Math 140: Week-in-Review 4 (3.1-3.3)

1. **Set up the following linear programming problem, but do not solve**

   Nash Furniture Company manufactures tables and chairs. Each table requires 45 feet of wood and 4 labor-hours. Each chair requires 20 feet of wood and 5 labor-hours. The profit from the sale of each table is \$65, and the profit for each chair is \$30. In a certain week, the company has 4300 feet of wood available and 480 labor-hours. How many tables and chairs should Nash Furniture Company manufacture and sell to maximize its profits?

   unknowns / variables

   objective

   Variables:

   t := number of tables manufactured and sold

   c := number of chairs manufactured and sold

   p := profit from sales (in \$)

   Objective: Maximize/Minimize $p = 65t + 30c$

   Subject to:

   constraints

   $45t + 20c \leq 4300$ (wood in feet)

   $4t + 5c \leq 480$ (labor-hours)

   $\left.\begin{array}{l} t \geq 0 \\ c \geq 0 \end{array}\right\}$ non-negativity constraints

2. **Set up the following linear programming problem, but do not solve**

A housing contractor wants to develop a 42 acre tract of land. He has three types of houses: a small 3 bedroom, a large 3 bedroom and a 4 bedroom house. The small three bedroom house requires \$70,000 of capital for a profit of \$20,000, the large three bedroom house requires \$84,000 of capital for a profit of \$25,000, and the four bedroom house requires \$100,000 of capital for a profit of \$24,000. The small three bedroom needs 3000 labor hours, the large three bedroom needs 3500 labor hours, and the 4 bedroom house needs 3900 labor hours. There are currently 250,000 labor hours available. If the small three bedroom house is on half an acre, the large 3 bedroom house is on 0.75 acres, the four bedroom house is on 1.5 acres and the contractor has \$6 million in capital, how many of each type should be built to maximize the profit?

variables   objective

Variables:

x := number of small 3 bedroom houses built and sold

y := number of large 3 bedroom houses built and sold

z := number of 4 bedroom houses built and sold

P := profit from sale of the houses (in thousands of \$)

Objective: Maximize/Minimize $P = 20x + 25y + 24z$

Subject to:

constraints

$70x + 84y + 100z \leq 6{,}000$ (capital in thousands of \$)

$3x + 3.5y + 3.9z \leq 250$ (labor hours in thousands)

$0.5x + 0.75y + 1.5z \leq 42$ (acres of land)

$x \geq 0$, $y \geq 0$, $z \geq 0$ (non-negativity)

3. **Set up the following linear programming problem, but do not solve**

Etina Mining Company operates two mines for the purpose of extracting gold and copper. The Lonekop Mine costs $20,000 per day to operate and it yields 1.5kg of gold and 35kg of copper each day. The Mimosa Mine costs $25,000 per day to operate and it yields 2kg of gold and 15kg of copper each day. The company has a target of at least 22kg of gold and 230kg of copper. How many days should each mine be operated so as to meet the target at a minimum cost?

variables

objective

Variables

l = number of days operating Lonekop Mine

m = number of days operating Mimosa Mine

C = cost of operating the two mines (in thousands of $)

Objective: Minimize $C = 20l + 25m$

Subject to:
$$\begin{cases} 1.5l + 2m \geq 22 & \text{(gold target in kg)} \\ 35l + 15m \geq 230 & \text{(copper target in kg)} \\ l \geq 0,\ m \geq 0 & \text{(non-negativity)} \end{cases}$$

4. **Set up the following linear programming problem, but do not solve**

You have \$15,000 to invest, some in Stock A and some in Stock B. You have decided that the money invested in Stock A must be at least twice as much as that in Stock B. However, the money invested in Stock A must not be greater than \$12,000. If Stock A earn 4% annual interest, and Stock B earn 5% annual interest, how much money should you invest in each to maximize your annual interest?

variables   objective

Variables

a = amount invested in Stock A (in \$)

b = amount invested in Stock B (in \$)

I = annual interest earned (in \$)

