Section 6.1: Interest and Effective Rates

Pr 1. You borrowed $1000 from a quick loan business for 52 days, at a simple interest rate of 42% per year. What is the interest you will pay on the loan?

simple interest

$I = Prt$, P = principal
r = rate (decimal)
t = time in years

$A = P + I$

$I = 1000 \times .42 \times \frac{52}{365}$

$I \approx \$59.83$

Pr 2. Suppose that $3000 accumulated to $4529.96 in an investment certificate, compounded weekly. If the annual interest rate is 2.3%, then how much time has passed? (in years)

- N = number of periods
- I% = interest rate (as %)
- PV = present value
- PMT = periodic payment = 0
- FV = future value
- P/Y = # of periods in a year
- C/Y =
- PMT: END BEGIN

In chapter 5,

$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

- use calculator instead

C/Y = n = 52

N = want to find
I% = 2.3
PV = −3000
PMT = 0
FV = 4529.96
P/Y = 52
C/Y = 52
END

amount is negative if you lose it (deposits, payments)

N = 931.91 periods → 932 periods

correct answer = $\frac{931.91}{52} = 17.92$

$\approx$ 18 years

Pr 3. How much should be invested now into an account paying 6.25% annual interest, compounded quarterly, for it to accumulate to $6,000 in 3 years?

N = 12
I% = 6.25
PV = ?
PMT = 0
FV = 6000
P/Y = 4
C/Y = 4
PMT: END BEGIN

n=4

t= 3 years

N = n·t = 4·3 = 12

PV = $ 4981.39

on TVM Solver,

-4981.39

Pr 4. How long will an investment need to remain in an account with a 2.028% annual interest rate, compounded continuously, in order for the investment to increase by 50%?

solving for t

n = ∞ → $A = Pe^{rt}$

t = ?

A =

P =

r = .02028

investment increase by 50%

$A = P + .5P$

$A = 1.5P$

$\frac{1.5P}{P} = \frac{Pe^{.02028t}}{P}$

$1.5 = e^{.02028t}$

$\ln(1.5) = \ln\left(e^{.02028t}\right) = .02028t \cdot \ln(e)$

$\ln(1.5) = .02028t$

$t = \frac{\ln(1.5)}{.02028} = \frac{\ln\left(\frac{A}{P}\right)}{r}$

$t \approx 20$ years

- effective interest rates
annual percentage yield[3]
↓
If you put one dollar into account, APY is the total interest after a year.

Pr 5. Consider the following three accounts:
A - interest rate 3.26%, compounded quarterly
B - interest rate 3.25%, compounded weekly
C - interest rate 3.24%, compounded continuously.

(a) Compute the effective interest rates for each of these accounts

A - interest rate 3.26%, $n=4$

calc → Eff (3.26, 4) = 3.3000 % A

B - Eff (3.25, 52) = 3.3023% B

C - effective interest rate r_{eff}

$e^r = \left(1 + \frac{r_{eff}}{1}\right)^1$ $r_e = e^r - 1$

Pe^{rt} $P\left(1+\frac{r}{n}\right)^{nt}$

$e^{.0324} - 1 = .032930$

→ 3.293% C

(b) Which account is the best one to use for investing?

B, because it has the highest effective interest rate.

Section 6.2: Annuities, Sinking Funds, and Amortization

Pr 1. You have just given birth to a child. You decide to save up for their college education. You make an initial deposit of $2000 into an account that earns 2.4 % interest, compounded monthly. You also put $500 per month into the account. How much will be in the account after 18 years?

N = 216
I% = 2.4
PV = -2000
PMT = -500
FV = ?
P/Y = 12
C/Y = 12
PMT: END BEGIN

→ we want FV.

I

N = 18 × 12 =

$137,997.02

Extra- Total amount we put into account?

