# Week in Review
## Math 152

**Week 07**

Common Exam 2

Preparation

# Common Exam II Prep

After an appropriate substitution, the integral $\int \sqrt{x^2 + x}\, dx$ is equivalent to which of the following?

(a) $\int \tan^2\theta \sec\theta \, d\theta$

(b) $\frac{1}{4}\int \sec^3\theta \, d\theta$

(c) $-\frac{1}{4}\int \sin^2\theta \, d\theta$

(d) $\frac{1}{4}\int \tan^2\theta \sec\theta \, d\theta$ ← correct

(e) $\int \cos^2\theta \, d\theta$

$$\int \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\, dx$$

$$= \frac{1}{2}\int \sqrt{\left[\frac{x+1/2}{1/2}\right]^2 - 1}\, dx$$

Let $\frac{x+1/2}{1/2} = \sec\theta$

$$\sqrt{\left[\frac{x+1/2}{1/2}\right]^2 - 1} = \tan\theta$$

$$dx = \frac{1}{2}\sec\theta \tan\theta \, d\theta$$

$$\frac{1}{2}\int \sqrt{\left[\frac{x+1/2}{1/2}\right]^2 - 1}\, dx$$

$$\frac{1}{4}\int \tan\theta \sec\theta \tan\theta \, d\theta$$

# Common Exam II Prep

The region bounded by $y = \cos x$ and the $x$-axis on the interval $\left[0, \frac{\pi}{2}\right]$ is rotated about the $x$-axis. Find the volume of the resulting solid.

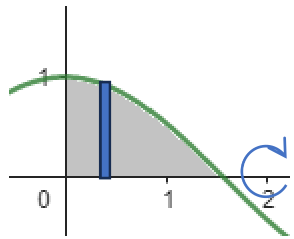
(a) $1$

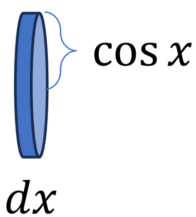
(b) $\frac{\pi^2}{2}$

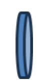
(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

(e) $\frac{\pi^2}{4}$ ← correct




$V($  $) = \pi \cos^2 x \, dx$

$$V = \int_0^{\pi/2} \pi \cos^2 x \, dx$$
$$= \int_0^{\pi/2} \pi \left[\frac{1+\cos 2x}{2}\right] dx$$
$$= \frac{\pi}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/2}$$
$$= \frac{\pi}{2}\left[\frac{\pi}{2}\right]$$
$$= \frac{\pi^2}{4}$$

# Common Exam II Prep

. Compute $\int_0^{\pi/2} \sin^2(\theta)\cos^3(\theta)\,d\theta$.

(a) $\frac{2}{5}$

(b) $\frac{2}{15}$ ← correct

(c) $\frac{4}{5}$

(d) $\frac{8}{5}$

(e) None of the above

$\int \sin^2\theta \cos^3\theta \, d\theta$

$= \int \sin^2\theta \cos^2\theta \, (\cos\theta \, d\theta)$ : odd man out

$= \int \sin^2\theta \, (1-\sin^2\theta)(\cos\theta \, d\theta)$

$u = \sin\theta \Rightarrow$

$du = \cos\theta d\theta$

$\int_{x=0}^{x=\pi/2} \Rightarrow$

$\int_{u=0}^{u=1}$

$\int_0^1 u^2(1-u^2)du$

$= \int_0^1 (u^2 - u^4)du$

$= \left[\frac{1}{3}u^3 - \frac{1}{5}u^4\right]_0^1$

$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

# Common Exam II Prep

Evaluate $\int \tan^3(x)\sec^5(x)\,dx$.