Objective: Maximize $I = 0.04a + 0.05b$

Subject to: $a + b \leq 15{,}000$ (\$ available to invest)

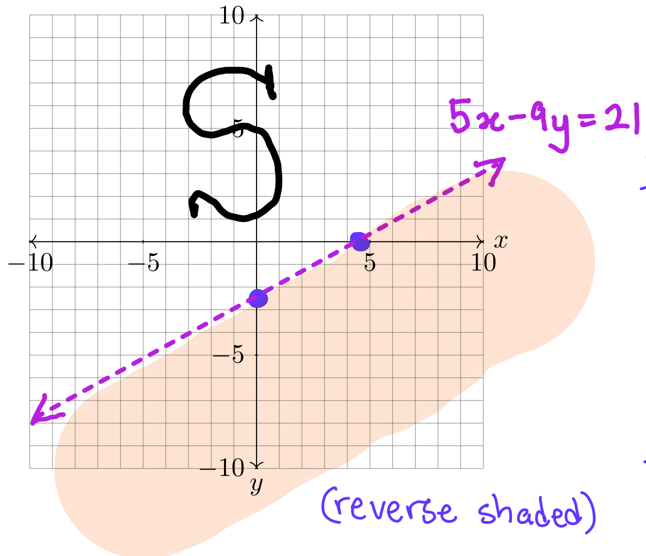
$a \geq 2b$ (ratio)

$a \leq 12{,}000$ (max investment in Stock A)

$a \geq 0,\ b \geq 0$ (non-negativity)

5. Graph the inequality $5x - 9y < 21$, labeling the boundary line and the solution set with **S**.

strict



(reverse shaded)

Boundary line (dashed)

$5x - 9y = 21$

x-intercept : $y = 0$

$\left(\frac{21}{5}, 0\right)$

y-intercept ; $x = 0$

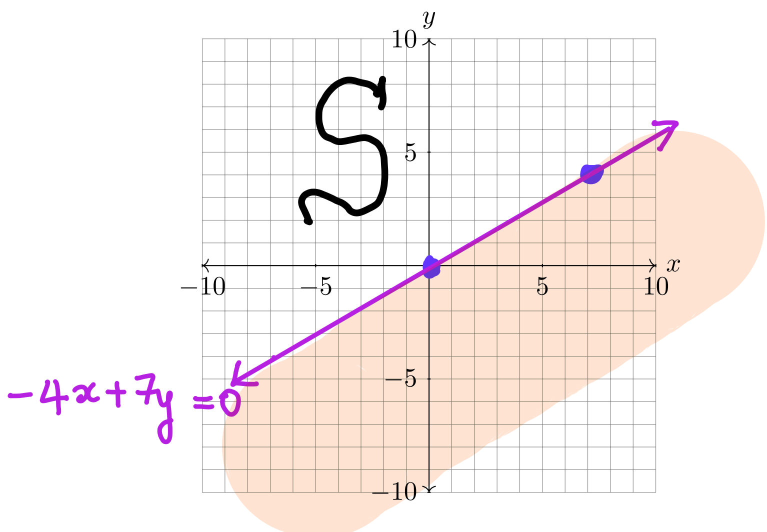
$\left(0, -7/3\right)$

Test point: $(0,0)$

$5(0) - 9(0) \overset{?}{<} 21 \Rightarrow 0 < 21$ ✓

6. Graph the inequality $-4x + 7y \geq 0$, labeling the boundary line and the solution set with **S**.

non-strict



Boundary line (solid)

