Math 150 - Week-In-Review 7

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Problem Statements

1. Solve each of the following for x. **Always check for extraneous solutions.**

(a) $e^x = \frac{5}{2}$

$\ln(e^x) = \ln\left(\frac{5}{2}\right)$

$x = \ln(5/2)$

$x \overset{\text{or}}{=} \ln 5 - \ln 2$

(b) $3^x + 7 = 15$ using the common logarithm

$3^x = 8$

$\ln(3^x) = \ln 8$

$x \ln 3 = \ln 8 \qquad x = \frac{\ln 8}{\ln 3}$

(c) $\frac{15}{100 + e^{2x}} = 3$

(Domain restriction $e^{2x} + 100 = 0$

$e^{2x} = -100 \longrightarrow$ No solution. since $e^{2x} > 0$)

$15 = 3(100 + e^{2x})$

$5 = 100 + e^{2x}$

$-95 = e^{2x} \longrightarrow$ No solutions

[note: $\ln(e^{2x}) = \ln(\overbrace{-95}^{\text{not in domain}})$]

(d) $e^{2x} + 7e^x - 18 = 0 \longrightarrow (e^x)^2 + 7(e^x) - 18 = 0$

let $t = e^x \rightsquigarrow t^2 + 7t - 18 = 0$

$(t+9)(t-2) = 0 \begin{cases} \to t = -9 \to e^x = -9 \text{ (no solution)} \\ \to t = 2 \longrightarrow e^x = 2 \end{cases}$

$\boxed{x = \ln 2}$

(e) $3^{x^2-1} = 27$

note : $27 = 3^3$

$3^{x^2-1} = 3^3$

$\Rightarrow x^2 - 1 = 3$

$x^2 = 4 \longrightarrow x = 2, -2$

$(x^2-1)\ln(3) = \ln(27)$

$(x^2-1) = \dfrac{\ln(27)}{\ln(3)} = \dfrac{\ln(3)^3}{\ln(3)} = \dfrac{3\ln(3)}{\ln(3)}$

2. The number of bacteria y in a culture after t days is given by the function $y(t) = 100e^{t/8}$.

(a) What is the initial number of bacteria in the culture?

i.e. $t = 0$ days

(answer will be the coefficient)

$y(0) = 100e^0 = 100$

(b) How many bacteria are there after 40 days?

$t = 40$

$y(40) = 100e^{40/8} = 100e^5$ bacteria

(c) After how many days will there be 4,000 bacteria?

$4000 = 100e^{t/8}$

$40 = e^{t/8}$

$t/8 = \ln(40) \longrightarrow t = 8\ln(40)$ days

3. Simplify each of the following without a calculator:

(a) $7^{\log_7(4)} + 2$

$4 + 2 = 8$

(b) $\log(10^{-5})$ $= -5 \cdot \log 10 = -5$

(c) $\log_{11}(3x+5) = \log_{11}(9)$

domain $3x+5 > 0$

$3x > -5$

$x > -\frac{5}{3}$

$3x + 5 = 9$

$3x = 4$ $x = \frac{4}{3}$ ✓

4. Change $\log_7(45)$ to base 5.

$$\log_7 45 = \frac{\log_5 45}{\log_5 7}$$

5. Change $\log_6(x)$ to base 10

$$\log_6 x = \frac{\log x}{\log 6}$$

6. Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)

(a) $\log_4(64x^2)$

$$= \log_4 64 + \log_4 x^2$$

$$= 3\log_4 4 + 2\log_4 x$$

$$= \boxed{3 + 2\log_4 x}$$

(b) $\ln \sqrt[3]{\dfrac{x^2}{x^2 - 8x - 20}}$

$$= \ln\left(\frac{x^2}{x^2-8x-20}\right)^{\frac{1}{3}} = \frac{1}{3}\left[\ln\left(\frac{x^2}{x^2-8x-20}\right)\right]$$

$$= \frac{1}{3}\left[\ln x^2 - \ln(x^2-8x-20)\right] = \frac{1}{3}\left[2\ln x - \ln((x-10)(x+2))\right]$$

$$= \frac{1}{3}\left[2\ln x - (\ln(x-10) + \ln(x+2))\right] = \frac{1}{3}\left(2\ln x - \ln(x-10) - \ln(x+2)\right)$$

7. Use the properties of logarithms to condense the expression as a single logarithm. (Assume all variables are positive.)

(a) $2\log_5(x-1) + 4\log_5(y) - 1$

$$= \log_5 (x-1)^2 + \log_5 y^4 - 1 = \log_5 (x-1)^2 \cdot y^4 - \log_5 5 = \log_5\left(\frac{(x-1)^2 y^4}{5}\right)$$

(b) $2\ln(6) - \ln(8) - \ln(81)$ $= \ln(6)^2 - \ln 8 - \ln 81$

$$\ln 36 - (\ln 8 + \ln 81) = \ln(36) - \ln(8\times 81) = \ln\left(\frac{\overbrace{36}^{9\times 4}}{\underbrace{8}_{4\times 2}\times\underbrace{81}_{9\times 9}}\right)$$

$$= \ln\left(\frac{1}{18}\right)$$

$$\left[\overset{\text{or}}{=} \ln(18)^{-1} = -\ln(18)\right]$$

$(2x+1)^2$ left by 1 shrink by 2

8. Describe the transformation(s) of the graph of $f(x) = e^x$ that yield(s) the graph of $g(x) = -e^{2x-1} + 2$.

$= -e^{2(x-\frac{1}{2})} + 2$

Transformations:

$e^x \to e^{2x}$ (1) Horizontal shrink by 2

$e^{2x} \to e^{2x-1} = e^{2(x-\frac{1}{2})}$ (2) Horizontal shift by $\frac{1}{2}$ right

$e^{2x-1} \to -e^{2x-1}$ (3) reflection w.r.t. x-axis

(4) Vertical shift up 2.