(a) $\frac{1}{7}\tan^7 x - \frac{1}{5}\sec^5 x + C$
(b) $\frac{1}{7}\sec^7 x - \frac{1}{5}\tan^5 x + C$
(c) $\frac{1}{4}\sec^6 x - \frac{1}{6}\tan^{10} x + C$
(d) $\frac{1}{4}\sec^4 x - \frac{1}{6}\tan^6 x + C$
(e) $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

$$\int \tan^2 x \sec^4 x\,(\sec x \tan x\,dx)$$
$$= \int (\sec^2 x - 1)\sec^4 x\,(\sec x \tan x\,dx)$$
$$u = \sec x$$
$$\Rightarrow du = \sec x \tan x\,dx$$

$$\int (u^2 - 1)u^4\,du$$
$$= \int (u^6 - u^4)\,du$$
$$= \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$$
$$= \frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$$

sec out; odd man out

# Common Exam II Prep

After an appropriate substitution, the integral $\int \sqrt{x^2 + x}\, dx$ is equivalent to which of the following?

(a) $\frac{1}{4} \int \tan^2 \theta \sec \theta \, d\theta$ ← correct

(b) $\int \tan^2 \theta \sec \theta \, d\theta$

(c) $\frac{1}{4} \int \sec^3 \theta \, d\theta$

(d) $-\frac{1}{4} \int \sin^2 \theta \, d\theta$

(e) $\int \cos^2 \theta \, d\theta$

$$\int \sqrt{\left(x + \frac{1}{2}\right)^2 - \frac{1}{2^2}}\, dx$$

$$\left(x + \frac{1}{2}\right) = \frac{1}{2} \sec \theta \;\Rightarrow dx = \frac{1}{2} \sec \theta \tan \theta \, d\theta$$

$$\sqrt{\left(x + \frac{1}{2}\right)^2 - \frac{1}{2^2}} = \sqrt{\frac{1}{2^2}(\sec^2 \theta - 1)} = \frac{1}{2} \tan \theta$$

$$= \int \frac{1}{2} \tan \theta \left(\frac{1}{2} \sec \theta \tan \theta \, d\theta\right)$$

$$= \frac{1}{4} \int \tan^2 \theta \sec \theta \, d\theta$$

# Common Exam II Prep

Which of the following is the form of the partial-fraction decomposition for the rational function?

$$\frac{1}{(x+1)(x^2-2x-3)(x^2-2x+2)}$$

(a) $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-2x-3} + \dfrac{Dx+E}{x^2-2x+2}$

(b) $\dfrac{A}{x+1} + \dfrac{Bx+C}{(x+1)^2} + \dfrac{D}{x-3} + \dfrac{Ex+F}{x^2-2x+2}$

(c) $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x-3} + \dfrac{D}{x^2-2x+2}$

(d) $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x-3} + \dfrac{Dx+E}{x^2-2x+2}$ ← correct

(e) $\dfrac{A}{x+1} + \dfrac{B}{x^2-2x-3} + \dfrac{C}{x^2-2x+2}$

$$\frac{1}{(x+1)(x-3)(x+1)(x^2-2x+2)}$$

$$= \frac{1}{(x+1)^2(x-3)(x^2-2x+2)}$$

$$= \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x-3)} + \frac{Dx+E}{(x^2-2x+2)}$$

$$\int_1^\infty xe^{-x^2}\,dx =$$

(a) $1$

(b) $2e$

(c) $\frac{1}{2e}$ $\leftarrow$ correct

(d) $\frac{1}{2}$

(e) $\infty$

Let $u = x^2$

$\frac{1}{2}du = dx$

$\int_{x=1}^{\infty} \Rightarrow \int_{u=1}^{\infty}$

$\frac{1}{2}\int_{u=1}^{\infty} e^{-u}du$

$= \frac{1}{2}[-e^{-u}]_1^\infty$

$= \frac{1}{2}[0 + e^{-1}]$

# Common Exam II Prep

$$\int_0^1 \frac{2}{x^2 - 1}\, dx =$$

(a) $-\infty$ $\leftarrow$ correct

(b) $\infty$

(c) $\ln 2$

(d) $-\ln 2$

(e) $0$

Improper integral (singularity at $x = 1$)

$$\lim_{a \to 1-} \int_0^a \frac{2}{(x-1)(x+1)}\, dx$$

$$= \lim_{a \to 1-} \int_0^a \frac{1}{2}\left[\frac{1}{x-1} - \frac{1}{x+1}\right] dx$$

$$= \lim_{a \to 1-} \frac{1}{2}\left[\ln|x-1| - \ln|x+1|\right]_0^a$$

$$= \lim_{a \to 1-} \frac{1}{2}\left[\ln\left|\frac{a-1}{a+1}\right|\right]$$

$$= -\infty$$

# Common Exam II Prep

$\lim_{n\to\infty} a_n = 0$ ?