$-4x + 7y = 0$

x-intercept : $y = 0$

$(0,0)$

y-intercept : $x = 0$

$(0,0)$

* need another point *

Let $y = 4$. Then $x = 7$

$(7,4)$

Test point : $(5,0)$

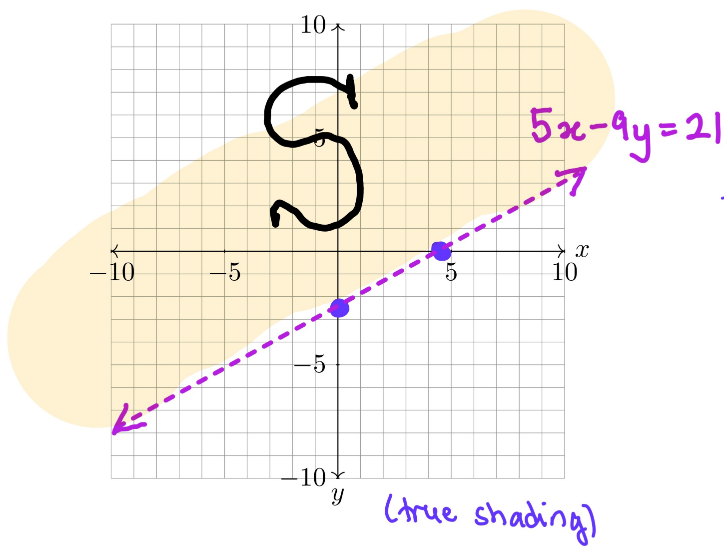
$-4(5) + 7(0) \overset{?}{\geq} 0$

$-20 \geq 0$ ✗

(reverse shaded)

5. Graph the inequality $5x - 9y < 21$, labeling the boundary line and the solution set with **S**.

strict



Boundary line (dashed)

$5x - 9y = 21$

x-intercept : $y = 0$

$\left(\frac{21}{5}, 0\right)$

y-intercept : $x = 0$

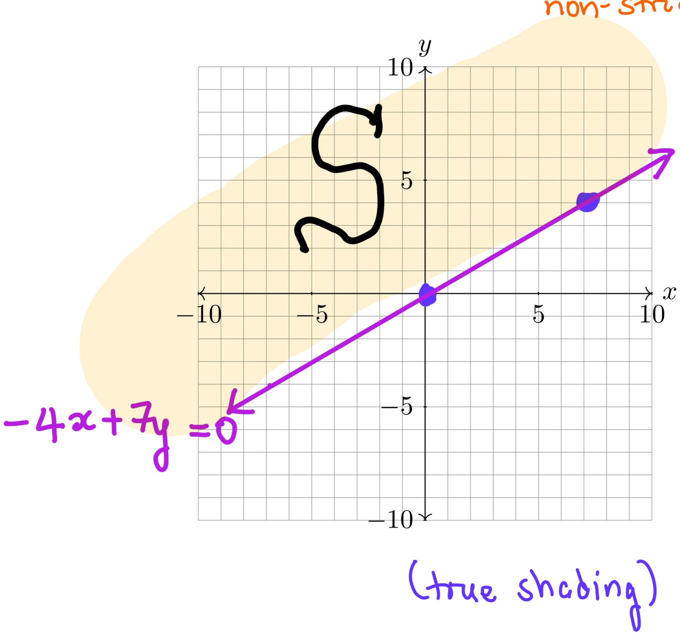
$\left(0, -\frac{7}{3}\right)$

Test point: $(0,0)$

$5(0) - 9(0) \overset{?}{<} 21 \Rightarrow 0 < 21$ ✓

6. Graph the inequality $-4x + 7y \geq 0$, labeling the boundary line and the solution set with **S**.

non-strict



Boundary line (solid)

