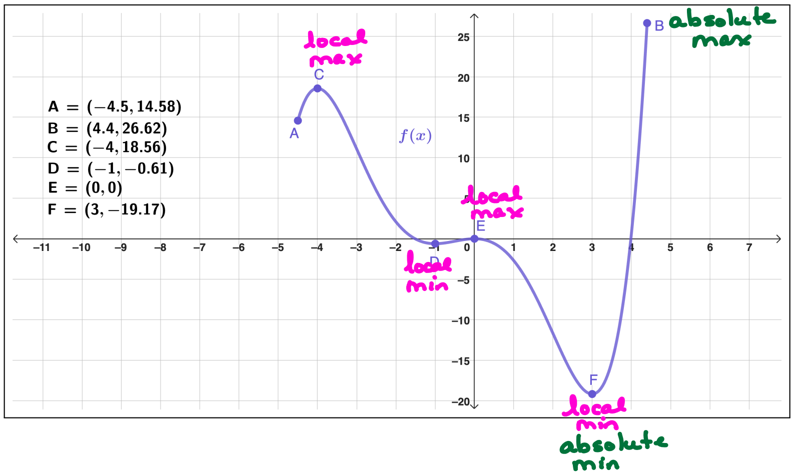
Session 8: Sections 3-4 and 3-5

1. For the graph of $f(x)$ shown below with domain $[-4.5, 4.4]$ find (a) any local extrema and (b) any absolute extrema, as well as where they occur, of $f(x)$. Specify whether an extremum is a minimum or maximum.



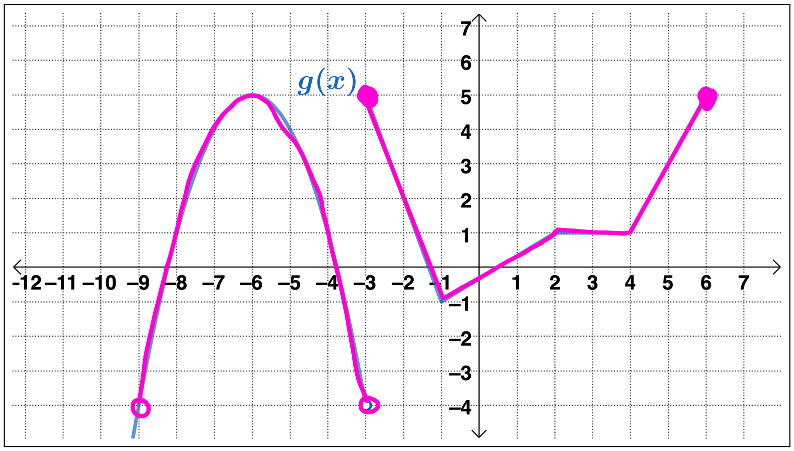
local max at
C (-4, 18.56)
E (0,0)

local min at
D (-1, -.61)
F (3, -19.17)

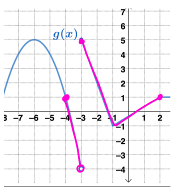
absolute max of 26.62 at x = 4.4.

absolute min of -19.17 at x = 3.

2. Use the graph of g below to find the absolute extrema of g on each of the given intervals, if they exist.



(a) $[-4, 2]$

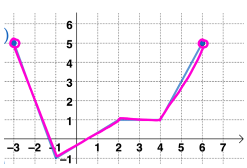


absolute max of 5 at $x = -3$
no absolute min

(b) $(-2, 6]$

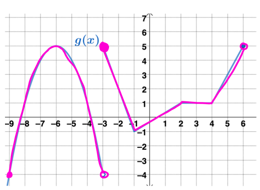
absolute min of -1 at $x = -1$
absolute max of 5 at $x = 6$

(c) $(-2, 6)$



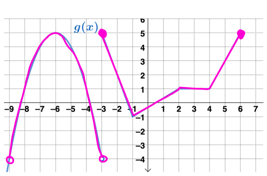
absolute min of -1 at $x = -1$
no absolute max.

(d) $[-9, 6)$



absolute min of -4 at $x = -9$
absolute max of 5 at $x = -3$

(e) $(-9, 6]$



no absolute min
absolute max of 5 at $x = -3, 6$.

