## Math 251/221

## WEEK in REVIEW 11.

Fall 2024

- 1. Find the local extrema/saddle points for  $f(x, y) = x^3 3x + 3xy^2$
- 2. Find the absolute maximum and minimum values of the function  $f(x, y) = x^2 + 2xy + 3y^2$  over the set D, where D is the closed triangular region with vertices (-1, 1), (2, 1), and (-1, -2).
- 3. Find the gradient vector field of the function  $f(x, y, z) = xy^2 yz^3$ .
- 4. Evaluate the line integral  $\int_C x^3 z ds$  if C is given by  $x = 2 \sin t$ , y = t,  $z = 2 \cos t$ ,  $0 \le t \le \pi/2$ .
- 5. Evaluate  $\int_C y dx + z dy + x dz$  if C consists of the line segments from (0,0,0) to (1,1,2) and from (1,1,2) to (3,1,4).
- 6. Evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F}(x, y) = x^2 y \mathbf{i} + e^y \mathbf{j}$  and C is given by  $\mathbf{r}(t) = t^2 \mathbf{i} t^3 \mathbf{j}, 0 \le t \le 1$ .
- 7. Show that  $\mathbf{F}(x, y) = (2x + y^2 + 3x^2y)\mathbf{i} + (2xy + x^3 + 3y^2)\mathbf{j}$  is conservative vector field. Use this fact to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  if C is the arc of the curve  $y = x \sin x$  from (0,0) to  $(\pi, 0)$ .
- 8. Show that  $\mathbf{F}(x, y, z) = yz(2x+y)\mathbf{i} + xz(x+2y)\mathbf{j} + xy(x+y)\mathbf{k}$  is conservative vector field. Use this fact to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  if C is given by  $\mathbf{r}(t) = (1+t)\mathbf{i} + (1+2t^2)\mathbf{j} + (1+3t^3)\mathbf{k}, \ 0 \le t \le 1$ .
- 9. Given the line integral  $I = \oint_C 4x^2y \, dx (2+x) \, dy$  where C consists of the line segment from (0,0) to (2,-2), the line segment from (2,-2) to (2,4), and the part of the parabola  $y = x^2$  from (2,4) to (0,0). Use Green's theorem to **evaluate** the given integral and **sketch** the curve C indicating the **positive direction**.
- 10. Find curl **F** and div **F** if  $\mathbf{F} = x^2 z \mathbf{i} + 2x \sin y \mathbf{j} + 2z \cos y \mathbf{k}$
- 11. Find the area of the surface with parametric equations  $x = u^2$ , y = uv,  $z = \frac{1}{2}v^2$ ,  $0 \le u \le 1$ ,  $0 \le v \le 2$ .
- 12. Find the area of the part of the paraboloid  $z = x^2 + y^2$  that lies inside the cylinder  $x^2 + y^2 = 4$ .
- 13. Find the mass of a thin funnel in the shape of a cone  $z = \sqrt{x^2 + y^2}$ ,  $1 \le z \le 4$  if its density function is  $\rho(x, y, z) = 10 z$ .
- 14. Evaluate  $\iint_S xy \, dS$  if S is the boundary of the region enclosed by the cylinder  $x^2 + z^2 = 1$  and the planes y = 0 and x + y = 2.
- 15. Evaluate  $\iint_S yz \, dS$  if S is the surface given by  $\mathbf{r}(u, v) = \langle uv, u + v, u v \rangle, u^2 + v^2 \le 1.$
- 16. Evaluate  $\iint_{S} \mathbf{F} \cdot d\mathbf{S}$ , if
  - (a)  $\mathbf{F}(x, y, z) = \langle x^2y, -3xy^2, 4y^3 \rangle$  and S is the part of the elliptic paraboloid  $z = x^2 + y^2 9$  that lies below the rectangle  $0 \le x \le 2, 0 \le y \le 1$  and has downward orientation.
  - (b)  $\mathbf{F}(x, y, z) = \langle x, y, 5 \rangle$  and S is the boundary of the region enclosed by the cylinder  $x^2 + z^2 = 1$  and the planes y = 0 and x + y = 2.
- 17. Use Stokes' Theorem to evaluate  $\oint_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F}(x, y, z) = \langle 3z, 5x, -2y \rangle$  and C is the ellipse in which the plane z = y + 3 intersects the cylinder  $x^2 + y^2 = 4$ , with positive orientation as viewed from above.

- 18. Use Stokes' Theorem to evaluate  $\iint_{S} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$  for the vector field  $\mathbf{F}(x, y, z) = \langle ze^y, x \cos y, xz \sin y \rangle$  and the hemisphere  $y = \sqrt{4 x^2 z^2}$  oriented in the direction of the positive y-axis.
- 19. Use the Divergence Theorem to find the flux of the vector field  $\mathbf{F} = \langle x, y, 1 \rangle$  across the surface S which is the boundary of the region enclosed by the cylinder  $y^2 + z^2 = 1$  and the planes x = 0 and x + y = 5.