$-4x + 7y = 0$

x-intercept : $y = 0$

$(0,0)$

y-intercept : $x = 0$

$(0,0)$

* need another point *

Let $y = 4$. Then $x = 7$

$(7,4)$

Test point : $(5,0)$

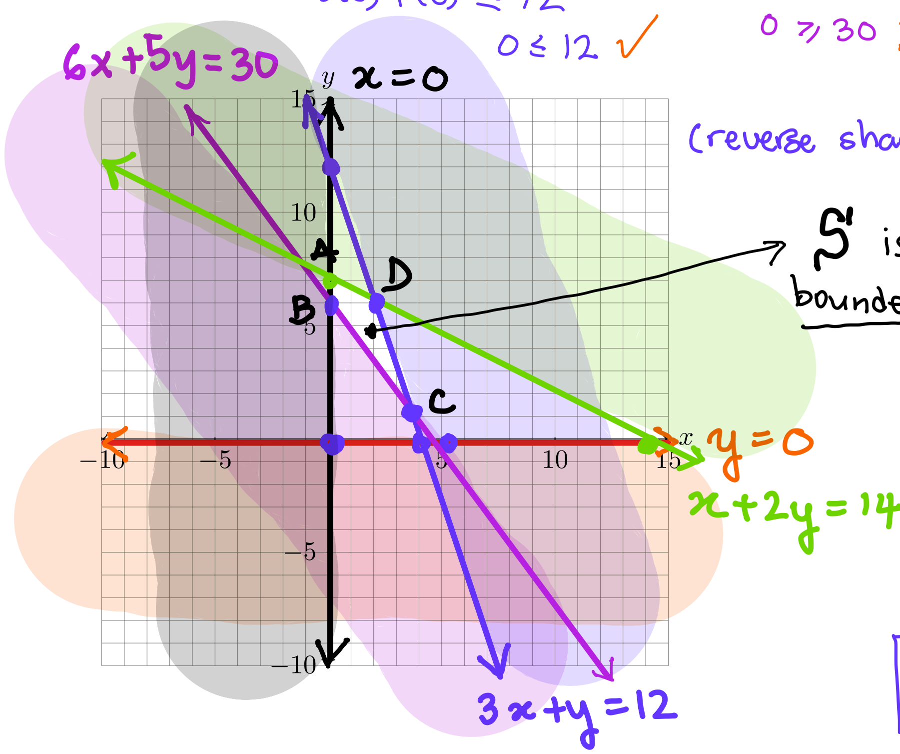
$-4(5) + 7(0) \overset{?}{\geq} 0$

$-20 \geq 0$ ✗

7. Graph the system of linear inequalities below, and then label the solution set with **S**. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

$$\begin{aligned} 3x + y &\leq 12 \\ 6x + 5y &\geq 30 \\ x + 2y &\leq 14 \\ x \geq 0,\ \ & y \geq 0 \end{aligned}$$

	$3x + y \leq 12$	$6x + 5y \geq 30$	$x + 2y \leq 14$	$x \geq 0$	$y \geq 0$
Boundary Line:	(solid) $3x+y=12$	(solid) $6x+5y=30$	(solid) $x+2y=14$	(solid) $x=0$	(solid) $y=0$
				Quadrant I	
***x*-intercept**:	$(4,0)$	$(5,0)$	$(14,0)$		
***y*-intercept**:	$(0,12)$	$(0,6)$	$(0,7)$		
Test Point:	$(0,0)$ $3(0)+(0) \overset{?}{\leq} 12$ $0 \leq 12$ ✓	$(0,0)$ $6(0)+5(0) \overset{?}{\geq} 30$ $0 \geq 30$ X	$(0,0)$ $0+2(0) \overset{?}{\leq} 14$ $0 \leq 14$ ✓		



(reverse shaded)

S is bounded

corner point C

$3x+y=12,\ 6x+5y=30$

$$\left[\begin{array}{cc|c} 3 & 1 & 12 \\ 6 & 5 & 30 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 10/3 \\ 0 & 1 & 2 \end{array}\right]$$

