those surveyed is shown in the table. Suppose a randomly selected veteran from the Air Force or Navy is interviewed. What is the probability the person chosen is

	Air Force	Navy	**Total**
Private	5	8	**13**
Corporal	11	8	**19**
Sergeant	5	4	9
Lieutenant	2	2	**4**
Captain	2	3	**5**
Total	**25**	**25**	**50**

$25-8-8-2-3=4$

$25-5-11-2-2=5$

$4-2=2$

$\rightarrow 5-2=3$

row sum = total on the right

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

(a) P(is a Corporal or in the Navy)

$$=\frac{19+25-8}{50}=\frac{36}{50}$$

(b) P(in the Air Force or in the Navy) $= 1$

(c) P(is a Private and a Lieutenant) $= 0$

impossible event
different rows
mutually exclusive.

(d) P(is not in the Air Force, but is a Sergeant) $= P(\text{Air}^c \cap \text{serg})$

"and"

$$= P(\text{Navy} \cap \text{serg})$$

$$= \frac{4}{50}$$

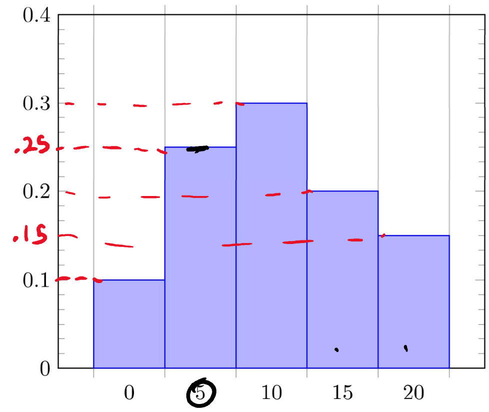
(e) P(is not a Captain and is in the service) $= P(\text{captain}^c \cap S)$

superfluous

$$= P(\text{captain}^c)$$

$$= \frac{50-5}{50} = \frac{45}{50}$$

Pr 4. Consider the following histogram for a random variable X :



(a) Compute $P(X = 5)$ $= .25$

(b) Compute $P(X > 10)$ $= P(X=15) + P(X=20) = .2 + .15$

$= .35$

(c) Compute $P(5 < X \leq 15)$ $= P(X=10) + P(X=15)$

$X \neq 5$ $\quad = .3 + .2 = .5$

$1+1=2$

$4+4=8$

Pr 5. Consider the experiment of rolling two distinguishable, fair four-sided die and then observing the sum of the two numbers appearing on the two ~~uppermost~~ bottommost faces.

(a) Write down the probability distribution for this experiment.

$S = \{2, 3, 4, 5, 6, 7, 8\}$

	2	3	4	5	6	7	8
Pr	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

sec \ first	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2				
3				
4				

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

entries $= 16$

(b) Is the probability distribution uniform?

No, because $Pr(\text{sum is } 2) = \frac{1}{16}$

$Pr(\text{sum is } 5) = \frac{4}{16}$

not equal

(c) Compute the probability that the sum is a multiple of 3?

$3, 6, \cancel{9}, \cancel{12}, \ldots$

$Pr(\text{sum is a mult of } 3)$

$= Pr(\text{sum} = 3) + Pr(\text{sum} = 6)$

$= \frac{2}{16} + \frac{3}{16} = \boxed{\frac{5}{16}}$

Pr 6. A student organization decides to raise money at Maroon Night in downtown Bryan, using a game. The organization has a bag with 100 plushies in it. A contestant pulls a plushie out of the bag uniformly at random. If the contestant pulls out the only plush Reveille, they earn \$100. If they pull out one of the twenty-eight plush fish, they earn a dollar. The remaining plushies are all longhorns, and pulling one of those earns a hiss (and earning zero dollars).

(a) Let X be the random variable representing the amount won playing the game. What are the outcomes of X?

$$X = \{100, 1, 0\}$$

(b) Write down the probability distribution for X.