Domain: $x \in (-\infty, \infty)$

x-intercept $y=0 \to e^{2x-1} = 2 \to 2x-1 = \ln 2 \to x = \frac{\ln 2 + 1}{2}$

y-intercept: i.e. let $x=0$ $g(0) = -e^{2(0)-1} + 2 = -e^{-1} + 2 = -\frac{1}{e} + 2$

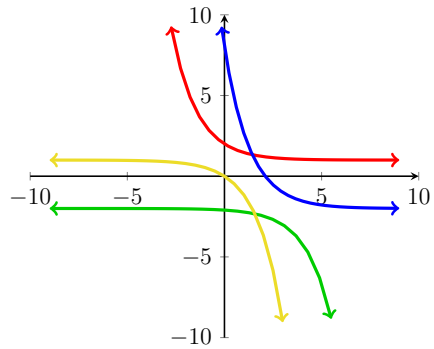
Horizontal Asymptote:

going through the transformation $y = 2$

9. Describe the transformation(s) of the graph of $f(x) = 3^x$ that yield(s) the graph of $g(x) = 3^{-0.7x} + 1$, then choose the graph that matches the function.

Transformations:

1. Horizontal stretch (by $\frac{1}{0.7}$ units)
2. Reflect over y-axis
3. Vertical shift up 1



Domain: $x \in (-\infty, \infty)$

red graph ✓

x-intercept(s): None

$3^{-0.7x} = -1 \rightsquigarrow \ln(3)^{-0.7x} = \ln(-1) \rightarrow$ No solution

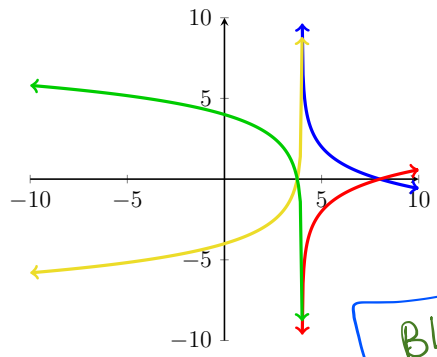
y-int: let $x = 0$ $\quad g(0) = 1 + 1 = 2 \quad (0, 2)$

Horizontal Asymptote(s): $y = 1$

10. Describe the transformations of $f(x) = \log_2(x)$ that yield $g(x) = -\log_2(x-4) + 2$. Then state the domain, x-intercept, and vertical asymptote of the logarithmic function $f(x)$, then choose the graph that matches the function.

Transformations:

1. Horizontal shift right 4
2. Reflection w.r.t x-axis
3. vertical shift up 2 units



Blue graph

Domain: $x - 4 > 0$ $\quad x > 4$

$x \in (4, \infty)$

x-intercept(s): let $y = 0$ $\quad 2 = \log_2(x-4) \iff 2^2 = x - 4 \rightarrow x = 8$ $\quad (8, 0)$

y-intercept(s): let $x = 0$ (we can't since zero is not in the domain)

Vertical Asymptote(s):

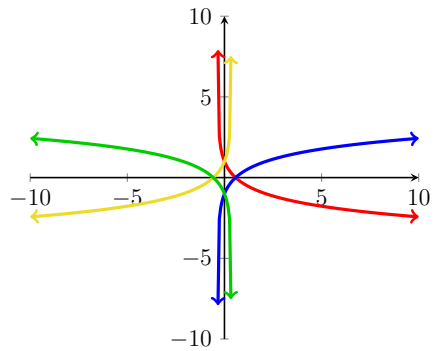
$x = 4$ (found by checking the transformations.

$g(x) = \ln\left(3\left(x+\frac{1}{3}\right)\right) - 1$

11. Describe the transformations of $f(x) = \ln(x)$ that yield $g(x) = \ln(3x+1) - 1$. Then state the domain, x-intercept, and vertical asymptote of the logarithmic function $g(x)$, then choose the graph that matches the function.

Transformations:

1. Horizontal shrink by 3
2. Horizontal shift left $\frac{1}{3}$
3. Vertical shift down by 1



$\boxed{\text{Blue}}$

Domain: $3x+1>0 \qquad x > -\frac{1}{3}$

$x \in \left(-\frac{1}{3}, \infty\right)$

x-intercept(s): let $y=0 \rightarrow \ln(3x+1) = 1 \iff 3x+1 = e \rightarrow x = \frac{e-1}{3}$

y-intercept: let $x=0 \rightarrow g(0) = \ln(1) - 1 = 0 - 1 = -1$

Vertical Asymptote(s): $x = -\frac{1}{3}$