No → Divergent

Yes ↓

Any special type series ?
- Geometric series
- P-series
- Telescoping series
- Alternating series

Yes → Convergent Condition
- $|r| < 1$
- $p > 1$
- Converges
- $\lim_{n\to\infty} (-1)^n a_n = 0$

No ↓

Integral test
Known antiderivative ?

→ Convergent Condition
$\int_1^\infty f(x)dx < \infty$

No ↓

Long term behavior

Comparison test
$\Sigma a_k < \Sigma b_k < L$
(convergent)
$\infty = \Sigma b_k < \Sigma a_k$
(divergent)

Limit Comparison test
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
$\Sigma b_k < \infty$ : convergent $\Sigma a_k$
$\Sigma b_k = \infty$ : divergent $\Sigma a_k$

Ratio/root test
$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho < 1$
$\lim_{n\to\infty} (a_n)^{1/n} = \rho < 1$
convergent

# Common Exam II Prep

Which of the following statements is true for the series $\sum_{n=1}^{\infty} \frac{2+\cos n}{n}$?

(a) The series converges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \leq \sum_{n=1}^{\infty} \frac{3}{n}$, which converges.

(b) The series converges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \leq \sum_{n=1}^{\infty} \frac{1}{n}$, which converges.

(c) The series diverges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \geq \sum_{n=1}^{\infty} \frac{3}{n}$, which diverges.

(d) The series diverges because $\sum_{n=1}^{\infty} \frac{\cos n + 2}{n} \geq \sum_{n=1}^{\infty} \frac{1}{n}$, which diverges. ← correct

(e) None of the above

# Common Exam II Prep

Which of the following statements is true regarding the improper integral $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx$?

(a) The integral converges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \le \int_1^\infty \frac{1}{x^2}\,dx$, which converges.

(b) The integral diverges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \ge \int_1^\infty \frac{3}{x}\,dx$, which diverges.

(c) The integral converges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \le \int_1^\infty \frac{3}{x^2}\,dx$, which converges. ← correct

(d) The integral diverges because $\int_1^\infty \frac{\sin x + 2}{x(x+1)}\,dx \ge \int_1^\infty \frac{1}{x}\,dx$, which diverges.

(e) The integral diverges by oscillation.

Let $s = \sum_{n=1}^{\infty} \frac{3}{n^4}$. Using the Remainder Estimate for the Integral Test, find the smallest value of $n$ that will ensure that $R_n = s - s_n \leq \frac{1}{100}$.

(a) $n = 2$

(b) $n = 3$

(c) $n = 4$

(d) $n = 5$ ← correct

(e) $n = 6$

$$R_n \leq \int_n^\infty \frac{3}{x^4}\,dx$$
$$= \left[-x^{-3}\right]_n^\infty$$
$$= \frac{1}{n^3} \leq \frac{1}{100}$$
$$n^3 \geq 100$$
$$n \geq 5$$

# Common Exam II Prep

The series $\sum_{n=1}^{\infty}\left(3^{1/n} - 3^{1/(n+1)}\right)$

(a) converges to 3

(b) converges to 2 $\leftarrow$ correct

(c) converges to 1

(d) converges to $\frac{1}{3}$

(e) diverges

Telescoping series

$$\sum_{n=1}^{\infty}\left[3^{\frac{1}{n}} - 3^{\frac{1}{n+1}}\right]$$

$$= \lim_{N\to\infty}\sum_{n=1}^{N}\left[3^{\frac{1}{n}} - 3^{\frac{1}{n+1}}\right]$$

$$= \lim_{N\to\infty}\left\{\left[3^{\frac{1}{1}} - 3^{\frac{1}{2}}\right] + \left[3^{\frac{1}{2}} - 3^{\frac{1}{3}}\right] + \cdots + \left[3^{\frac{1}{N}} - 3^{\frac{1}{N+1}}\right]\right\}$$

$$= \lim_{N\to\infty}\left\{3^{\frac{1}{1}} - 3^{\frac{1}{N+1}}\right\} = 3 - 1 = 2$$

# Common Exam II Prep

Which of the following is true for the three sequences below?