	Reveille	Fish	tu
X	100	1	0
$P(x)$	$\frac{1}{100}$	$\frac{28}{100}$	$\boxed{\frac{71}{100}}$

\# Longhorns $= 100 - 1 - 28 = 71$

(c) Compute the expected winnings of playing the game.

$$\text{Expected winnings} = 100 \cdot \frac{1}{100} + 1 \cdot \frac{28}{100} + 0 \cdot \frac{71}{80}$$

$$= 1 + \frac{28}{100} + 0 = \$1.28$$

(d) How much should the student organization charge to play if the want the game to be fair? → expectation $= 0$

$\boxed{\$1.28}$

Net winnings = expected winnings − cost to play

fair game, Net winnings $= 0$

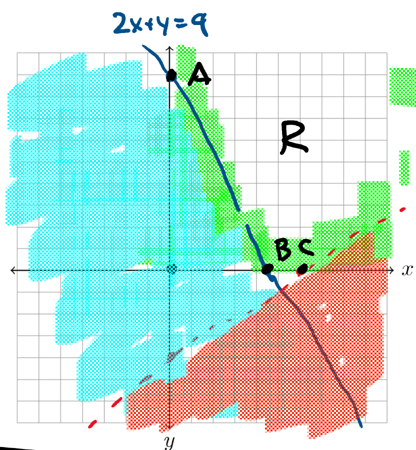
Chapter 3

Pr 7. Graph the system of inequalities below. Then determine if the solution set is bounded or unbounded and find all corner points of the solution set.

$$2x + y \geq 9$$
$$2x - 3y < 12 \quad \text{strict}$$
$$x \geq 0, y \geq 0 \quad \text{don't graph these}$$

Reverse shading

	Solid	dashed
	$2x + y = 9$	$2x - 3y = 12$
x-intercept: $y=0$	$2x+0=9$ $2x=9$ $x=9/2$ $(9/2, 0)$	$2x-3\cdot0=12$ $2x=12$ $x=6$ $(6,0)$
y-intercept: $x=0$	$2\cdot0+y=9$ $y=9$ $(0,9)$	$2\cdot0-3y=12$ $-3y=12$ $y=\frac{12}{-3}=-4$ $(0,-4)$
Test point: $(0,0)$	$2\cdot0+0\geq9$ $0\geq9$ False	$2\cdot0-3\cdot0<12$ $0<12$ True
True shading	opposite side from $(0,0)$	same side
Reverse	same side	opp. side



corner points

$A=(0,9)$, $B=(\frac{9}{2},0)$

$C=(6,0)$

R is unbounded

Pr 8. A breakfast diner sells a few different versions of biscuits and gravy. Their small plate consists of one biscuit, and two servings of gravy. Their large plate has two biscuits, and three servings of gravy. They currently have 300 biscuits and 600 servings of gravy, and they always sell at least twice as many large plates as small plates.

If they charge \$6 for the small plate, and \$12 for the large plate, then how many of each type of order of biscuits and gravy should they sell to maximize their revenue?

x = # of small plates

y = # of large plates

R = total revenue

2 variables 7, for each type of plate

1 for Revenue

(a) Set up the linear programming problem.

$y \geq 2x$

or: $2y \geq x$

?

Maximize $R = 6x + 12y$

subject to:

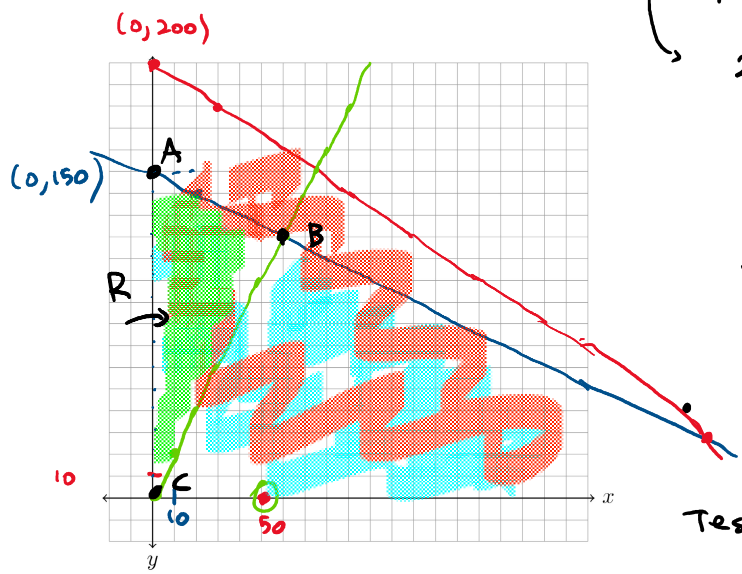
$1 \cdot x + 2y \leq 300$ (biscuits)

$2x + 3y \leq 600$ (gravy)

$y \geq 2x$ (ratio)

$x \geq 0,\ y \geq 0$ (non-neg)

(b) Graph the system of inequalities.



