Math 150 - Week-In-Review 2

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Problem Statements

$(x+3)(x-2) - (x-3)(x-4)$

$\cancel{x^2} + x - 6 - (\cancel{x^2} - 7x + 12)$

$8x - 18$

1. Perform the operations and simplify.

(a) $\dfrac{2x^2-5x-3}{6x^2+3x} \cdot \dfrac{3x^2+12x-15}{x^2+2x-15}$

(b) $\dfrac{x^2+5x-14}{x^2+8x+7} \div \dfrac{x^2-x-2}{x-3}$

check last week

(c) $\dfrac{x+1}{\underbrace{x^2-2x-8}_{(x-4)(x+2)}} - \dfrac{x-3}{\underbrace{x^2-4}_{(x-2)(x+2)}} = \dfrac{(x+1)(x-2) - (x-3)(x-4)}{(x-4)(x+2)(x-2)}$

$$\frac{(x^2-x-2)-(x^2-7x+12)}{(x-4)(x+2)(x-2)} = \frac{-x-2+7x-12}{(x-4)(x+2)(x-2)} = \frac{8x-14}{(x-4)(x+2)(x-2)}$$

(d) $\dfrac{\frac{1}{x} - \frac{1}{2x^2}}{\frac{2}{x} - 1 \cdot \frac{x}{x}}$ $\quad \frac{2x}{2x}$

$$= \frac{\frac{2x-1}{2x^2}}{\frac{2-x}{x}} = \frac{2x-1}{2x^{\cancel{2}}} \cdot \frac{\cancel{x}}{2-x} = \frac{(2x-1)}{2x(2-x)}$$

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2. Determine whether the function is even, odd, or neither. Then describe the symmetry.

(a) $f(x) = \dfrac{x(x^2-1)}{5x^4+1}$

$$f(-x) = \frac{-x[(-x)^2-1]}{5(-x)^4+1} = \frac{-x(x^2-1)}{5x^4+1} = -\frac{x(x^2-1)}{5x^4+1} = -f(x)$$

$\Rightarrow$ odd

(b) $g(x) = \sqrt[3]{x^2-1}$

$$g(-x) = \sqrt[3]{(-x)^2-1} = \sqrt[3]{x^2-1} = g(x) \Rightarrow \text{even}$$

(c) $h(x) = \dfrac{x^3-1}{x^4+2}$

$$h(-x) = \frac{(-x)^3-1}{(-x)^4+2} = \frac{-x^3-1}{x^4+2} \neq -h(x) = -\frac{x^3-1}{x^4+2} = \frac{-x^3+1}{x^4+2}$$

Neither!

3. If $h(x) = \dfrac{3x}{2} + 1$ evaluate the following:

(a) $h(a)$

$$h(a) = \frac{3a}{2} + 1$$

(b) $h(a+b)$ $= \dfrac{3(a+b)}{2} + 1 = \dfrac{3a}{2} + \dfrac{3b}{2} + 1$

(c) $\dfrac{h(a+b) - h(a)}{b}$ $= \dfrac{\left(\cancel{\frac{3a}{2}} + \frac{3b}{2} \cancel{+1}\right) - \left(\cancel{\frac{3a}{2}} \cancel{+1}\right)}{b} = \dfrac{3}{2}$

4. Consider the complex numbers $z_1 = 1 + \sqrt{-27}$ and $z_2 = 2 - \sqrt{-12}$.

(a) Write z_1 and z_2 in standard form.

$$z_1 = 1 + \sqrt{-3^2 \times 3} = 1 + 3\sqrt{-3} = 1 + 3\sqrt{3}\, i$$

$$z_2 = 2 - \sqrt{-4 \times 3} = 2 - 2\sqrt{-3} = 2 - 2\sqrt{3}\, i$$

(b) Find $z_1 + z_2$, $z_1 - z_2$, and $z_1 z_2$.

$$z_1 + z_2 = 3 + \sqrt{3}\, i$$

$$z_1 - z_2 = -1 + 5\sqrt{3}\, i$$

$$z_1 z_2 = (1 + 3\sqrt{3}\, i)(2 - 2\sqrt{3}\, i) = 2 - 2\sqrt{3}\, i + 6\sqrt{3}\, i - 6 \times 3\, i^2$$

$$= 2 + 4\sqrt{3}\, i + 18 = 20 + 4\sqrt{3}\, i$$

(c) Find the complex conjugate of z_1.

