FINAL EXAM REVIEW

Exercise 1

Find the solution. Where is the solution defined?

$$y' - ty^2 = t, \qquad y(0) = 1.$$

Exercise 2

Find the general solution.

$$(\sin(x) + x^2 e^y)y' + y\cos(x) + 2xe^y - 2 = 0$$

Exercise 3

Without solving the equation, determine where a unique solution is guaranteed to exist.

$$\ln(t)y'' - y' + \tan(t)y = \sqrt{6-t}, \qquad y(6/5) = 16/5.$$

Exercise 4

Consider the differential equation

$$y' = y^3 - 5y^2 + 6y.$$

(a) Find the equilibrium solutions.

(b) Plot the direction field.

- Draw a few example solutions on the direction field.

(c) Draw the phase line diagram.

(d) Determine the stability of each equilibrium point.

Exercise 5

Suppose you take out a $20,000 loan with a 7% annual interest rate that is compounded continuously. Suppose you pay $300 per month on the loan. Write down a differential equation to describe the amount of the loan. How long will it take you to pay off the loan?

Exercise 6

Find an integrating factor that makes the following equation exact.

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

Exercise 7

Find the general solution.

$$u'' + 4u' + 4u = 0$$

Exercise 8

Find the general solution. Prove that it is indeed the general solution.

$$u'' + 4u' + 5u = 0$$

Exercise 9

Find a particular solution to

$$t^2y'' - 2y = 3t^2, \qquad t > 0,$$

given that t^2 and t^{-1} are solutions to the corresponding homogeneous equation.

Exercise 10

Find a particular solution.

$$y'' - 3y' - 10y = 2e^{5t} + 1$$

Exercise 11

Suppose there is a 20 N mass hanging on a spring. When the mass was attached to the spring, the spring stretched by 30 cm. When the mass is moving 3 m/s, it experiences a damping force of 12 N. There is an external upward force of 7 N acting on the mass for the first 9 seconds, after which there is no external force. Initially the mass sent into motion with a downward velocity of 40 cm/s from the equilibrium position. (Use $g = 10\,\mathrm{m/s^2}$.)

(a) Write down an initial value problem that describes the motion of the mass.

(b) Is the system over, under, or critically damped?

Exercise 12

Solve the initial value problem.

$$y'' - 3y' + 2y = u_1(t), \qquad y(0) = 0, \qquad y'(0) = 0.$$

Exercise 13

Using the definition of the Laplace transform, show that $\mathcal{L}\{t\} = \dfrac{1}{s^2}$.

Exercise 14

Find the general solution in the form of a power series centered at $x = 1$.

$$y'' - xy = 0.$$

Exercise 15

Find the general solution to the system of differential equations.

$$x_1' = x_1 - 4x_2$$
$$x_2' = 4x_1 - 7x_2$$