3. Find the absolute maximum and minimum of $f(x) = x^3 + \frac{3}{2}x^2 - 18x$ on each of the given intervals.

$$f'(x) = 3x^2 + 3x - 18 = 3(x^2 + x - 6) = 3(x+3)(x-2)$$

$$0 = 3(x-2)(x+3)$$

$$x = 2, -3$$

(a) $[-5, 5]$

x	$f(x) = x^3 + \frac{3}{2}x^2 - 18x$
-5	2.5
-3	40.5
2	-22 absol. min
5	72.5 absol max

(b) $[-6, 4]$

x	$f(x)$
-6	-54 ← absolute min
-3	40.5 ← absolute max
2	-22
4	16

(c) $[-4, 0]$

x	$f(x)$
-4	32
-3	40.5 ← absolute max
0	0 ← absolute min

4. Determine the absolute extrema of $g(x) = e^{(x-2)^2}$ on $(1, 3)$.

$g'(x) = e^{(x-2)^2}\,(2(x-2))$

$0 = e^{(x-2)^2}\,(2x-4)$

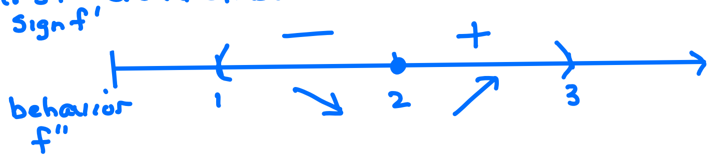
$e^{(x-2)^2} \neq 0$
Never

$2x - 4 = 0$
$2x = 4$
$x = 2$

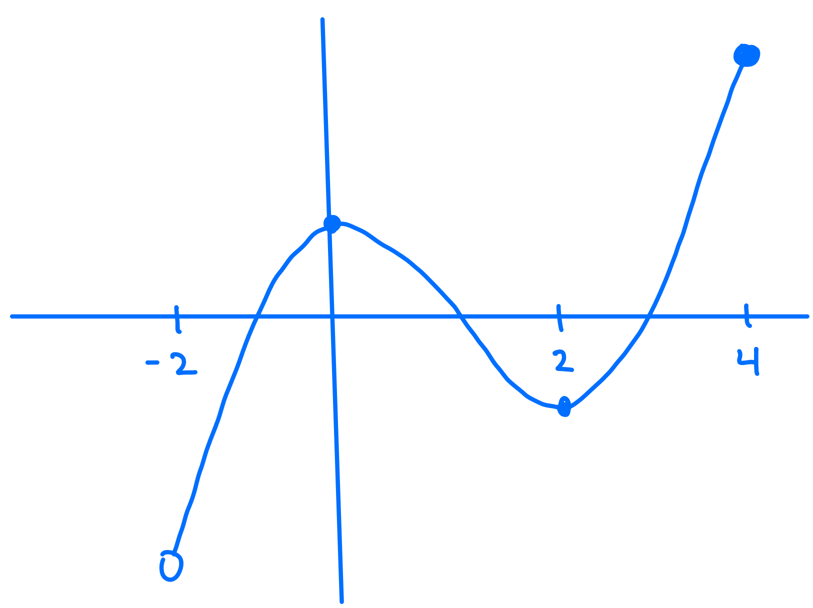
$\boxed{\text{absolute min of 1 at } x = 2. \quad \text{no absolute max}}$

Second derivative test
$g''(2) \approx 2 > 0 \Rightarrow$ absolute min.

first derivative test



5. Draw a graph of a function on the interval $(-2, 4]$ that has an absolute maximum at $x = 4$, a local maximum at $x = 0$, a local minimum at $x = 2$, and no absolute minimum.



6. Find two positive numbers such that the sum of the one and the square of the other is 200 and whose product is a maximum.

① maximize product

② no picture

③ objective function

$$P = xy$$

④ constraints

$$x + y^2 = 200$$

⑤ objective function in one variable

$$x = 200 - y^2 \longrightarrow P = (200 - y^2)y$$
$$= 200y - y^3$$

⑥ interval: $(0, \sqrt{200})$

y^2 cannot be bigger than 200

⑦ Derivative

$$P'(y) = 200 - 3y^2$$
$$0 = 200 - 3y^2$$
$$\sqrt{\frac{-200}{-3}} = y^2$$
$$\frac{10\sqrt{2}}{\sqrt{3}} = y$$

⑧ $x = 200 - y^2$

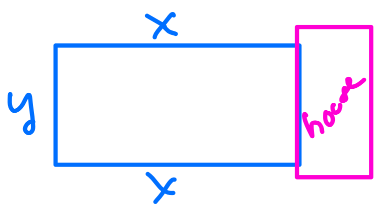
$$= 200 - \frac{200}{3}$$

$$\boxed{x = \frac{400}{3} \quad y = \sqrt{\frac{200}{3}}}$$

7. Mike has a 450 square foot area that he intends to use to build a rectangular enclosure for his Scottish terrier (a dog). He plans to build the enclosure against one side of his house, so fencing is needed on just three sides of the enclosure. Determine the dimensions that will minimize the amount of feet of fencing Mike can purchase to enclose the area as he intends.