$x + 2y = 300$

$y = 2x$

$2y = -x + 300$

$y = \dfrac{-x + 300}{2}$

$y = -\frac{1}{2}x + 150$

y-intercept $= (0, 150)$

slope $= -\frac{1}{2}$

$2x + 3y = 600$

$3y = -2x + 600$

$y = -\frac{2}{3}x + 200$

(0, 200)

$y = 2x$

"True shading"

Test point (50, 0)

$x + 2y \leq 300$

$50 + 2 \cdot 0 \leq 300$

True

$2x + 3y \leq 600$

$2 \cdot 50 + 3 \cdot 0 \leq 600$

$100 \leq 600$ ✓

True

$y \geq 2x \rightarrow 0 \geq 2 \cdot 50$

False

(c) Does the corresponding linear program have a solution? Why or why not?

Yes, because we are maximizing over a bounded region.

(d) Identify the corner points of the feasible region.

A = (0, 150)

B = ? (60, 120)

C = (0,0)

intersection of blue + green

$y = 2x$

$x+2y = 300$

$$\left.\begin{aligned} x + 2y &= 300 \\ y &= 2x \end{aligned}\right\} \text{RREF}$$

$x + 2(2x) = 300$

$x + 4x = 300$

$5x = 300$

$x = \frac{300}{5} = 60$

$y = 2 \cdot 60 = 120$

(e) Use the method of corners to solve the linear programming problem.

		$R = 6x + 12y$
A	(0, 150)	$6 \cdot 0 + 12 \cdot 150 = 1800$
B	(60, 120)	$6 \cdot 60 + 12 \cdot 120 = 1800$
C	(0, 0)	$6 \cdot 0 + 12 \cdot 0 = 0$ ✓

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\begin{bmatrix} 0 & 150 \\ 60 & 120 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 12 \end{bmatrix} =$$

Solution: Maximum profit is \$1800 on the line segment from (0,150) to (60, 120).

(f) Suppose that they change the price to charge \$8 for the small plate. Now what is the optimal solution? Are there any leftovers?

B → $8 \cdot 60 + 12 \cdot 120 = 1920$

A, C value is unchanged (x=0)

The maximum profit is \$1920 for selling 60 small plates and 120 large plates.

# of biscuits used = $1 \cdot 60 + 2 \cdot 120 \leq$ first constraint

$60 + 240 = 300$

No left over biscuits,

# of gravy servings = $2 \cdot 60 + 3 \cdot 120$ ← 2nd constraint

$= 120 + 360$

$= 480$

600 total − 480 = $\boxed{\text{120 servings of gravy left over}}$

600 total - 480 = 480

$\boxed{\text{120 servings of gravy left over}}$

Pr 9. For the following simplex tableau, identify the pivot row, pivot column, and pivot element. Also, identify the basic variables, non-basic variables, and the corresponding solution. Is the solution optimal?

or entry

(a)
$$
\begin{array}{cccccc}
x & y & s_1 & s_2 & P & \text{constant} \\
\end{array}
\left[\begin{array}{ccccc|c}
0 & 2 & 1 & 0 & 0 & 8 \\
1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 5 \\
\hline
0 & -1 & 0 & -\frac{3}{2} & 1 & 15
\end{array}\right]
$$

$\cancel{8/0}$

$5/\frac{1}{2} = 10$

pivot column = 4

pivot row = 2

pivot entry = 1/2

no pivots

$-\frac{3}{2} < -1$

basic variables: x, s_1, P

non-basic var's: y, s_2

x = 5

P = 15

y = 0

P = 15 @ (5, 0) Not optimal

(b)
$$
\begin{array}{ccccccc}
x & y & s_1 & s_2 & s_3 & P & \text{constant} \\
\end{array}
\left[\begin{array}{cccccc|c}
\frac{2}{3} & 0 & 1 & 0 & 0 & 0 & 2 \\
-1 & -1 & 0 & 1 & 0 & 0 & 1 \\
3 & \frac{1}{3} & 0 & 0 & 1 & 0 & 4 \\
\hline
-3 & -2 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
$$

$2/\frac{2}{3}$

$\cancel{1/-1}$

4/3 ← smallest

pivot column is col. 1

pivot row is 3

pivot element is 3

Solution is not optimal

$2 \div \frac{2}{3} = 2 \times \frac{3}{2} = 3$

$4/3 \approx 1.33$

basic : s_1, s_2, s_3, P

non-basic : x, y

Solution : P = 0 @ (0, 0)

(c)
$$
\begin{array}{ccccccc}
x & y & z & s_1 & s_2 & P & \text{constant} \\
\end{array}
\left[\begin{array}{cccccc|c}
0 & 2 & 1 & 0 & 0 & 0 & 8 \\
1 & \frac{1}{2} & 0 & \frac{1}{3} & 0 & 0 & 5 \\
\hline
0 & \frac{1}{2} & 0 & 2 & \frac{3}{2} & 1 & 15
\end{array}\right]
$$

← no negatives

No Pivot...