$$\bar{z}_1 = 1 - 3\sqrt{3}\, i$$

(d) Find $z_2 \div z_1$.

$$\frac{z_2}{z_1} = \frac{2 - 2\sqrt{3}\, i}{1 + 3\sqrt{3}\, i} \cdot \frac{1 - 3\sqrt{3}\, i}{1 - 3\sqrt{3}\, i} = \frac{(2 - 2\sqrt{3}\, i)(1 - 3\sqrt{3}\, i)}{1 + 27}$$

$$= \frac{2 - 6\sqrt{3}\, i - 2\sqrt{3}\, i + 18\, i^2}{28} = \frac{-16 - 8\sqrt{3}\, i}{28} = -\frac{4}{7} - \frac{2\sqrt{3}}{7}\, i$$

5. Solve the equation by using the quadratic formula. $x^2 = 5 - 2x$

$ax^2 + bx + c = 0 \qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 + 2x - 5 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2} = \frac{-2 \pm \sqrt{4 + 20}}{2} = \frac{-2 \pm \overbrace{\sqrt{24}}^{2\sqrt{6}}}{2}$$

$$= -1 \pm \sqrt{6}$$

$x = -1 + \sqrt{6}$ & $x = -1 - \sqrt{6}$

6. Solve the equation $5x^2 + 2x - 1 = 0$ by completing the square.

$$5\left[x^2 + \frac{2}{5}x - \frac{1}{5}\right] = 0$$

$$5\left[x^2 + \frac{2}{5}x + \left(\frac{2/5}{2}\right)^2 - \left(\frac{2/5}{2}\right)^2 - \frac{1}{5}\right] = 0$$

$$5\left[x^2 + \frac{2}{5}x + \frac{1}{25} - \frac{1}{25} - \frac{1}{5}\right] = 0$$

$$5\left[\left(x + \frac{1}{5}\right)^2 + \frac{-1-5}{25}\right] = 0$$

$$5\left(x + \frac{1}{5}\right)^2 - \frac{6}{5} = 0 \longrightarrow 5\left(x + \frac{1}{5}\right)^2 = \frac{6}{5} \longrightarrow \left(x + \frac{1}{5}\right)^2 = \frac{6}{25}$$

$$\Longrightarrow x + \frac{1}{5} = \pm\sqrt{\frac{6}{25}} \Longrightarrow x = -\frac{1}{5} \pm \sqrt{\frac{6}{25}} = \boxed{-\frac{1}{5} \pm \frac{\sqrt{6}}{5}}$$

7. Perform the indicated operation on the functions $f(x) = \dfrac{x-1}{x+2}$ and $g(x) = \sqrt{2x+3}$ and determine the domain of each new function.

$\text{Dom}(f) = (-\infty, -2) \cup (-2, \infty)$

$\text{Dom}(g) = [-\frac{3}{2}, \infty)$

a. $(f+g)(x) = f(x) + g(x) = \dfrac{x-1}{x+2} + \sqrt{2x+3}$

Domain: $[-\frac{3}{2}, +\infty)$

b. $(fg)(x) = \left(\dfrac{x-1}{x+2}\right)\left(\sqrt{2x+3}\right) = \dfrac{(x-1)\sqrt{2x+3}}{x+2}$

Domain: $[-\frac{3}{2}, \infty)$

c. $\left(\dfrac{f}{g}\right)(x)$ $\dfrac{\frac{x-1}{x+2}}{\sqrt{2x+3}} = \dfrac{x-1}{\underbrace{(x+2)(\sqrt{2x+3})}_{\text{Can't be zero}}}$

Domain: $(-\frac{3}{2}, \infty)$

d. $(f \circ g)(x)$ $= f(g(x)) = \dfrac{\sqrt{2x+3} - 1}{\sqrt{2x+3} + 2} = \dfrac{\sqrt{2x+3} - 1}{\sqrt{2x+3} + 2} \rightarrow$ $\sqrt{2x+3} \neq -2 \rightarrow$ No solution