$\left(\frac{10}{3}, 2\right)$

corner point D

$x+2y=14,\ 3x+y=12$

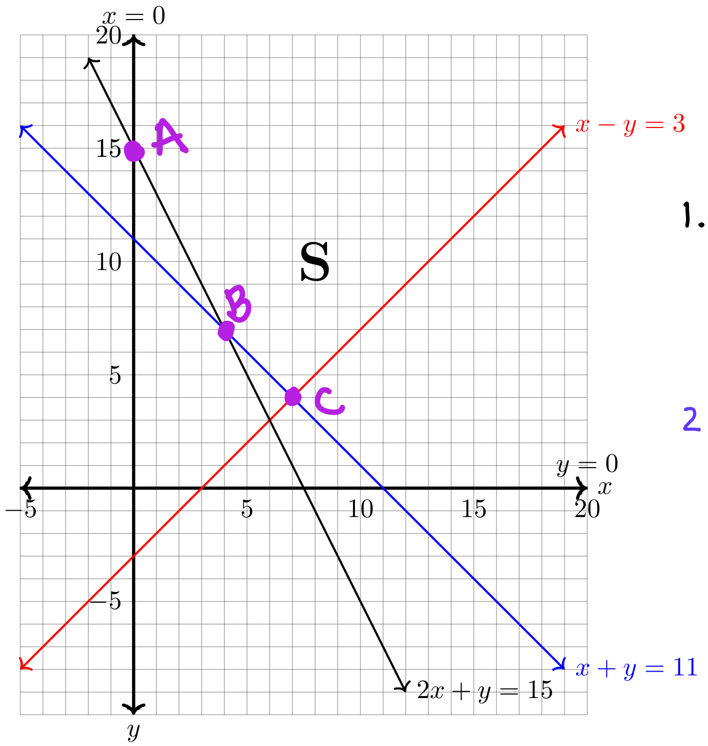
$$\left[\begin{array}{cc|c} 1 & 2 & 14 \\ 3 & 1 & 12 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 6 \end{array}\right]$$

Corner Points:

A: $(0,7)$ C: $\left(\frac{10}{3}, 2\right)$

B: $(0,6)$ D: $(2,6)$

8. Use the graph below to write the corresponding system of linear inequalities.



* all lines are solid ⇒ non-strict inequalities

* use test points to test for type of inequality ($\leq$ or $\geq$)

1. $2x + y = 15$

$2(0) + 0 \ \boxed{\geq} \ 15$

(0,0) not in solution set

2. $x + y = 11$

$0 + 0 \ \boxed{\geq} \ 11$

(0,0) not in solution set

3. $x - y = 3$

$0 - 0 \ \boxed{\leq} \ 3$

(0,0) is in solution set

4. non-negativity constraints satisfied in S

$x \geq 0, y \geq 0$

Inequalities

$2x + y \geq 15$

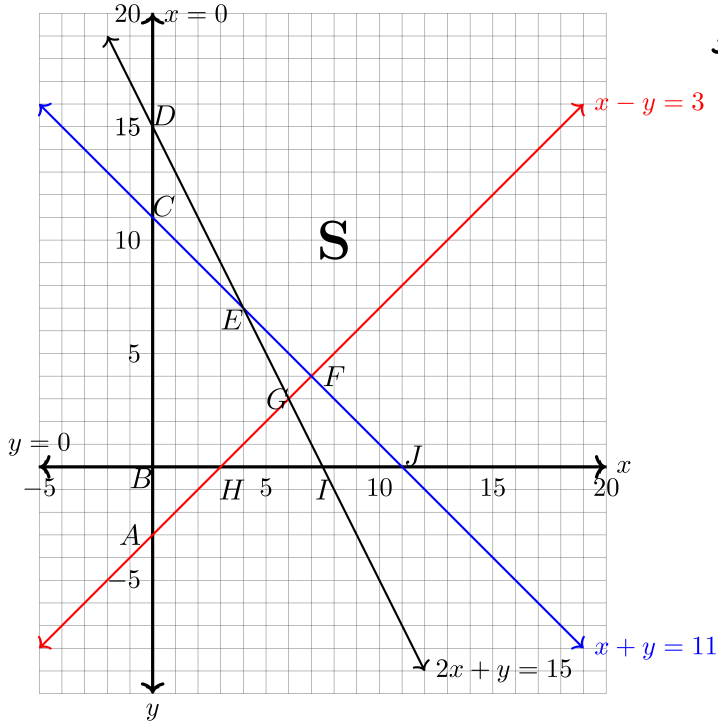
$x + y \geq 11$

$x - y \leq 3$

$x \geq 0, y \geq 0$

* unless a line passes through the origin, it is encouraged to use (0,0) as a test point *

9. Use the feasible region to determine the maximum and minimum values of the objective function $z = 2x + y$ over the region, if they exist and where they occur.