(I) $a_n = \ln(2n+3) - \ln n$ (II) $a_n = \dfrac{\ln(n^2)}{n}$ (III) $a_n = n\sin\left(\frac{1}{n}\right)$

(a) Only (I) and (II) converge.

(b) Only (II) converges.

(c) Only (II) and (III) converge.

(d) Only (I) and (III) converge.

(e) All three converge. ← correct

(1) $a_n = \ln\left|\frac{2n+3}{n}\right| = \ln\left|2+\frac{3}{n}\right| \to \ln 2$ as $n \to \infty$

(2) Recall $\ln x \ll x^{\frac{1}{n}} \ll x \ll x^n \ll e^x$ for large $x$

$a_n = \dfrac{2\ln n}{n} \to 0$ as $n \to \infty$

(3) $\lim\limits_{n\to\infty} n\sin\left(\frac{1}{n}\right)$

Let $x = \frac{1}{n}$ then as $n \to \infty$, $x \to 0+$

$$\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right) = \lim_{x\to 0+}\frac{\sin x}{x} = 1$$

# Common Exam II Prep

Find the 2023rd term, $a_{2023}$, of the sequence $\{a_n\}$, assuming that the pattern of the first few terms continues beginning with $n = 1$.

$$\left\{ -\frac{4}{4}, \frac{8}{9}, -\frac{16}{16}, \frac{32}{25}, -\frac{64}{36}, \cdots \right\}$$

(a) $a_{2023} = -\dfrac{2^{2023}}{(2023)^2}$

(b) $a_{2023} = \dfrac{2^{2024}}{(2023)^2}$

(c) $a_{2023} = -\dfrac{2^{2024}}{(2024)^2}$ ← correct

(d) $a_{2023} = \dfrac{2^{2024}}{(2024)^2}$

(e) $a_{2023} = \dfrac{2^{2023}}{(2024)^2}$

Arithmetic sequence with
- The common difference $= d$
- Initial term $= a$

$a_n = a + d(n-1)$

Geometric sequence with
- The common ratio $= r$
- Initial term $= a$

$a_n = ar^{n-1}$

$4, 8, 16, 32, 64, \cdots$: Geometric sequence w/ $a = 4$ and $r = 2 \Rightarrow 4 \cdot 2^{n-1} = 2^{n+1}$

$4, 9, 16, 25, 36, \cdots$: Quadratic sequence $(n+1)^2$

Putting together $a_n = (-1)^n \dfrac{2^{n+1}}{(n+1)^2}$

# Common Exam II Prep

Find the sum of series $\sum_{n=1}^{\infty} a_n$ if

$$a_1 + a_2 + \cdots + a_n = \frac{2}{n(n+1)}$$

(a) 0 $\leftarrow$ correct

(b) $\frac{1}{2}$

(c) 1

(d) 2

(e) This series diverges

If $s_N = \sum_{n=1}^{N} a_n$ then $\lim_{N\to\infty} s_N = \sum_{n=1}^{\infty} a_n$

$\lim_{N\to\infty} \frac{2}{n(n+1)} = 0$

Suppose a sequence $\{a_n\}$ is increasing and bounded above by 5 for all positive integers $n$. Determine the convergence if the sequence satisfies

$$a_1 = 3 \quad \text{and} \quad a_{n+1} = 5 - \frac{4}{a_n}$$

(a) convergent to 4 $\leftarrow$ correct

(b) convergent to 1

(c) convergent to 5

(d) convergent to $\frac{2}{9}$

(e) divergent

Bounded Monotone Sequences are convergent to LUB/GLB

Let $\lim_{n\to\infty} a_n = a$ then

$a = 5 - \frac{4}{a}$

$a^2 = 5a - 4$

$a^2 - 5a + 4 = 0$

$(a-4)(a-1) = 0$

$a = 1,4$

# Common Exam II Prep

Find TRUE statements in the following.

I. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

II. If the $n$th partial sum, $\{s_n\}$, converges, then $\sum_{n=1}^{\infty} a_n$ converges.

III. The geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$.

IV. If $\{a_n\}$ is decreasing and $a_n \geq 0$ for all $n$, then $\lim_{n\to\infty} a_n = 0$.

(a) I and II only

(b) II and III only $\leftarrow$ correct

(c) III and IV only

(d) I, II, and III only

(e) II, III, and IV only

(I) False (Harmonic series)
(II) True (definition)
(III) True (formula $\frac{a}{1-r}$)
(IV) False (converges to the greatest lower bound)

# Common Exam II Prep

Which of the following series diverges by the Test for Divergence?

(I) $\sum_{n=1}^{\infty} \cos\left(\frac{n}{2n+3}\right)$ (II) $\sum_{n=1}^{\infty} \frac{2}{3+2^{4n}}$ (III) $\sum_{n=1}^{\infty} \frac{1}{\arctan n}$

(a) I only

(b) II only

(c) III only

(d) I and II only

(e) I and III only $\leftarrow$ correct

Divergent test (positive sequence)

If $\lim_{n\to\infty} a_n = a > 0$ then $\sum_{n=1}^{\infty} a_n = \infty$

(1) $\cos\left(\frac{n}{2n+3}\right) \to \cos 0 = 1$ divergent

(2) $\frac{2}{3+2^{4n}} \to 0$ undetermined

(3) $\frac{1}{\arctan(n)} \to \frac{1}{\pi/2}$ divergent

(8 points) Determine whether the series converges or diverges. If it converges, find the sum. Simplify your final answer.

$$\sum_{n=1}^{\infty} \frac{(-3)^n + 2^{2n}}{5^n}$$

Sum of two convergent geometric series ⇒ convergent

$$\sum_n \left\{ \left(-\frac{3}{5}\right)^n + \left(\frac{2^2}{5}\right)^n \right\}$$

$$= \sum_n \left(-\frac{3}{5}\right)^n + \sum_n \left(\frac{4}{5}\right)^n$$

$$= \frac{-\frac{3}{5}}{1+\frac{3}{5}} + \frac{\frac{4}{5}}{1-\frac{4}{5}}$$

$$= -\frac{3}{5+3} + \frac{4}{5-4}$$

$$= -\frac{3}{8} + 4$$

$$= \frac{29}{8}$$

# Common Exam II Prep

(11 points) Compute the following integral showing all necessary work clearly.

$$\int \frac{1}{(x^2+9)^{5/2}}\,dx$$

$$\int \frac{dx}{\{3^2((x/3)^2+1)\}^{5/2}}$$

$$= \int \frac{dx}{3^5((x/3)^2+1)^{5/2}}$$

Let $\frac{x}{3} = \tan\theta$ then $\left((x/3)^2+1\right) = \sec^2\theta$ and $dx = 3\sec^2\theta\, d\theta$

$$= \frac{1}{3^5}\int \frac{3\sec^2\theta\, d\theta}{[\sec^2\theta]^{5/2}}$$

$$= \frac{1}{3^4}\int \frac{\sec^2\theta\, d\theta}{\sec^5\theta}$$

$$= \frac{1}{3^4}\int \frac{d\theta}{\sec^3\theta} = \frac{1}{3^4}\int \cos^3\theta\, d\theta$$

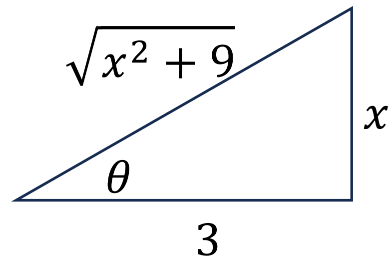
$$= \frac{1}{3^4}\int (1-\sin^2\theta)\cos\theta\, d\theta$$

