Math 308: Week-in-Review 4

1. Solve the initial value problems

   (a)
   $$y'' + 3y' + 2y = 0, \quad y(0) = 0, \;\; y'(0) = -1.$$

   (b)
   $$y'' - 6y' + 9y = 0, \quad y(0) = 2, \;\; y'(0) = -1.$$

(c)

$$y'' - 4y' + 13y = 0, \quad y(0) = 0, \;\; y'(0) = 3.$$

2. Find the initial value problems (equations and initial conditions) that have the solutions

(a)

$$y(t) = 4e^{-t} - e^{-2t}$$

(b)

$$y(t) = e^{4t} + 2te^{4t}$$

(c)

$$y(t) = 2e^{-t}\cos(2t) + 3e^{-t}\sin(2t)$$

3. Verify that $y_1(t) = \cos(\ln t)$ and $y_2(t) = \sin(\ln t)$ are solutions of the differential equation

$$t^2y'' + ty' + y = 0, \quad t > 0.$$

Do they constitute a fundamental set?

4. Suppose $y_1(t) = t^{-2}$ is a solution of the differential equation

$$t^2y'' + 5ty' + 4y = 0,\ t > 0.$$

Determine a second linearly independent solution $y_2(t)$.

5. If the differential equation

$$t^2y'' - 3ty' + 4y = 0,\ t > 0$$

has a fundamental set of solutions $y_1(t)$ and $y_2(t)$ and $W[y_1, y_2](2) = 8$, find the value of $W[y_1, y_2](3)$ without solving the differential equation.

6. Find a general solution of

$$4t^2y'' + 4ty' + (4t^2 - 1)y = 0$$

given that $y = t^{-1/2}\cos(t)$ is one solution.