Note: As sections 14.1 - 14.5 were covered in the WIR session last week, this WIR session focuses on the remaining sections (that is, 14.6 - 14.8). Students are strongly encouraged to review last week's WIR session.

**Example 1** (14.1). *Sketch the level curves of*

*(a)* $f(x, y) = e^x + y$ *at* $z = 1, 2, 3$.

*(b)* $f(x, y) = e^{x^2+y^2}$ *at* $z = 1, 2, 3$.

*(b)* $f(x, y) = \ln(x^2 + 9y^2)$ *at* $z = 1, 2, 3$.

 

**Example 2** (14.4). *Consider the function*

$$f(x, y) = ye^{xy}.$$

*(a) Find the linearization of the function at the point* $(0, 3)$.

*(b) Use differentials or the linearization to estimate* $(2.98)e^{(0.03)(2.98)}$.

**Example 3** (14.4). *The radius and height of a right circular cone are measured as* $6\,ft$ *and* $10\,ft$*, respectively, with a possible error of at most* $0.1\,ft$*. Use differentials to estimate the maximum error in the calculated volume of the cone.*

**Example 4** (14.5). *Let* $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, $x = re^s$, $y = se^r$ *and* $z = e^{rs}$*. Find* $\dfrac{\partial f}{\partial r}$ *when* $r = 0$ *and* $s = 2$.

**Example 5** (14.5). *Find* $\dfrac{\partial z}{\partial x}$ *and* $\dfrac{\partial z}{\partial y}$ *if* $x^2 + y^2 + z^2 = 2e^{xyz}$.

**Example 6** (14.6). *Find the directional derivative of* $f(x, y) = x\sin(xy)$ *at* $P(1, \pi)$ *in the direction of the vector* $\mathbf{v}$ *that makes an angle* $\theta = \pi/3$ *with positive* $x$*-axis.*

**Example 7** (14.1/14.6). *Suppose* $f(x,y) = 2xy + \ln(4x + y)$.

*(a) Sketch the domain of the function.*

*(b) Find the directional derivative of* $f$ *at* $P(-1/4, 2)$ *in the direction from* $P$ *to* $Q(3/4, 1)$.

*(c) In what direction does* $f$ *increase fastest at* $P$*? What is the maximum rate of change?*

*(d) In what direction does* $f$ *decrease fastest at* $P$*? What is the minimum rate of change?*

**Example 8** (14.6). *Find equations of (a) the tangent plane and (b) the normal line to the surface* $x^2 + y^2 + yz = xz^2$ *at the point* $P(1, -1, 1)$.

**Example 9** (14.7). *Find the local minimum and maximum values and saddle points of the function*

$$f(x, y) = 3xy - x^2y - xy^2 + 2.$$

**Example 10** (14.7). *Find the absolute maximum and minimum values of* $f(x, y) = x^2 + y^2 - 2x$ *over a triangular region* $D$ *with vertices* $(0, 2), (0, -2)$ *and* $(4, -2)$.

 

**Example 11** (14.7). *Find the point on the plane*

$$x - 2y + 3z = 6$$

*that is closest to the point* $(0, 1, 1)$.

**Example 12** (14.8)**.** *Use Lagrange multipliers method to find the point on the plane*

$$x - 2y + 3z = 6$$

*that is closest to the point* $(0, 1, 1)$.

**Example 13** (14.8). *Use Lagrange multipliers to find the extreme values the function*

$$f(x, y) = 2xe^y + 5$$

*subject to*

$$x^2 + y^2 = 2.$$