Let $u = \sin\theta$ then $du = \cos\theta\, d\theta$

$$= \frac{1}{3^4}\int (1-u^2)du = \frac{1}{3^4}\left[u - \frac{1}{3}u^3\right] + C$$

$$= \frac{1}{3^4}\left[\sin\theta - \frac{1}{3}\sin^3\theta\right] + C$$

$$= \frac{1}{3^4}\left[\frac{x}{\sqrt{x^2+9}} - \frac{1}{3}\left(\frac{x}{\sqrt{x^2+9}}\right)^3\right] + C$$




$\frac{x}{3} = \tan\theta$

$\sin\theta = \frac{x}{\sqrt{x^2+9}}$

# Common Exam II Prep

(11 points) Compute the following integral showing all necessary work clearly.

$$\int \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)}\,dx$$

Partial fraction

$$\frac{4x^2-5x+11}{(x+1)(x-1)(x^2+4)} = \frac{\left[\frac{4x^2-5x+11}{(x-1)(x^2+4)}\right]_{x=-1}}{(x+1)} + \frac{\left[\frac{4x^2-5x+11}{(x+1)(x^2+4)}\right]_{x=1}}{(x-1)} + \frac{Ax+B}{(x^2+4)}$$

$$= \frac{\frac{4+5+11}{-2\cdot 5}}{(x+1)} + \frac{\frac{4-5+11}{2\cdot 5}}{(x-1)} + \frac{Ax+B}{(x^2+4)} = \frac{-2}{(x+1)} + \frac{1}{(x-1)} + \frac{Ax+B}{(x^2+4)}$$

$$= \frac{-2x+2+x+1}{(x+1)(x-1)} + \frac{Ax+B}{(x^2+4)}$$

$$\frac{4x^2-5x+11}{(x+1)(x-1)(x^2+4)} + \frac{x-3}{(x+1)(x-1)} = \frac{Ax+B}{(x^2+4)}$$

$$\frac{Ax+B}{(x^2+4)} = \frac{4x^2-5x+11+(x-3)(x^2+4)}{(x+1)(x-1)(x^2+4)} = \frac{4x^2-5x+11+x^3-3x^2+4x-12}{(x+1)(x-1)(x^2+4)}$$

$$= \frac{x^3+x^2-x-1}{(x+1)(x-1)(x^2+4)} = \frac{x^2(x+1)-(x+1)}{(x+1)(x-1)(x^2+4)}$$

$$= \frac{(x^2-1)(x+1)}{(x+1)(x-1)(x^2+4)} = \frac{x+1}{(x^2+4)}$$

$$\int \frac{4x^2-5x+11}{(x+1)(x-1)(x^2+4)}\,dx = \int \left[\frac{-2}{(x+1)} + \frac{1}{(x-1)} + \frac{x+1}{(x^2+4)}\right] dx$$

$$-2\ln|x+1| + \ln|x-1| + \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\arctan\frac{x}{2} + C$$

# Common Exam II Prep

(10 points) Compute the following integral or show it diverges. Correct mathematical notations must be shown throughout the work for full credit.

$$\int_0^1 x^2 \ln x \, dx$$

| $u$ | $v'$ |
|---|---|
| $\ln x$ | $x^2$ |
| $1/x$ | $2x$ |

$+$ , $-\int$

$$\lim_{a \to 0+} \int_a^1 x^2 \ln x \, dx$$

$$= \lim_{a \to 0+} \left\{ [2x \ln x]_a^1 - \int_a^1 2 dx \right\}$$

$$= \lim_{a \to 0+} \{ [2x \ln x - 2x]_a^1 \}$$

$$= \lim_{a \to 0+} \{ -2 - 2a \ln a + 2a \}$$

$$= \lim_{a \to 0+} (2a - 2) - 2 \lim_{a \to 0+} a \ln a$$

$$= -2 - 2 \lim_{n \to \infty} \frac{1}{n} \ln \frac{1}{n}$$

$$= -2 + 2 \lim_{n \to \infty} \frac{\ln n}{n}$$

$$= -2$$