Domain of Composition = domain of inside function $\cap$ domain of the result

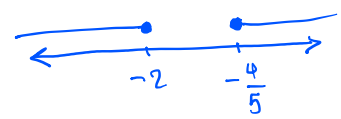
$\text{Dom}(f \circ g(x)) = \left[-\frac{3}{2}, \infty\right)$

e. $(g \circ f)(x) = g(f(x)) = \sqrt{2\left(\frac{x-1}{x+2}\right) + 3} = \sqrt{\frac{2x-2}{x+2} + 3} = \sqrt{\frac{5x+4}{x+2}}$

$x \neq -2$

$\frac{5x+4}{x+2} \geq 0 \rightarrow$ & $5x+4 \geq 0$, $x+2 \geq 0$ or $5x+4 \leq 0$, $x+2 \leq 0$

$(-\infty, -2] \cup \left[-\frac{4}{5}, \infty\right)$



$\text{Dom}(g \circ f) = (-\infty, -2) \cup \left[-\frac{4}{5}, \infty\right)$

f. $(f \circ f)(x)$ $= f(f(x))$

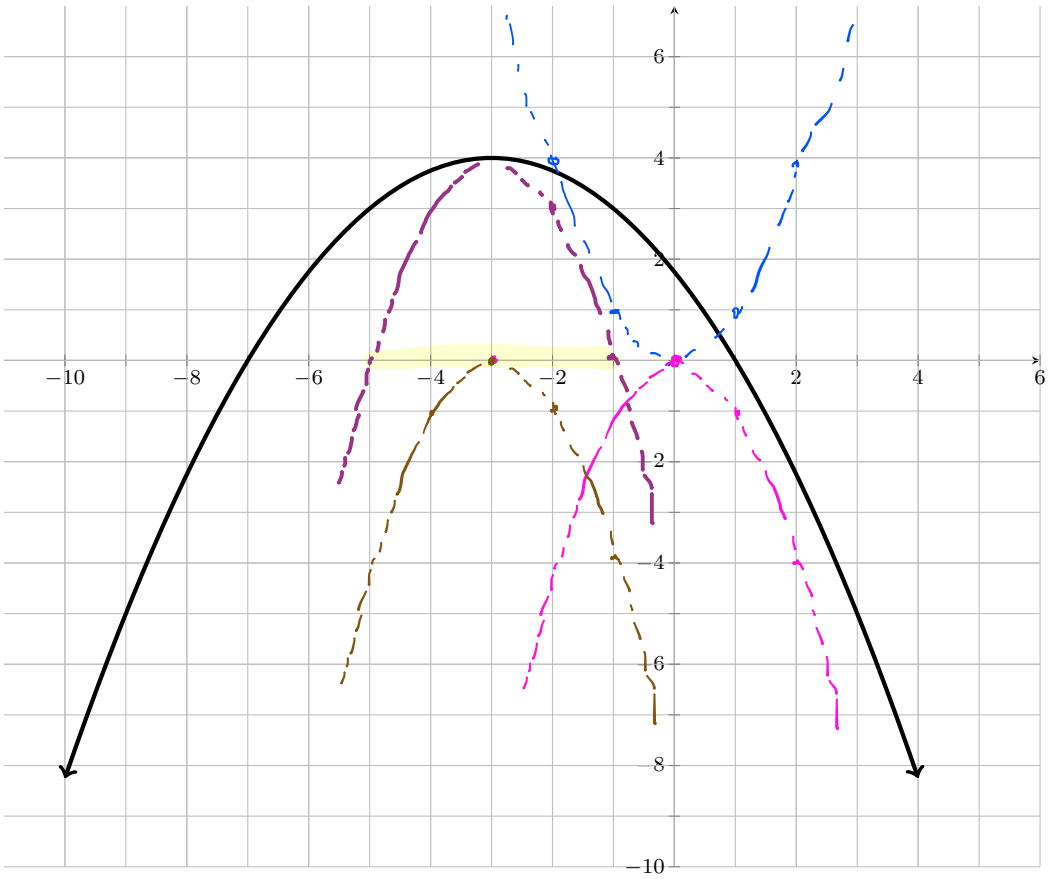
$$\frac{\left(\frac{x-1}{x+2}\right) - 1}{\frac{x-1}{x+2} + 2} = \frac{\frac{x-1-x-2}{x+2}}{\frac{x-1+2x+4}{x+2}} = \frac{-3\,(\cancel{x+2})}{(\cancel{x+2})\,(3x-3)} = \frac{-1}{x-1}$$

$x \neq -2$

$x \neq 1$

$\text{Dom}(f \circ f) = (-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

8. The graph of a function g is given below.



a) Identify the parent function f.