* S is unbounded
* S is in Quadrant I
* The coefficients of the objective function $z = 2x + y$ are positive ← positive
* Hence, by the Fundamental Theorem of Linear Programming, z has a MINIMUM in S, but not a MAXIMUM

(x, y)	corner pnt	z
A: $(0, -3)$	X	
B: $(0, 0)$	X	
C: $(0, 11)$	X	
D: $(0, 15)$	✓	$z = 2(0) + 15 = 15$ * minimum
E: $(4, 7)$	✓	$z = 2(4) + 7 = 15$ *
F: $(7, 4)$	✓	$z = 2(7) + 4 = 18$
G: $(6, 3)$	X	
H: $(3, 0)$	X	
I: $(7.5, 0)$	X	
J: $(11, 0)$	X	

The minimum value is $z = 15$ and it occurs at all points on the line segment joining the corner points D and E.
The FTLP says z has no maximum in S.

10. Use the Method of Corners to solve the following linear programming problem.

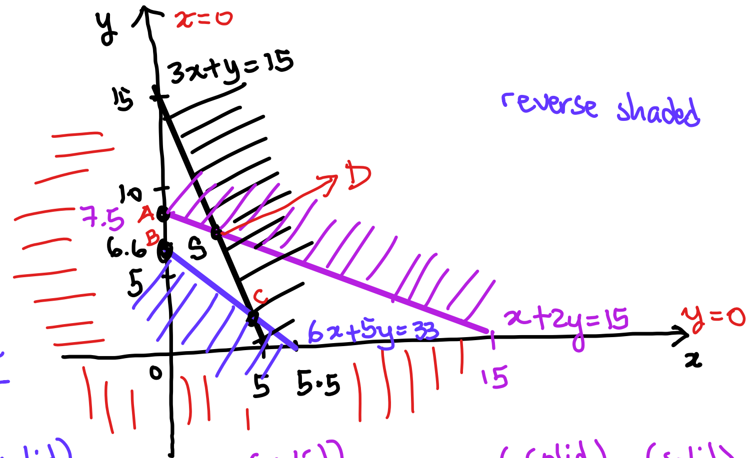
Objective: Maximize $P = 12x + 8y$

Subject to: $3x + y \leq 15$

$6x + 5y \geq 33$

$x + 2y \leq 15$

$\boxed{x \geq 0,\ y \geq 0}$ → Quadrant I



	(solid)	(solid)	(solid)	(solid)	(solid)
Boundary line:	$3x+y=15$,	$6x+5y=33$,	$x+2y=15$	$X=0$	$y=0$
x-int:	$(5,0)$	$(\frac{11}{2},0)$	$(15,0)$	y-axis	x-axis
y-int:	$(0,15)$	$(0,\frac{33}{5})$	$(0,\frac{15}{2})$	$X \geq 0,\ y \geq 0$	
Test point:	$(0,0)$	$(0,0)$	$(0,0)$		
	$3(0)+0 \overset{?}{\leq} 15$	$6(0)+5(0) \overset{?}{\geq} 33$	$0+2(0) \overset{?}{\leq} 15$		
	$0 \leq 15$ ✓	$0 \geq 33$ ✗	$0 \leq 15$ ✓		

Corner pnt	P
A: $(0,7.5)$	$P = 12(0)+8(7.5) = 60$
B: $(0,6.6)$	$P = 12(0)+8(6.6) = 52.8$
C: $(\frac{14}{3},1)$	$P = 12(\frac{14}{3})+8(1) = 64$
D: $(3,6)$	$P = 12(3)+8(6) = 84$ * max

Corner point C

intersection of $6x+5y=33$, $3x+y=15$

$$\left[\begin{array}{cc|c} 6 & 5 & 33 \\ 3 & 1 & 15 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 14/3 \\ 0 & 1 & 1 \end{array}\right]$$

corner point D

intersection of $3x+y=15$, $X+2y=15$

$$\left[\begin{array}{cc|c} 3 & 1 & 15 \\ 1 & 2 & 15 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 6 \end{array}\right]$$