500 × 216 + 2000

= 110,000

Extra- Total interest = FV - Total paid

≈ 27,997.02

Pr 2. You have just started your career. Your goal is to have one million dollars in your savings account when you retire 30 years from now. Your savings account earns 1.2% interest, compounded monthly. How much do you need to put into the account each month to reach your retirement goals?

N = 12

N = 360
I% = 1.2
PV = 0
PMT =
FV = 1000000
P/Y = 12
C/Y = 12
PMT: END BEGIN

→ want to solve for PMT

N = 12 × 30 = 360

PMT = $2309.08

Pr 3. You borrowed $ 27,000 in subsidized loans to pay tuition for four years of college. Now that you have been out of college for a year, your loan company is going to start charging annual interest at a rate of 3.5%, compounded monthly. You find that you can afford to pay $300 per month. How long will it take to pay off the loan?

N =
I% = 3.5
PV = +27000
PMT = -300
FV = 0
P/Y = 12
C/Y = 12
PMT: END BEGIN

for loans
PV = original loan amount
FV = 0

N = 104.5 periods
= 104.5 months → 105 months

$\frac{105}{12}$ = 8.708 years

Pr 4. Congratulations, you won 10 million in the lottery. The lottery gives you 30 annual payments. Suppose that the account has a 2.7% annual interest, compounded monthly. How much money will the lottery give you if you choose to take the lump sum instead?

N = 30
I% = 2.7
PV = ?
PMT = 333333.33
FV = 0
P/Y = 1
C/Y = 1
PMT: END BEGIN

PMT = $\frac{10000000}{30}$ = 333333.33

PV = lump sum

lump sum = $ 6,794,289.78

Pr 5. Ten years ago, we decided to buy a house for $280,000. To avoid having to pay mortgage insurance, we decided to make a 20% down payment. We took out a 30-year loan for the rest. The loan charges 4.6% interest, compounded monthly.

(a) How much was the original amount of the loan?

problem involving down payment.

Most common answer = 280,000 ← wrong

Loan Amount = Purchase Price − down payment

= Purchase × (1 − d%)

$280000(1-.2) = .8(280000)$

= $224000

(b) How much have we paid in interest on the loan so far? (10 years have passed)

↘ Solve (c) first

Total paid = 1148.32 × 120 + .2 × 280000

1148.32: PMT; 120: # of payments so far; .2 × 280000: down payment

A = Total paid − 280000 + outstanding balance

= $93769.4

= A + B = A + (280000 − outstanding balance)

A: interest; B: amount towards principal

(c) What is the outstanding principal on the loan? (outstanding balance) → amount we still owe.

↗ Solve

Preliminary work:

N = 30 × 12

I = 4.6

PV = 224000

PMT = ? −1148.32

FV = 0

P/Y = C/Y = 12

Approach 1: Solve for "FV"

N = ← number of periods so far = 10 × 12 = 120

I = 4.6

PV = 224000

PMT = ? −1148.32 →

FV = ?

P/Y = 12

C/Y = 12

FV = 179,971.05

Approach 2: solve for PV

FV = 0

N = # of periods remaining

↳ N = (30 − 10) × 12

= 20 × 12

= 240

Outstanding Balance = $179,971.05

Ten years ago, we decided to buy a house for $280,000. To avoid having to pay mortgage insurance, we decided to make a 20% down payment. We took out a 30-year loan for the rest. The loan charges 4.6% interest, compounded monthly.

(d) What is the current equity for the house?

How much value is in the house

= Purchase Price − outstanding balance

= 280000 − 179971.05

= $100,028.95

(e) How much of the first payment went towards interest, and how much went towards the principal?

first Payment = 1148.32

first interest charge = $P\frac{r}{n}(1)$

ith interest amount = $P_{i-1} \cdot \frac{r}{n}$

$224000 \times \frac{.046}{12} = \$858.67$ towards interest

1148.32
− 858.67
amount towards principal

= $289.65

You can use the TVM Solver to solve for similar questions