Parabola $\rightsquigarrow$ $f(x) = x^2$ $\circlearrowleft$ blue dot

b) Describe the sequence of transformations from f to g.

① reflect w.r.t. x-axis Pink dot

② left 3 (Horizontal shift) brown dot

③ up 4 units Purple dots

④ Horizontal stretch 2 $\rightsquigarrow$ (Vertical shrink $\frac{1}{4}$)

c) Find the function g.

① $-x^2$

② $-(x+3)^2$

③ $-(x+3)^2+4$

④ $-\left(\frac{x+3}{2}\right)^2+4 = -\frac{1}{4}(x+3)^2+4$

d) Use function notation to write g in terms of f.

$g(x) = -\frac{1}{4} f(x+3) + 4$

9. Consider the function $g(x) = -2|-x+3| - 4$. $= -2\,|x-3| - 4$

a) Identify the parent function f.

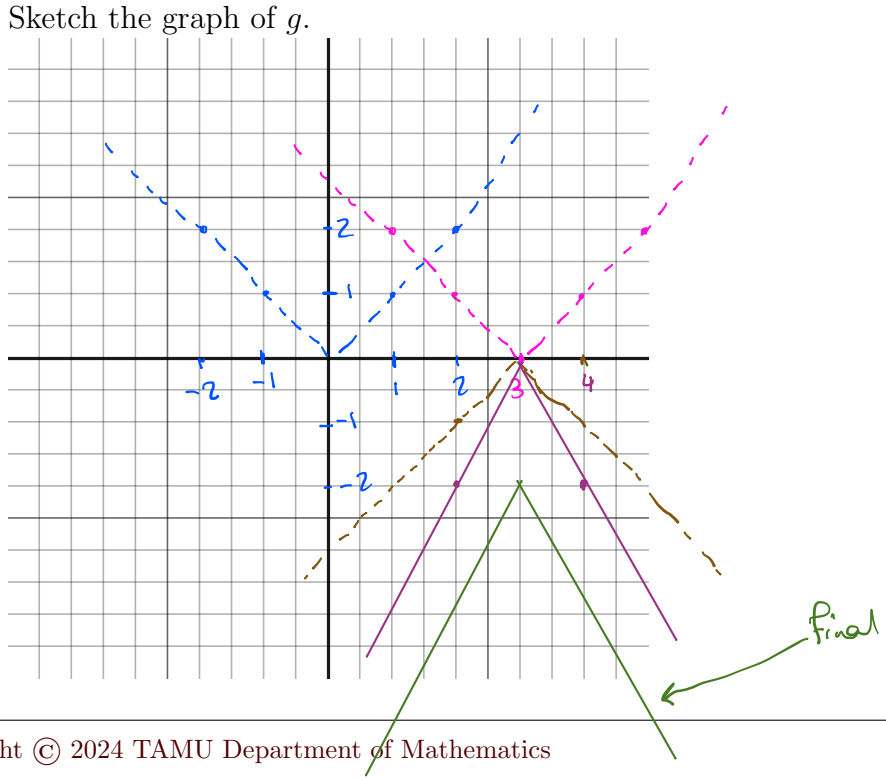
$f(x) = |x|$

b) Describe the sequence of transformations from f to g.

1. $|-x| \rightsquigarrow$ reflect ? $|-x| = |x|$

2. $|x-3| \rightsquigarrow$ right 3 units pink dots
3. $-|x-3| \rightsquigarrow$ reflect w.r.t x-axis brown dots
4. $-2\,|x-3| \rightsquigarrow$ vertical stretch 2 units purple solid
5. $-2\,|x-3| - 4 \rightsquigarrow$ vertical shift (down 4) Green Solid

c) Use function notation to write g in terms of f.

$g(x) = -2\,f(x-3) - 4$

d) Sketch the graph of g.