① minimize fencing

② Picture



③ objective function

$$F = 2x + y$$

④ constraint equations

$$xy = 450$$

⑤ objective function in one variable

$$y = \frac{450}{x} \Longrightarrow F = 2x + 450x^{-1}$$

⑥ Interval

$(0, \infty)$

⑦ Derivative of F

$$F' = 2 - 450x^{-2}$$
$$0 = 2 - \frac{450}{x^2}$$
$$-2x^2 = -450$$
$$x^2 = \frac{450}{2} = 225$$
$$x = 15$$

⑧ $F''(15) = .26 > 0$

absolute min

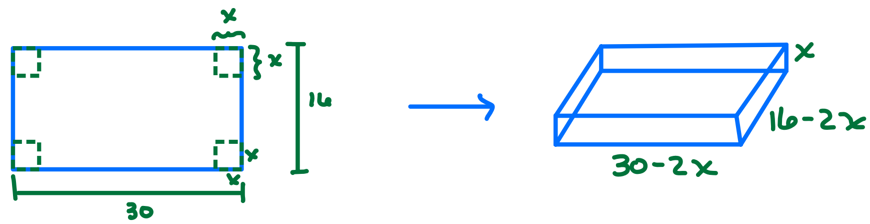
⑨ $\boxed{\text{15 feet by 30 feet}}$

$$y = \frac{450}{15} = 30$$

8. You have a piece of cardboard that is 30 cm by 16 cm and you want to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.

① maximize Volume

②



③ objective function: $V = (30-2x)(16-2x)x$

$= (30-2x)(16x-2x^2)$

$= 480x - 60x^2 - 32x^2 + 4x^3$

$V = 4x^3 - 92x^2 + 480x$

④ no constraint equations

⑤ interval: $(0, 8)$ ← $\frac{1}{2}$ of 16

⑥ Take derivative

$V' = 12x^2 - 184x + 480$

$= 4(3x^2 - 46x + 120)$

$= 4(3x-10)(x-12)$

12 — 3
10 — 1

$0 = 3x - 10$ $\qquad$ $x - 12 = 0$

$\frac{10}{3} = x$ $\qquad$ $\cancel{x = 12}$ side length is only 16

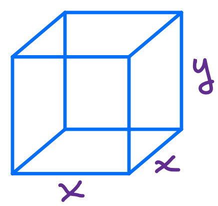
⑦ $c''(10/3) = -104 < 0$ ⟸ absolute max

⑧ Height of box should be $\frac{10}{3}$ cm.

9. A closed box is made with a square base and must have a volume of 343 cubic inches. The material for the sides and the top cost \$0.02 per square inch, and the material for the base costs \$0.04 per square inch. Determine the dimensions of the box that minimize the cost of the materials. Round to two decimals if necessary,

① minimize cost of materials

② Picture



③ objective function: $.02x^2 + .02(4xy) + .04x^2$

Cost of top — Cost of sides — Cost of bottom

$$C = .06x^2 + .08xy$$

④ Constraint equation Volume is 343 in^3.

$$x^2y = 343$$

⑤ objective function in terms of one variable

$$y = \frac{343}{x^2} = 343x^{-2}$$

$$C = .06x^2 + .08x(343x^{-2})$$

$$= .06x^2 + 27.44x^{-1}$$

⑥ Interval: $(0, \infty)$

⑦ Take derivative:

$$C'(x) = .12x - 27.44x^{-2} = 0$$

$$.12x = \frac{27.44}{x^2}$$

$$.12x^3 = 27.44$$

$x^3 = 228.67$

$x \approx 6.12$ inches

8 $c''(6.12) \approx .359 > 0 \Rightarrow$ absolute min

9 answer the question

$$y = \frac{343}{x^2} = \frac{343}{(6.12)^2} \approx 9.16$$

Dimensions are 6.12 in x 6.12 in x 9.16 in