Review of Sections 4.4 and 4.7

1. Find the limit.

   (a) $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x-1}$

   (b) $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$

(c) $\lim_{x \to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1} \right)$

(d) $\lim_{x \to 0^+} x^2 \ln x$

(e) $\lim_{x \to \infty} \sqrt{x} e^{-x/2}$

(f) $\lim_{x \to 0} (\sin x)^{\tan x}$

(g) $\lim_{x\to\infty}\left(\dfrac{2x-3}{2x+5}\right)^{2x+1}$

(h) $\lim_{x\to 0^+}(1+\sin 3x)^{1/x}$

2. A farmer with 750 ft of fencing wants to enclosed a rectangular field and then divide it in four parts with a fence parallel to one of the sides of the rectangle. What is the largest possible total area of the four pens?

3. A box with a square base and open top must have a volume of 32000 cm^3. Find the dimensions of the box that minimize the amount of material used.

4. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of the cheapest such container.

5. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest area.

6. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 6 - x^2$.

7. Find the point on the line $6x + y = 5$ that are closest to the point $(-5, 3)$.

8. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of a triangle.