The solution is optimal

basic: x, z, P

non-basic: y, s_1, s_2

x = 5 y = 0

z = 8

P = 15

Maximum is P = 15 at (5, 0, 8)

Pr 10. A 4-H member raises only goats and pigs. She has pen space for no more than 18 animals. She spends \$30 to raise each goat and \$60 to raise each pig and she has \$900 available for this project. Each goat produces \$15 in profit and each pig \$30 in profit. We find that the corresponding linear programming problem is given by:

x = # goats

y = # pigs

$$\text{maximize } P = 15x + 30y$$
$$\text{subject to: } y \le 18 - x \checkmark$$
$$900 - 30y \ge 60x$$
$$x \ge 0, y \ge 0$$

(a) Is this a standard maximization problem?

var's $\le$ constant c

$c \ge 0$

$y \le 18 - x$
$+x \quad\quad +x$

$\rightarrow \boxed{x + y \le 18} \checkmark$

$a \ge 6$
$\downarrow$
$-a \le -6$

$900 - 30y \ge 60x$
$-900 \quad -60x \quad -60x$
-900

$-60x - 30y \ge -900 \rightarrow \boxed{60x + 30y \le 900}$

Yes

(b) Convert the constraints of a linear programming problem to linear equations with slack variables.

slack var's = # of constraints − # of non-negativity constraints.

$s_1 = 18 - x - y$ ✓

- $x + y + s_1 = 18$
- $60x + 30y + s_2 = 900$

$x \ge 0,\ y \ge 0$

$s_1 \ge 0,\ s_2 \ge 0$

$P = 15x + 30y$

$-15x - 30y + P = 0$

(c) Construct the simplex tableau for this linear programming problem.

$x + y + s_1 + 0s_2 + 0P = 18$

$60x + 30y + 0s_1 + s_2 + 0P = 900$

$-15x - 30y + 0s_1 + 0s_2 + P = 0$

$$\begin{array}{c}
\\ \\ \\ \text{Objective function} \rightarrow
\end{array}
\begin{array}{ccccc|c}
x & y & s_1 & s_2 & P & \text{constant} \\
\hline
1 & 1 & 1 & 0 & 0 & 18 \\
60 & 30 & 0 & 1 & 0 & 900 \\
\hline
-15 & -30 & 0 & 0 & 1 & 0
\end{array}
\begin{array}{c}
\\ 18/1 \\ 900/ \\ \\
\end{array}$$

(d) Identify the pivot row, pivot column, and pivot entry for the tableau.

pivot column = 2

A 4-H member raises only goats and pigs. She has pen space for no more than 18 animals. She spends \$30 to raise each goat and \$60 to raise each pig and she has \$900 available for this project. Each goat produces \$15 in profit and each pig \$30 in profit. Suppose that, after one pivot, we obtain the following tableau:

$$\begin{array}{cccccc} x & y & s_1 & s_2 & P & \text{constant} \end{array}$$
$$\left[\begin{array}{ccccc|c} \frac{1}{2} & 0 & 1 & -\frac{1}{60} & 0 & 3 \\ \frac{1}{2} & 1 & 0 & \frac{1}{60} & 0 & 15 \\ \hline 0 & 0 & 0 & \frac{1}{2} & 1 & 450 \end{array}\right] \rightarrow \begin{array}{ccc|c} y & s_1 & & \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 450 \end{array} \Bigg\} \begin{array}{l} s_1 = 3 \\ y = 15 \\ P = 450 \end{array}$$

(e) What are the basic and non-basic variables?

basic = y, s_1, P

non-basic = x, s_2

$P = 450$

$x = 0 \quad y = 15$

(f) What is the corresponding solution for this tableau? Is this solution an optimal solution?

Solution is optimal

Profit is \$450 for raising 0 goats and 15 pigs.

(g) Identify all leftovers, if there are any.

$s_1 = 3 \quad s_2 = 0$

The pen has space for 3 more animals.