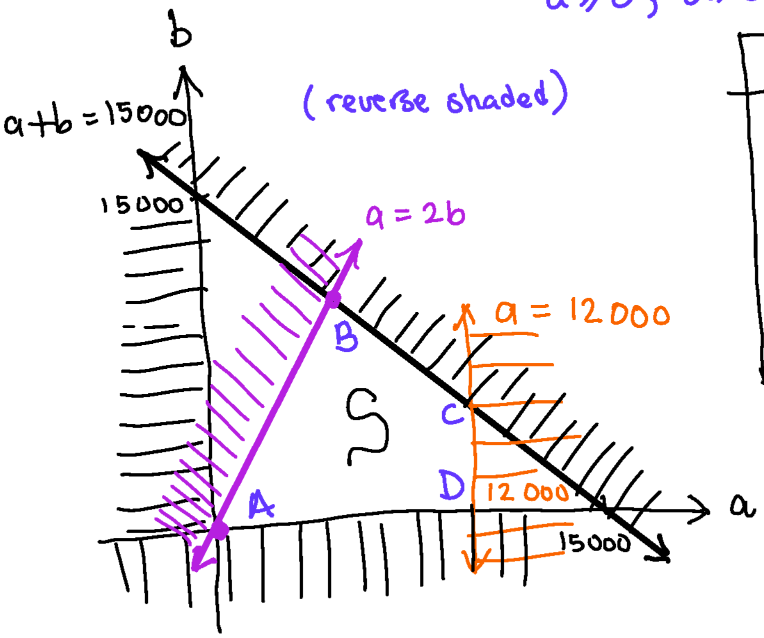
SOLUTION: The maximum value is $P = 84$ and it occurs at the point when $(x,y) = (3,6)$

11. **Solve using the Method of Corners**

You have \$15,000 to invest, some in Stock A and some in Stock B. You have decided that the money invested in Stock A must be at least twice as much as that in Stock B. However, the money invested in Stock A must not be greater than \$12,000. If Stock A earn 4% annual interest, and Stock B earn 5% annual interest, how much money should you invest in each to maximize your annual interest?

From problem 4,
a = amnt invested in Stock A (in \$)
b = amnt invested in Stock B (in \$)
I = total annual interest

Objective: max $I = 0.04a + 0.05b$
Subject to: $a + b \leq 15000$ (\$ available)
$a \geq 2b$ (ratio)
$a \leq 12000$ (maximum investment in Stock A)
$a \geq 0,\ b \geq 0$ (non-negativity)



corner pnt (a,b)	$I = 0.04a + 0.05b$
A: $(0,0)$	$I = 0$
B: $(10000, 5000)$	$I = 650$ * MAX
C: $(12000, 3000)$	$I = 630$
D: $(12000, 0)$	$I = 480$

SOLN: The maximum interest is $I = \$650$ when \$10,000 is invested in Stock A and \$5,000 is invested in Stock B.

12. A company manufactures two types of furniture: chairs and rockers. Each chair requires 1 box of screws, 2 units of plastic, and 4 units of wood. Each rocker takes 2 boxes of screws, 3 units of plastic, and 2 units of wood. The company has 65 boxes of screws, 110 units of plastic, and 105 units of wood on hand.

    In order to maximize their profit using these materials on hand, the company has determined that they must make 13 chairs and 22 rockers. How many of each of the materials (boxes of screws, units of plastic, and units of wood) are left over when the company makes 13 chairs and 22 rockers?

* c = number of chairs made and sold

* r = number of rockers made and sold

* to figure out the left-overs, we need the constraints

subject to:

(screws) $c + 2r \leq 65$

(plastic) $2c + 3r \leq 110$

(wood) $4c + 2r \leq 105$

* To maximize profit, we need $c = 13$ and $r = 22$ *

Resource	Amount used	Amount available	Amount left over (unused)
screws	$13 + 2(22) = 57$	65	8
plastic	$2(13) + 3(22) = 92$	110	18
wood	$4(13) + 2(22) = 96$	105	9

SOLUTION: To maximize profit, 13 chairs and 22 rockers are made and 8 boxes of screws, 18 units of plastic, and 9 units of wood are left over.