Math 152 - Exam 2 Review

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1. Given the sequence $\{a_n\} = \left\{2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, ...\right\}$,

   (a) find the n_{th} term.

   (b) find the limit of the sequence.

   (c) Is this a monotonic sequence?

   (d) Is this a bounded sequence?

2. Is the sequence given by $a_n = \frac{\ln n}{n}$ increasing or decreasing? Find the limit of the sequence.

Find the limit of the sequence.

3. $a_n = \dfrac{n^2 + 2n - 5}{2n^2 + 1}$

4. $\left(1 + \dfrac{3}{n}\right)^{2n}$

5. $a_n = \dfrac{4n + (-1)^n}{n}$

6. Given the recursive sequence $\{a_n\}$ where $a_1 = 2$ and $a_{n+1} = 1 - \dfrac{1}{a_n}$, find the limit of the sequence if it converges.

7. Find the limit of the sequence $a_n = \dfrac{(-1)^n(2n^2+2)}{3n^2+1}$ or show that it diverges.

8. Find the 3^{rd} partial sum S_3 of the series $\sum a_n = \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$.

9. If the nth partial sum of a series is given by $S_n = 3 - n2^{-n}$,

   (a) find a_n

   (b) find $\sum_{n=1}^{\infty} a_n$

10. Find the nth partial sum S_n of the series $\sum_{n=1}^{\infty} \frac{3}{n^2+n}$ and then determine if the series converges or diverges.

11. Find the sum s of the series $\sum_{n=3}^{\infty} 10\left(\frac{2}{5}\right)^{n-1}$.

12. Compute the sum of the series $\sum_{n=0}^{\infty}\left\{\left(\frac{1}{2}\right)^{n}+\left(\frac{2}{3}\right)^{n}\right\}$

Do the following series converge or diverge? If they converge, find the sum of the series.

13. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4+e^{-n}}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+1}{e^n}$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^3}$

16. Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges or diverges.

17. Explain why the Integral Test can NOT be used to determine whether the series $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{1+n^2}$ is convergent.

18. Given that the 10th partial sum for the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is $s_{10} = 1.64522$,

(a) Find the error when using the 10th partial sum to approximate the sum of the series.

(b) How many terms n would be required so that the error $s \approx s_n$ is less than 0.001?

Use the Comparison test to determine whether the following integrals converge or diverge.

19. $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$

20. $\displaystyle\int_1^\infty \frac{\cos^2 x}{x^2}\,dx$

21. $\displaystyle\int_1^\infty \frac{2+\cos x}{\sqrt{x^4+x^2}}\,dx$

22. $\displaystyle\int_1^\infty \frac{2+e^{-x}}{x}\,dx$

23. Is the integral $\displaystyle\int_{-1}^{2} \frac{x}{(x+1)^2}\, dx$ convergent or divergent?

24. Is the integral $\displaystyle\int_{-\infty}^{0} \frac{1}{3-4x}\, dx$ convergent or divergent?

25. Use partial fractions to evaluate the integral $\int \frac{4x}{x^3+x^2+x+1}\,dx$

26. Evaluate the integral $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$